CHAPTER 9

SOLUTION BY THE METHOD OF CHARACTERISTICS

The history of water hammer analysis is marked by various clever and practical techniques for solving the Euler and conservation of mass equations derived in Chapters 7 and 8. Those methods were a reflection of the level of sophistication of the numerical analysis capabilities of their time as well as the ingenuity of the practitioners. In recent years the availability of low-cost, high-performance desktop computers has led to the creation of solution methods for these equations which are numerically very accurate and are capable of incorporating a wide range of boundary and initial conditions.

At this time the most general and widely-used technique for solving these equations is the *method of characteristics*. It is no coincidence that this method is very compatible with numerical solution by digital computer. For this reason we consider only the method of characteristics approach to problem solving in this and following chapters.

9.1 METHOD OF CHARACTERISTICS, APPROXIMATE GOVERNING EQUATIONS

9.1.1. DEVELOPMENT OF THE CHARACTERISTIC EQUATIONS

Anticipating that many engineers are even today unfamiliar with the method of characteristics as a solution technique, we first introduce the method using approximate versions of Eqs. 8.57 and 8.58. These approximate equations are obtained by neglecting the spatial variation of V and p whenever both space- and time-varying terms appear in the same equation. We do this because, in general, the spatial variations are much less significant in determining the solution behavior than are the time-varying terms.

Following this approach, Eq. 8.57 becomes

$$\frac{\partial V}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{f}{2D}V|V| = 0 \tag{9.1}$$

and Eq. 8.58 becomes

$$a^2\frac{\partial V}{\partial s} + \frac{1}{\rho}\frac{\partial p}{\partial t} = 0 \tag{9.2}$$

The essence of the method of characteristics is the successful replacement of a pair of *partial* differential equations by an equivalent set of *ordinary* differential equations. The development of the method begins by presuming that the pair of Eqs. 9.1 and 9.2 may be replaced by some linear combination of themselves. Using l as a constant linear scale factor, sometimes called a Lagrange multiplier, one possible combination is

$$\lambda\left(\frac{\partial V}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{f}{2D}V|V|\right) + \left(a^2\frac{\partial V}{\partial s} + \frac{1}{\rho}\frac{\partial p}{\partial t}\right) = 0 \tag{9.3}$$

Regrouping terms,

$$\left(\lambda\frac{\partial V}{\partial t}+a^2\frac{\partial V}{\partial s}\right)+\left(\frac{1}{\rho}\frac{\partial p}{\partial t}+\frac{\lambda}{\rho}\frac{\partial p}{\partial s}\right)+\lambda g\frac{dz}{ds}+\frac{\lambda f}{2D}V|V|=0 \tag{9.4}$$

We note that if $\lambda\frac{\partial V}{\partial t}+a^2\frac{\partial V}{\partial s}$ is to be replaced by $\lambda\frac{dV}{dt}$, then $\lambda\frac{ds}{dt}=a^2$. Further, if $\frac{1}{\rho}\frac{\partial p}{\partial t}+\frac{\lambda}{\rho}\frac{\partial p}{\partial s}$ is to be replaced by $\frac{1}{\rho}\frac{dp}{dt}$, then $\frac{\lambda}{\rho}=\frac{1}{\rho}\frac{ds}{dt}$. To satisfy these two requirements for $\frac{ds}{dt}$, we discover that $\lambda^2=a^2$ or

$$\lambda = \pm\, a \tag{9.5}$$

The scale factor l is linear and constant, as required, so long as a is constant, and we have succeeded in combining Eqs. 9.1 and 9.2. We first rewrite Eq. 9.3 with $l = +a$ as a replacement for the first equation. Then we rewrite Eq. 9.3 with $l = -a$ as a replacement for the second equation. Upon dividing the resulting equations by the wave speed a, we have a pair of ordinary differential equations rather than partial differential equations:

$$\frac{dV}{dt}+\frac{1}{a\rho}\frac{dp}{dt}+g\frac{dz}{ds}+\frac{f}{2D}V|V|=0 \tag{9.6}$$

$$\frac{dV}{dt}-\frac{1}{a\rho}\frac{dp}{dt}+g\frac{dz}{ds}+\frac{f}{2D}V|V|=0 \tag{9.7}$$

