Chapter

8

Statistical Tolerancing

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8.1 Introduction

Statistical tolerancing is an alternative to worst-case tolerancing. In worst-case tolerancing, the designer aims for 100% interchangeability of parts in an assembly. In statistical tolerancing, the designer abandons this lofty goal and accepts at the outset some small percentage of failures of the assembly.

Statistical tolerancing is used to specify a population of parts as opposed to specifying a single part. Statistical tolerances are usually, but not always, specified on parts that are components of an assembly. By specifying part tolerances statistically the designer can take advantage of cancellation of geometrical errors in the component parts of an assembly — a luxury he does not enjoy in worst-case tolerancing. This results in economic production of parts, which then explains why statistical tolerancing is popular in industry that relies on mass production.

In addition to gain in economy, statistical tolerancing is important for an integrated approach to statistical quality control. It is the first of three major steps - specification, production, and inspection - in any quality control process. While national and international standards exist for the use of statistical methods in production and inspection, none exists for product specification. For example, ASME Y14.5M-1994 focuses mainly on the worst-case tolerancing. By using statistical tolerancing, an integrated statistical approach to specification, production, and inspection can be realized.

Since 1995, ISO (International Organization for Standardization) has been working on developing standards for statistical tolerancing of mechanical parts. Several leading industrial nations, including the US, Japan, and Germany are actively participating in this work which is still in progress. This chapter explains what ISO has accomplished thus far toward standardizing statistical tolerancing. The reader is cautioned that everything reported in this chapter is subject to modification, review, and voting by ISO, and should not be taken as the final standard on statistical tolerancing.

8.2 Specification of Statistical Tolerancing

Statistical tolerancing is a language that has syntax (a symbol structure with rules of usage) and semantics (explanation of what the symbol structure means). This section describes the syntax and semantics of statistical tolerancing.

Statistical tolerancing is specified as an extension to the current geometrical dimensioning and tolerancing (GD&T) language. This extension consists of a statistical tolerance symbol and a statistical tolerance frame, as described in the next two paragraphs. Any geometrical characteristic or condition (such as size, distance, radius, angle, form, location, orientation, or runout, including MMC, LMC, and envelope requirement) of a feature may be statistically toleranced. This is accomplished by assigning an actual value to a chosen geometrical characteristic in each part of a population. Actual values are defined in ASME Y14.5.1M-1994. (See Chapter 7 for details about the Y14.5.1M-1994 standard that provides mathematical definitions of dimensioning and tolerancing principles.) Some experts think that statistically toleranced features should be produced by a manufacturing process that is in a state of statistical control for the statistically toleranced geometrical characteristic; this issue is still being debated.

The statistical tolerance symbol first appeared in ASME Y14.5M-1994. It consists of the letters ST enclosed within a hexagonal frame as shown, for example, in Fig. 8-1. For size, distance, radius, and angle characteristics the ST symbol is placed after the tolerances specified according to ASME Y14.5M-1994 or ISO 129. For geometrical tolerances (such as form, location, orientation, and runout) the ST symbol is placed after the geometrical tolerance frame specified according to ASME Y14.5M-1994 or ISO 1101. See Figs. 8-2 and 8-3 for further examples.

The statistical tolerance frame is a rectangular frame, which is divided into one or more compartments. It is placed after the ST symbol as shown in Figs. 8-1, 8-2, and 8-3. Statistical tolerance requirements can be indicated in the ST frame in one of the three ways defined in sections 8.2.1, 8.2.2, and 8.2.3.

8.2.1 Using Process Capability Indices

Three sets of process capability indices are defined as follows.

- Cp $= \dfrac{U-L}{6s}$,
- Cpk = min(Cpl,Cpu), where Cpl = $\dfrac{m-L}{3s}$ and Cpu = $\dfrac{U-m}{3s}$, and
- Cc = max(Ccl,Ccu) where Ccl = $\dfrac{t-m}{t-L}$ and Ccu = $\dfrac{m-t}{U-t}$.

In these definitions L is the lower specification limit, U is the upper specification limit, t is the target value, m is the population mean, and σ is the population standard deviation.

The process capability indices are nondimensional parameters involving the mean and the standard deviation of the population. The nondimensionality is achieved using the upper and lower specification limits. Cp is a measure of the spread of the population about the average. Cc is a measure of the location of the average of the population from the target value. Cpk is a measure of both the location and the spread of the population.

All of these five indices need not be used at the same time. Numerical lower limits for Cp, Cpk (or Cpu, Cpl) and numerical upper limit for Cc (or Ccu, Ccl) are indicated as shown in Fig. 8-1 using the $\geq$ and $\leq$ symbols. Cpu and Ccu are used instead of Cpk and Cc, respectively, for all geometrical tolerances (form, location, orientation, and runout) specified at RFS (Regardless of Feature Size). The requirement here is that the mean and the standard deviation of the population of actual values should be such that all the specified indices are within the indicated limits.

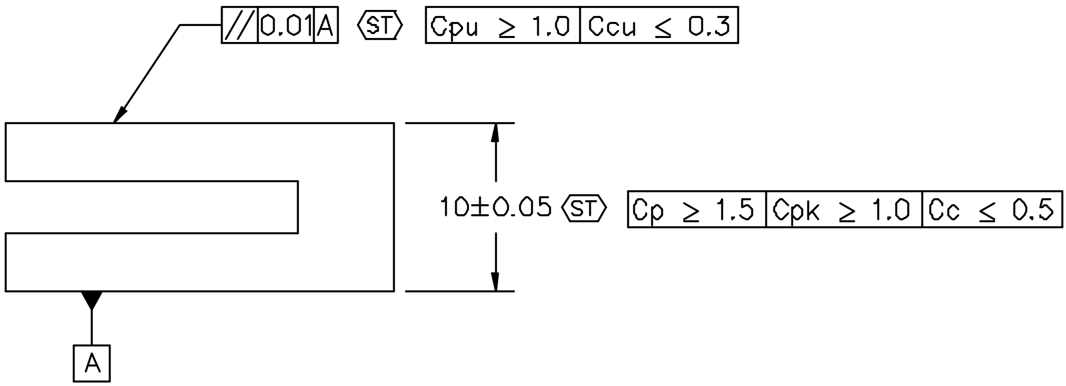

Figure 8-1 Statistical tolerancing using process capability indices

For the example illustrated in Fig. 8-1, the population of actual values for the specified size should have its Cp value at or above 1.5, Cpk value at or above 1.0, and Cc value at or below 0.5. For the indicated parallelism, the population of out-of-parallelism values (that is, the actual values for parallelism) should have its Cpu value at or above 1.0, and its Ccu value at or below 0.3.

Limits on the process capability indices also imply limits on the mean and the standard deviation of the population of actual values through the formulas shown at the beginning of this section. Such limits on m and s can be visualized as zones in the m-s plane, as described in section 8.3.1. To derive the limits on m and s, values of L, U, and t should be obtained from the specification. For the example illustrated in Fig. 8-1, consider the size first. From the size specification, the lower specification limit $L = 9.95$, the upper specification limit $U = 10.05$, and the target value $t = 10.00$ because it is the midpoint of the allowable size variation. Next consider the specified parallelism, from which it can be inferred that $L = 0.00$, $U = 0.01$, and $t = 0.00$ because zero is the intended target value.

Using Cpl, Cpu, or Cpk in the ST tolerance frame implies only that these values should be within the limits indicated. Caution must be exercised in any further interpretation, such as the fraction of population lying outside the L and/or U limits, because it requires further assumption about the type of distribution, such as normality, of the population. Note that such additional assumptions are not part of the specification, and their invocation, if any, should be separately justified.

Process capability indices are used quite extensively in industrial production, both in the US and abroad, to quantify manufacturing process capability and process potential. Their use in product specification may seem to be in conflict with the time-honored “process independence” principle of the ASME Y14.5. This apparent conflict is false; the process capability indices do not dictate what manufacturing process should be used — they place demand only on some statistical characteristics of whatever process that is chosen.

Issues raised in the last two paragraphs have led to some rethinking of the use of the phrase “process capability indices” in statistical tolerancing. We will come back to this point in section 8.5, after the introduction of a powerful concept called population parameter zones in section 8.3.1.