However, there are now special restrictive conditions on the independent variables in each equation. Equation 9.6 is subject to the requirement that $\lambda\frac{ds}{dt}=a^2$, so $\frac{ds}{dt}=\frac{a^2}{\lambda}=+a$. Therefore, Eq. 9.6 is valid only when $\frac{ds}{dt}=+a$. Similarly, Eq. 9.7 is valid only when $\frac{ds}{dt}=-a$. Thus we have replaced two partial differential equations by two pairs of ordinary differential equations, and we must follow these rules which relate the independent variables s and t. As we believe it is easier to visualize the propagation of pressure waves in terms of the piezometric head $p = g(H - z)$ the height of the EL-HGL above a horizontal datum (commonly sea level), we convert from p to H.

The new form of Eqs. 9.6 and 9.7 is now

$$\frac{dV}{dt}+\frac{g}{a}\frac{dH}{dt}+\frac{f}{2D}V|V|=0 \quad \text{only when} \quad \frac{ds}{dt}=+a \tag{9.8}$$

$$\frac{dV}{dt}-\frac{g}{a}\frac{dH}{dt}+\frac{f}{2D}V|V|=0 \quad \text{only when} \quad \frac{ds}{dt}=-a \tag{9.9}$$

From the fact that special relations must be maintained between s and t in Eqs. 9.8 and 9.9, the equations $\frac{ds}{dt}=+a$ and $\frac{ds}{dt}=-a$ have come to be called the *characteristics* of Eqs. 9.8 and 9.9, hence the name of the analysis procedure.

To see how we use these characteristic equations in a solution, we work with a graph having s as the abscissa and t as the ordinate, referred to as the s-t plane. Figure 9.1 shows how the s-t plane is related to the physical problem. Here the s-coordinate is the

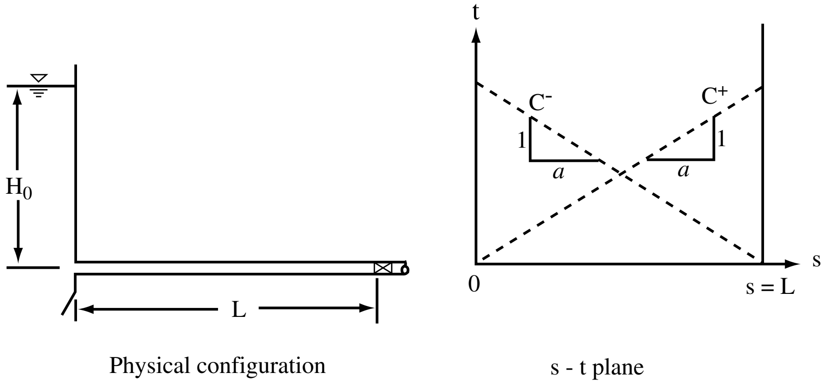


Figure 9.1 The s-t plane for the simple flow of Chapter 7.

distance along the pipe from the upstream end. With a as a constant, the characteristic equation for Eq. 9.8, $\frac{ds}{dt} = +a$, can easily be integrated (after inverting to get into proper form) to yield $t = s/a + \text{constant}$. This equation describes a family of straight lines of slope $1/a$ on the s-t plane; the position of any one line depends on the constant of integration. Because these lines are associated with the characteristic equation having the + sign for a, they are referred to as C^+ characteristics. Figure 9.1 depicts a C^+ characteristic passing through the origin. Similarly, the characteristic equation for Eq. 9.9 describes a family of straight lines on the s-t plane with a slope $-1/a$. The characteristics of Eq. 9.9 are referred to as C^- characteristics; one is plotted in Fig. 9.1 passing through the point $s = L$.