8.2.2 Using RMS Deviation Index

RMS (root-mean-square) deviation index is defined as $Cpm = \dfrac{U-L}{6\sqrt{s^2+(m-t)^2}}$. A numerical lower limit for Cpm is indicated as shown in Fig. 8-2 using the $\geq$ symbol. The requirement here is that the mean and standard deviation of the population of actual values should be such that the Cpm index is within the specified limit.

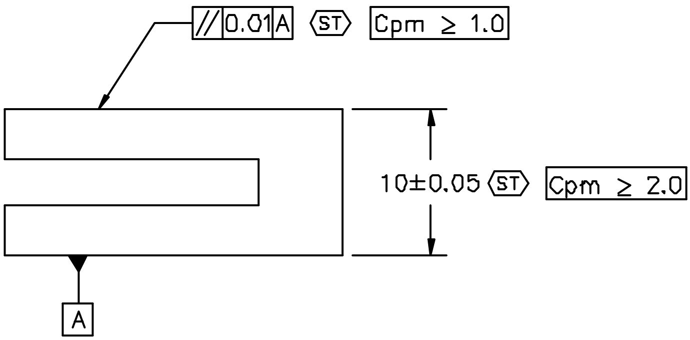

Figure 8-2 Statistical tolerancing using RMS deviation index

For the example illustrated in Fig. 8-2, the population of actual values for the size should have a Cpm value that is greater than or equal to 2.0. For the specified parallelism, the population of out-of-parallelism values (that is, the actual values for parallelism) should have a Cpm value that is greater than or equal to 1.0.

Cpm is called the RMS deviation index because $\sqrt{s^2+(m-t)^2}$ is the square root of the mean of the square of the deviation of actual values from the target value t. Limiting Cpm also limits the mean and the standard deviation, and this can be visualized as a zone in the μ–σ plane. Section 8.3.1 describes such zones. To derive the limits on μ and σ, values for *L, U,* and *t* should be obtained from the specification of Fig. 8-2 as explained in section 8.2.1.

Cpm is closely related to Taguchi’s quadratic cost function, which states that the total cost to society of producing a part whose actual value deviates from a specified target value increases quadratically with the deviation. Specifying an upper limit for Cpm is equivalent to specifying an upper limit to the average cost of parts according to the quadratic cost function. This methodology is popular in some Japanese industries.

8.2.3 Using Percent Containment

A tolerance interval or upper limit followed by the P symbol and a numerical value of the percent ending with a % symbol is indicated as shown in Fig. 8-3. The tolerance range indicated inside the ST frame should be smaller than the tolerance range indicated outside the ST frame before the ST symbol. The requirement here is that the entire population of actual values should be contained within the limits indicated before the ST symbol; the percentage following the P symbol inside the ST frame indicates the minimum percentage of the population of actual values that should be contained within the limits indicated within the ST frame before the ST symbol; the remaining population should be contained in the remaining tolerance range proportionately.

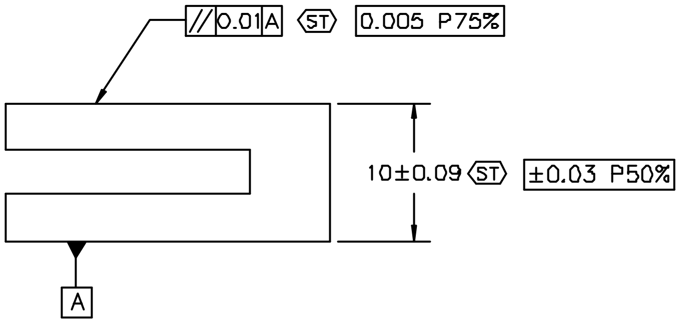

Figure 8-3 Statistical tolerancing using percent containment

In the example illustrated in Fig. 8-3 for the specified size, the entire population should be contained within 10 ± 0.09; at least 50% of the population should be contained within 10 ± 0.03; no more than 25% should be contained within $10^{-0.03}_{-0.09}$ and no more than 25% should be contained within $10^{+0.09}_{+0.03}$. For the specified parallelism, the entire population of out-of-parallelism values (that is, the actual values for the parallelism) should be less than 0.01 and at least 75% of this population of values should be less than 0.005.