Let us revisit the simple water hammer illustration of Chapter 7 to understand further the concept of characteristics. In that example a valve at the downstream end of a pipeline was suddenly closed, causing a pressure wave to propagate upstream at speed a. Friction was neglected. For this situation Eqs. 9.8 and 9.9, with negligible friction, can be written

$$\frac{dV}{dt} \pm \frac{g}{a}\frac{dH}{dt} = 0 \tag{9.10}$$

Multiplying by dt and rearranging,

$$dV = \pm\frac{g}{a}dH \qquad \text{or} \qquad dH = \pm\frac{a}{g}dV \tag{9.11}$$

This equation has the same form (replacing dV and dH with ΔV and ΔH) as the equation for pressure head increment that was derived earlier as Eq. 8.8. Tracing the C^- characteristic which has a slope of $1/a$ from right to left, we note from Fig. 9.1 that the wave reaches the origin (upstream end of the pipe) at $t = L/a$. Both of these results validate the simple water hammer analysis of Chapter 7 and show that pressure waves propagate along the characteristic lines. As we will see later, this important physical fact is crucial in obtaining reliable results from the numerical analysis.

With a physical grasp of Eqs. 9.8 and 9.9 now in hand, we will proceed to formalize the solution process more carefully. To compute values of H and V at various locations along the pipe as functions of time, we must begin with a knowledge of *initial* conditions along the s-axis of the s-t plane and *boundary* conditions for all time at the pipe ends $s = 0$ and $s = L$. Then a solution for values of H and V can "march" forward (upward) in the s-t plane.

To see how this marching process works, refer to Fig. 9.2, which is an s-t plane for some as yet undefined problem. At any point on the s-t plane, say point P, the values

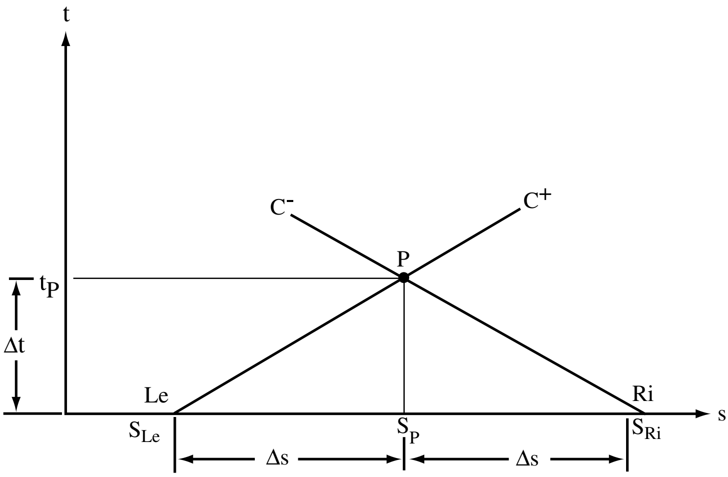


Figure 9.2 The s-t plane showing characteristics for Eqs. 9.8 and 9.9.

of the continuous variables H and V are unique (i.e., the H and V values are independent of the characteristic with which they are associated). We next draw the C^+ and C^- characteristic lines through point P and extend them to intersect the s-axis at points on the left and right sides of P, here called Le and Ri, respectively. Note that these two points are in this approximate case each the same distance Δs from P. Equation 9.8 applies along the C^+ characteristic, and Eq. 9.9 applies along the C^- characteristic. The information which determines H and V will propagate forward in time along these two characteristics from Le and Ri.