Percent containment statements are best visualized using distribution functions. A distribution function, denoted $\Pr[X \leq x]$, is the probability that the random variable X is less than or equal to a value x. Distribution functions are also known as cumulative distribution functions in some engineering literature. A distribution function is a nondecreasing function of x, and it varies between 0 and 1. It is possible to visually represent the percent containment requirements as zones that contain acceptable distribution functions, as shown in section 8.3.2.

Using percent containment is popular in some German industries. It is a simple but powerful way to indicate directly the percentage of populations that should lie within certain intervals.

8.3 Statistical Tolerance Zones

Statistical tolerance zone is a useful tool to visualize what is being specified and to compare different types of specifications. It is also a powerful concept that unifies several seemingly disparate practices of statistical tolerancing in industry today. A statistical tolerance zone can be either a population parameter zone (PPZ) or distribution function zone (DFZ). PPZs are based on parametric statistics, and DFZs are based on nonparametric statistics.

8.3.1 Population Parameter Zones

A PPZ is a region in the mean - standard deviation plane, as shown in Fig. 8-4. In this example, the shaded PPZ on the left is the zone that corresponds to the statistical specification of size in Fig. 8-1, and the shaded PPZ on the right is the zone that corresponds to the statistical specification of parallelism in Fig. 8-1. Vertical lines that limit the PPZ arise from limits on Cc, Ccu or Ccl because they limit only the mean; the top horizontal line comes from limiting Cp because it limits only the standard deviation; the slanted lines are due to limits on Cpk, Cpu or Cpl because they limit both the mean and the standard deviation. If the (m,s) point for a given population of geometrical characteristics lies within the PPZ, then the population is acceptable; otherwise it is rejected.

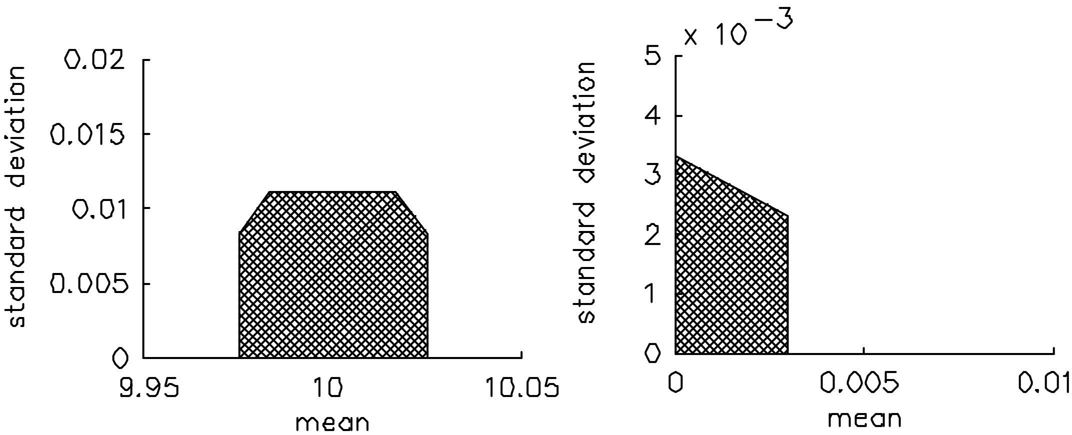

Figure 8-4 Population parameter zones for the specifications in Fig. 8.1

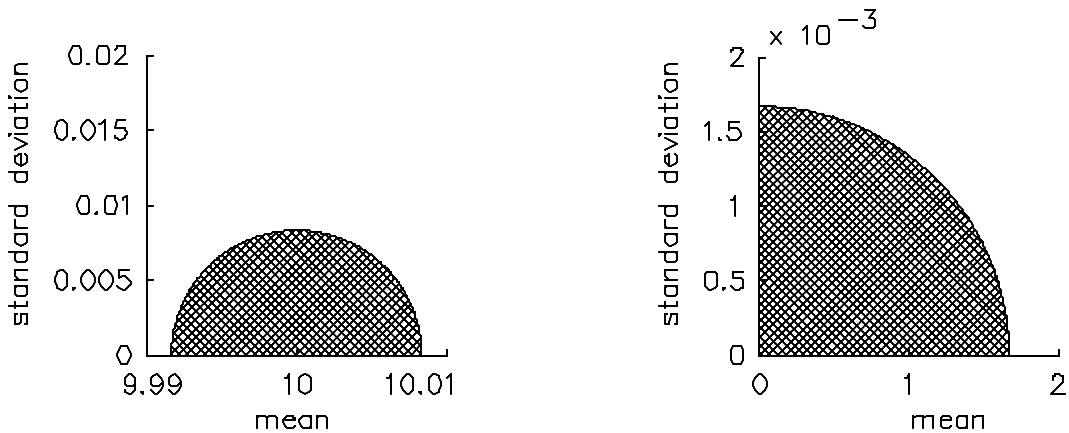

Figure 8-5 Population parameter zones for the specifications in Fig. 8.2

PPZs can be defined for specifications that use the RMS deviation index as well. Fig. 8-5 illustrates the PPZs for the specifications in Fig. 8-2. Here the zones are bounded by circular arcs. Again, the interpretation is that all (m,s) points that lie inside the zone correspond to acceptable populations, and points that lie outside the zone correspond to populations that are not acceptable per specification.

8.3.2 Distribution Function Zones

A DFZ is a region that lies between an upper and a lower distribution function, as shown in Fig. 8-6. Any population whose distribution function lies within the shaded zone is acceptable; if not, it is rejected.

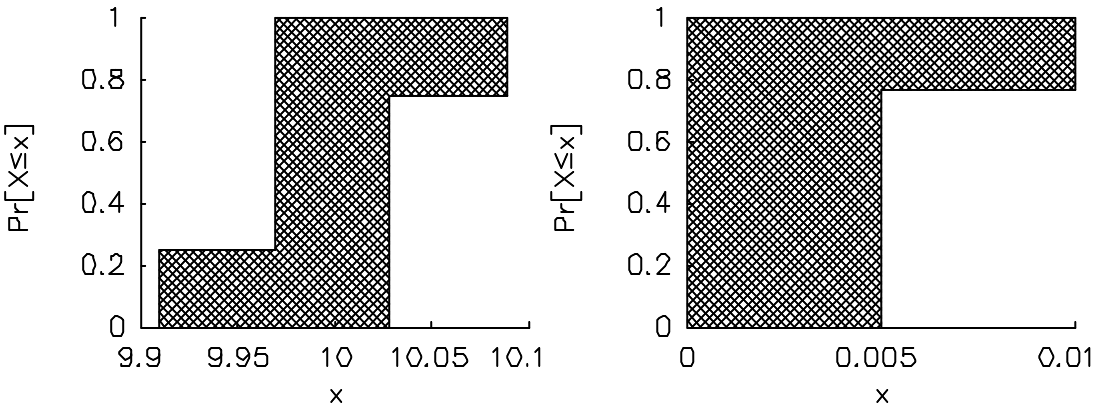

Figure 8-6 Population parameter zones for the specifications in Fig. 8.3

8.4 Additional Illustrations

Figs. 8-7 through 8-10 illustrate valid uses of statistical tolerancing in several examples. Though not exhaustive, these illustrations help in understanding valid specifications of statistical tolerancing.