9.1.2. THE FINITE DIFFERENCE REPRESENTATION

In seeking a numerical solution to our problem, we write Eqs. 9.8 and 9.9 in finite difference form. Equation 9.8 becomes

$$\frac{V_P - V_{Le}}{t_P - 0} + \frac{g}{a}\frac{H_P - H_{Le}}{t_P - 0} + \frac{f}{2D}V_{Le}\left|V_{Le}\right| = 0 \tag{9.12}$$

and Eq. 9.9 becomes

$$\frac{V_P - V_{Ri}}{t_P - 0} - \frac{g}{a}\frac{H_P - H_{Ri}}{t_P - 0} + \frac{f}{2D}V_{Ri}\left|V_{Ri}\right| = 0 \tag{9.13}$$

We have made two significant assumptions in developing these equations. First we assume that the velocity at the beginning of the time interval, rather than an average velocity over the interval, adequately represents the frictional effect. The computational implications are significant. If we were to include the unknown value V_P in the friction term, the difference equations would become nonlinear and require an iterative solution. In view of the tremendous number of times that we will solve these equations and the sometimes troublesome nature of nonlinear solution techniques, we choose not to employ that approach. With the generally small time increments in the solution of transient problems, we intuitively expect that this simplification will not cause significant inaccuracies in our results.

We also assume that the steady-state friction coefficient can adequately represent friction losses in a transient flow. The assumption of a non-transient constant friction coefficient in transient analyses has always been an approximation. The use of the steady-state Darcy-Weisbach f implies that the flow in the pipe is behaving as a wholly rough flow. That is, the f which would normally be changing with Reynolds number as the transient velocity changes, is kept constant. Use of the Hazen-Williams C partially compensates for this problem by letting an equivalent "f" adjust somewhat with the transient velocity change. However, neither method is based on the fundamental behavior of transient flow.

As the velocity changes relatively rapidly, even reversing direction, the velocity profile becomes quite complex. The calculation of the shear stress and energy dissipation is difficult. Silva-Araya and Chaudhry (1997) provide a state-of-the-art assessment of this problem; they retain the friction coefficient (the Darcy-Weisbach f) but multiply it by an energy dissipation factor to account for the additional friction loss in a transient flow. The method employs a dissipation function for axisymmetric flow which includes the effects of both viscous and turbulent stresses. While this approach shows promise, the extra computational effort currently appears excessive for practical use. Consequently we continue to use the traditional approach of employing steady-state friction coefficients.

We now replace $t_P - 0$ with Δt in the above equations so the analysis will apply to more than the first time interval. Multiplying these equations by Δt gives

$$C^+ : \ \left(V_P - V_{Le}\right) + \frac{g}{a}\left(H_P - H_{Le}\right) + \frac{f\Delta t}{2D} V_{Le}\left|V_{Le}\right| = 0 \tag{9.14}$$

and

$$C^- : \ \left(V_P - V_{Ri}\right) - \frac{g}{a}\left(H_P - H_{Ri}\right) + \frac{f\Delta t}{2D} V_{Ri}\left|V_{Ri}\right| = 0 \tag{9.15}$$

These equations will be referred to as the C^+ and C^- equations, respectively.

The characteristic equations can also be written in finite difference form as

$$\Delta s = \pm\ a\,\Delta t \tag{9.16}$$

Now, proceeding with the finite difference numerical solution, we must select a spatial interval in the s-direction, i.e., the number of sections into which the pipe will be divided. If we decide to divide the pipe into N sections, then each section will be of length $\Delta s = L/N$. This decision fixes Δs, and Eq. 9.16 is then used to compute Δt. We can now construct the *grid of characteristics* shown in Fig. 9.3 atop the next page.

Grid points along the s-axis represent points spaced Δs apart along the pipe axis, and the values of H and V at these points are *initial conditions*. Usually these initial conditions are a set of values of H and V which describe a steady flow in the pipeline at the moment a transient begins. With the known values from points Le and Ri we can now solve Eqs. 9.14 and 9.15 simultaneously to obtain the values H_P and V_P at points 2 through N at time $t = \Delta t$. The boundary conditions at $s = 0$ and $s = L$ must be used in conjunction with the appropriate C^+ or C^- equation to compute the values of H_{P_1} and $H_{P_{N+1}}$. This completes the solution for all the values of H and V at time $t = \Delta t$. We next compute the values of H and V at time $t = 2\Delta t$ using the just-computed values at $t = \Delta t$ as the known values in Eqs. 9.14 and 9.15. This process is repeated continuously as we march ahead in the s-t plane.

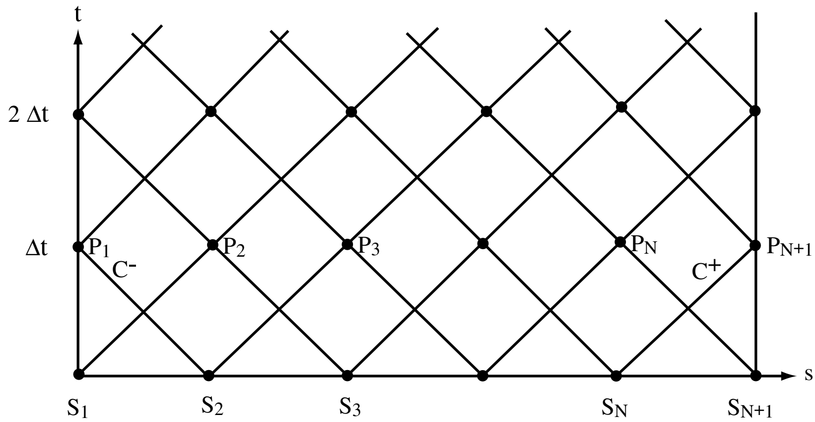


Figure 9.3 The characteristic grid for a single pipe.

Finally, we should emphasize an important conceptual point arising from our analysis. Any change in the velocity or head at a point in the pipeline cannot be sensed at another point in the pipeline until the pressure wave has had time to propagate at the wave speed to that section. This effect is illustrated in Fig. 9.4 showing where and when a disturbance at

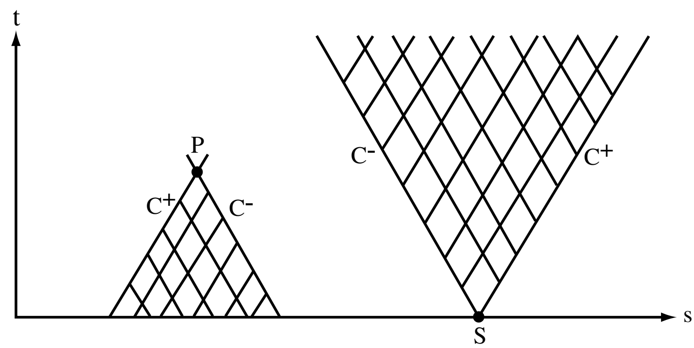


Figure 9.4 Disturbance propagation in the s-t plane.

S can be sensed at subsequent times. A corollary to this concept is also illustrated in Fig. 9.4; the values of H and V at a point P can only be affected by events contained within the zone formed by the subtended C^+ and C^- characteristics.

9.1.3. SETTING UP THE NUMERICAL PROCEDURE

In the previous section we have developed finite difference equations which permi t us to calculate H and V at predetermined intersections of the C^+ and C^- characteristics. The values of H and V at the ends of the pipe were determined by using boundary conditions. Now we will arrange the solution procedure so it can be conveniently implemented on a computer.