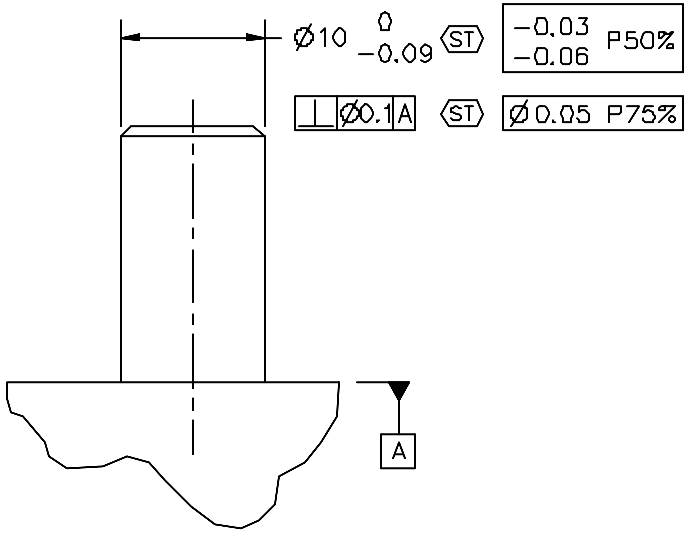

Figure 8-7 Additional illustration of specifying percent containment

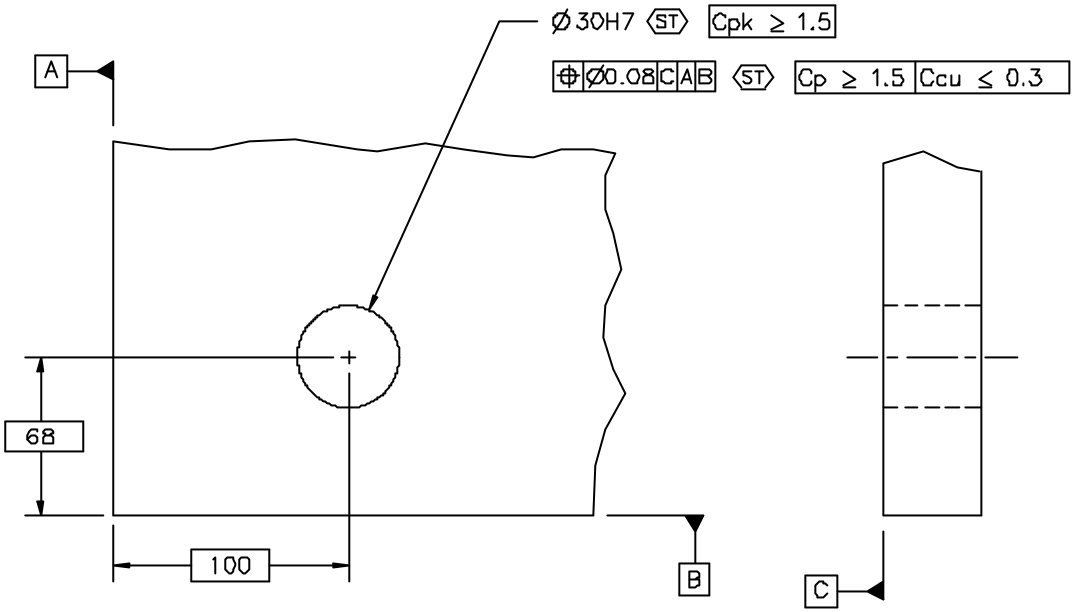

Figure 8-8 Illustration specifying process capability indices

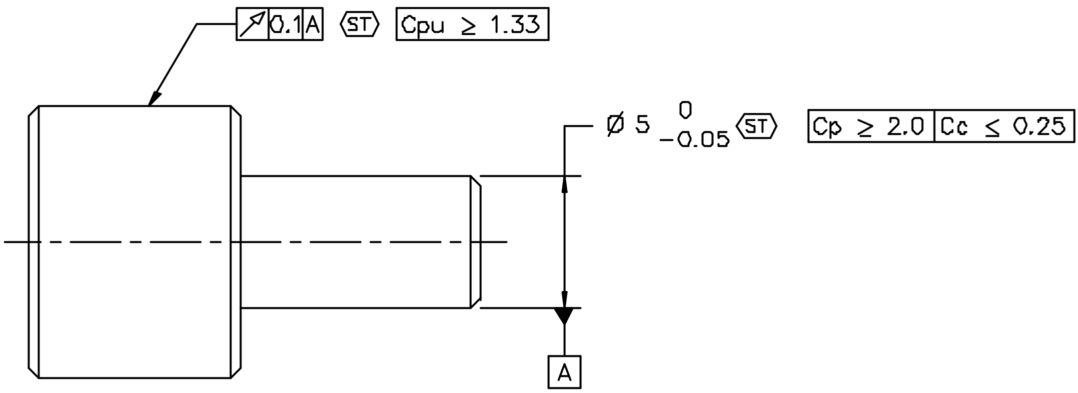

Figure 8-9 Additional illustration specifying process capability indices

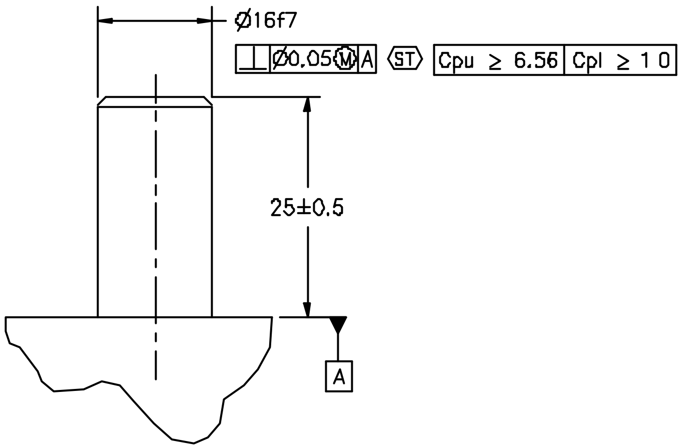

Figure 8-10 Illustration of statistical tolerancing under MMC

8.5 Summary and Concluding Remarks

This chapter dealt with the language of statistical tolerancing of mechanical parts. Statistical tolerancing is applicable when parts are produced in large quantities and assumptions about statistical composition of part deviations while assembling products can be justified. The economic case for statistical tolerancing can indeed be very compelling. In this chapter, three ways of indicating statistical tolerancing were described, and the associated statistical tolerance zones were illustrated. Population parameter zone (PPZ) and distribution function zone (DFZ) are the two most relevant new concepts that are driving the design of the ISO statistical tolerancing language.

Statistical tolerancing is deliberately designed as an extension to the current GD&T language. This has some disadvantages. It might be, for example, a better idea to indicate the statistical tolerance zones directly in the specifications. However, acceptance of statistical tolerancing by industry is greatly enhanced if it is designed as an extension to an existing popular language.

It was indicated earlier that some believe that statistically controlled parts should be produced by a manufacturing process that is in a state of statistical control. Strictly speaking, this is not a necessary condition for the success of statistical tolerancing. However, it is a good practice to insist on a state of statistical control, which can be achieved by the use of statistical process control methodologies for the manufacturing process. This is particularly true if a company has implemented just-in-time delivery, a practice in which one may not have the luxury of drawing a part at random from an existing bin full of parts. As mentioned in the body of this chapter, this issue is still being debated within ISO.