First we develop a pair of equations to find H and V at the interior points (points 2 through N). We do this by solving Eqs. 9.14 and 9.15 simultaneously to obtain

$$V_P = \frac{1}{2}\left[\left(V_{Le} + V_{Ri}\right) + \frac{g}{a}\left(H_{Le} - H_{Ri}\right) - \frac{f\Delta t}{2D}\left(V_{Le}\left|V_{Le}\right| + V_{Ri}\left|V_{Ri}\right|\right)\right] \qquad (9.17)$$

$$H_P = \frac{1}{2}\left[\frac{a}{g}\left(V_{Le} - V_{Ri}\right) + \left(H_{Le} + H_{Ri}\right) - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_{Le}\left|V_{Le}\right| - V_{Ri}\left|V_{Ri}\right|\right)\right] \qquad (9.18)$$

The boundary conditions at each end of the pipe describe externally-imposed conditions on velocity and/or pressure head. To aid the reader in understanding how boundary conditions are applied, we will examine a few common ones now.

Reservoir boundary condition (upstream end of pipe)

Where a pipe exits from a reservoir, the head H assumes the value corresponding to the head of the reservoir water surface. If the water surface elevation is constant in time, then H is constant. If the reservoir water surface elevation changes with time, so too does H, if the local pipe entrance loss is neglected. This is represented in equation form as

$$H_{P_1} = H_0 \qquad (9.19)$$

This value for H_{P_1} is substituted (R = 2) into Eq. 9.15 to yield an expression for velocity:

$$V_{P_1} = V_2 + \frac{g}{a}\left(H_0 - H_2\right) - \frac{f\Delta t}{2D} V_2\left|V_2\right| \qquad (9.20)$$

If the reservoir were at the downstream end of the pipe, the same approach using the C^+ equation would give a similar expression for $V_{P_{N+1}}$.

Velocity boundary condition (downstream end of pipe)

When the velocity is known at the downstream end of a pipe, this information can be combined with the C^+ characteristic equation to develop an equation for $H_{P_{N+1}}$. For example, suppose a valve is closed so that the velocity decreased linearly from V_0 to zero in T_c seconds. The velocity behavior is

$$\begin{aligned} V_{P_{N+1}} &= V_0\left(1 - \frac{t}{T_c}\right), \quad && 0 \le t \le T_c \\ V_{P_{N+1}} &= 0, && t \ge T_c \end{aligned} \qquad (9.21)$$

The equation for $H_{P_{N+1}}$ can be found by substituting Eqs. 9.21 into Eq. 9.14 to give

$$H_{P_{N+1}} = H_N - \frac{a}{g}\left(V_{P_{N+1}} - V_N\right) - \frac{a}{g}\frac{f\Delta t}{2D} V_N\left|V_N\right| \qquad (9.22)$$

for any value of $V_{P_{N+1}}$, including zero.

Constant speed pump boundary condition (upstream end of pipe)

This boundary condition offers the added complexity of having both H_{P_1} and V_{P_1} in the boundary equation. Consequently the boundary equation must be solved simultaneously with Eq. 9.15 to produce equations for H_{P_1} and V_{P_1}.

We now choose a way to represent the pump boundary condition. The simplest approach that is reasonably general is to represent the pump discharge characteristics by a quadratic equation of the form

$$h_p = A_p' Q^2 + B_p' Q + C_p' \tag{9.23}$$

in which Q is the pump discharge and h_p is the head increase across the pump. These variables are not identical to those in the C^- equation, so we make some adjustments. We replace Q with $V_{P_1} A$ and h_p with $H_{P_1} - H_{sump}$. Incorporating H_{sump} into C_p' and A into A_p' and B_p' leads to

$$H_{P_1} = A_p V_{P_1}^2 + B_p V_{P_1} + C_p \tag{9.24}$$

We note for future use that if this curve is to be concave down and always sloping downward for increasing Q (generally it should do this), then $A_p<0$, $B_p<0$, and $C_p>0$. This information is needed in computerizing this boundary condition.

When Eq. 9.24 is solved simultaneously with the C^- characteristic equation, Eq. 9.15, the elimination of H_{P_1} leads to the following equation for V_{P_1};

$$V_{P_1} - V_2 - \frac{g}{a}\left(A_p V_{P_1}^2 + B_p V_{P_1} + C_p\right) + \frac{g}{a} H_2 + \frac{f\Delta t}{2D} V_2 |V_2| = 0 \tag{9.25}$$

Rearranging, we get

$$\left(\frac{g}{a} A_p\right) V_{P_1}^2 + \left(\frac{g}{a} B_p - 1\right) V_{P_1} + \left(V_2 + \frac{g}{a} C_p - \frac{g}{a} H_2 - \frac{f\Delta t}{2D} V_2 |V_2|\right) = 0 \tag{9.26}$$

This quadratic equation can now be solved for V_{P_1}. Then a back substitution into Eq. 9.24 will yield H_{P_1}.

Incidentally, if a (loss-free) check valve were installed downstream of the pump, we could model it mathematically by first computing V_{P_1} from Eq. 9.26 and then checking the sign of the velocity; if it were negative, we would set V_{P_1} to zero before calculating H_{P_1} from the C^- characteristic equation, Eq. 9.15.

9.1.4. COMPUTERIZING THE NUMERICAL PROCEDURE

The problem-solving approach we have developed is relatively easy to program for the computer. Since we divided the pipe into N sections, the node points between sections can be numbered sequentially from 1 to N+1, beginning at the upstream end of the pipe. Keeping in mind the connection between the subscripts in our equations and the indices of the subscripted variables in computer programs, we rewrite Eqs. 9.14 and 9.15 as

$$C^+ : \left(V_{P_i} - V_{i-1}\right) + \frac{g}{a}\left(H_{P_i} - H_{i-1}\right) + \frac{f\Delta t}{2D} V_{i-1} |V_{i-1}| = 0 \tag{9.27}$$

$$C^- : \left(V_{P_i} - V_{i+1}\right) - \frac{g}{a}\left(H_{P_i} - H_{i+1}\right) + \frac{f\Delta t}{2D} V_{i+1} |V_{i+1}| = 0 \tag{9.28}$$

The solutions for the interior values of H_P and V_P (Eqs. 9.17 and 9.18) are now

$$V_{P_i} = \frac{1}{2}\left[\left(V_{i-1} + V_{i+1}\right) + \frac{g}{a}\left(H_{i-1} - H_{i+1}\right) - \frac{f\Delta t}{2D}\left(V_{i-1}\left|V_{i-1}\right| + V_{i+1}\left|V_{i+1}\right|\right)\right] \tag{9.29}$$

$$H_{P_i} = \frac{1}{2}\left[\frac{a}{g}\left(V_{i-1} - V_{i+1}\right) + \left(H_{i-1} + H_{i+1}\right) - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_{i-1}\left|V_{i-1}\right| - V_{i+1}\left|V_{i+1}\right|\right)\right] \tag{9.30}$$

for $2 \le i \le N$.

All boundary conditions must also be written in a form that is consistent with the subscripted-variable approach. The boundary conditions for the reservoir and the linearly-varying velocity are already in the proper form; however, the constant-speed pump boundary condition, which requires the solution of two simultaneous equations, needs further work. The practitioner must perform the algebra and then program the computer. We will now examine this process in detail because boundary conditions which lead to pairs of equations which must be solved are very common in transient problems. This example also employs a technique for handling the bulky C^+ and C^- equations in an efficient way which can be directly transported to the computer code.

Constant-speed pump revisited

For subsequent algebraic manipulation it is convenient to simplify the equations by representing a collection of known terms by a single symbol. Here we can write Eq. 9.28, applicable to the upstream end of the pipe, as

$$V_{P_1} = C_1 + C_2 H_{P_1} \tag{9.31}$$

where

$$C_1 = V_2 - \frac{g}{a}H_2 - \frac{f\Delta t}{2D}V_2\left|V_2\right| \qquad C_2 = \frac{g}{a} \tag{9.32}$$

In the computer C_1 and C_2 will just be numbers because they were calculated by using known values from the previous time.