Similarly, there is a vigorous debate within ISO on the use of the phrase “process capability indices” indicated symbolically by Cp, Cpl, Cpu, Cpk, Ccl, Ccu, Cc, and Cpm. This debate is fueled by a current lack of ISO standardized interpretation of the meaning of these indices. To circumvent this controversy, these symbols may be replaced by Fp, Fpl, Fpu, Fpk, Fcl, Fcu, Fc, and Fpm, respectively, but without changing their functional relationship to $L, U, m, s,$ and t. The intent is to preserve the powerful notion of population parameter zones, which is an important concept for statistical tolerancing, while avoiding the use of the nonstandard phrase “process capability indices.” This move may also open up the syntax to accept any user-defined function of population parameters.

A typical design problem is a tolerance allocation (also known as tolerance synthesis) problem. Here, given a tolerable variation in an assembly-level characteristic, the designer decides what are the tolerable variations in part-level geometrical characteristics. In general, this is a difficult problem. A more tractable problem is that of tolerance analysis, wherein given part-level geometrical variations the designer predicts what is the variation in an assembly-level characteristic. These are the types of problems that a designer faces in industry everyday. Both analytical and numerical (e.g., Monte-Carlo simulations) methods have been developed to solve the statistical tolerance analysis problem. Discussion of statistical tolerance analysis or synthesis is, however, beyond the scope of this chapter.

Acknowledgment and a Disclaimer

The author would like to express his deep gratitude to numerous colleagues who participated, and continue to participate, in the ASME and ISO standardization efforts. Standardization is a truly community affair, and he has merely reported their collective effort. Although the work described in this chapter draws heavily from the ongoing ISO efforts in standardization of statistical tolerancing, opinions expressed here are his own and not that of ISO or any of its member bodies.

8.6 References

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