Combining Eq. 9.31 with Eq. 9.24 to eliminate H_{P_1} gives

$$\frac{V_{P_1} - C_1}{C_2} = A_p V_{P_1}^2 + B_p V_{P_1} + C_p \tag{9.33}$$

Preparing the equation in standard quadratic form,

$$V_{P_1}^2 + \frac{B_p - 1/C_2}{A_p}V_{P_1} + \frac{C_p + C_1/C_2}{A_p} = 0 \tag{9.34}$$

Letting $C_3 = \dfrac{B_p - 1/C_2}{A_p}$ and $C_4 = \dfrac{C_p + C_1/C_2}{A_p}$, this equation becomes

$$V_{P_1}^2 + C_3 V_{P_1} + C_4 = 0 \tag{9.35}$$

The solution is

$$V_{P_1} = \frac{C_3}{2}\left[-1 \pm \sqrt{1 - \frac{4C_4}{C_3^2}}\right] \tag{9.36}$$

It only remains to determine which of the $\pm$ signs to use.

This sign decision must be made many more times, so we examine the process in detail now. We begin by determining the sign (where possible) of the C-terms. From Eq. 9.32 one can see that

$$C_1 = \text{ unknown sign} \qquad\qquad C_2 = (+) \tag{9.37}$$

Assuming the usual behavior for the coefficients of the pump model, Eq. 9.23, one has

$$A_p = (-) \qquad B_p = (-) \qquad C_p = (+) \tag{9.38}$$

From the definition equations for C_3 and C_4,

$$C_3 = \frac{(-)-(+)}{(-)} = (+) \qquad\qquad C_4 = \frac{(+)+(\text{unknown})}{(-)} = (\text{unknown}) \tag{9.39}$$

We conclude that C_3 is always positive, and we are not sure of the sign of C_4. Equation 9.36 can be written in terms of signs as

$$\left(\text{sign } V_{P_1}\right) = (+)\left[-1 \pm \sqrt{1 - \frac{(?)}{(+)}}\right] \tag{9.40}$$

At the beginning of a steady flow process V_{P_1} is positive; thus we must be able to obtain some positive values from this equation. This can only happen if the term in the brackets is positive. Because there must also be the possibility of negative velocities (see Chapter 7), the term in the brackets must also take on negative values. Because the square root must be positive, the only possibility that could lead to [ ] being either positive or negative would occur when a + sign is selected from the $\pm$ option. We are left with the equation for V_{P_1} as

$$V_{P_1} = \frac{C_3}{2}\left[-1 + \sqrt{1 - \frac{4C_4}{C_3^2}}\right] \tag{9.41}$$

and

$$H_{P_1} = \frac{V_{P_1} - C_1}{C_2} \tag{9.42}$$

Revisiting the issue of backflow, we might assume a check valve is installed. To simulate the check valve, we would test V_{P_1} and, if it were negative, set $V_{P_1} = 0$. We then go ahead and calculate H_{P_1} from Eq. 9.42. As before, we could not use Eq. 9.24 to compute H_{P_1} because the check valve has isolated the pump from the pipeline.

9.1.5. ELEMENTARY COMPUTER PROGRAMS

The first elementary computer program is presented in Fig. 9.5 to demonstrate the structure of a water hammer analysis computer code. The program is written in FORTRAN and provided in dynamic array dimensional form both as source code and executable elements on the enclosed CD ROM. The source listing here will enable the reader to study the various blocks of code comprising the analysis.

The program has NAMELIST input which makes the input data file easier to read. The input parameters are identified in the program under the NAMELIST /SPECS/ statement. Each input parameter is defined in the COMMENT statements at the beginning of the program listing.

In this basic program the user must perform the steady-state hydraulic calculations required to provide some of the input data to the program. These steady-state values are the initial conditions which are entered into a data file created by the user; it is to be read by the program at execution time. The boundary conditions in this program are written into the source code and consist of a constant-head reservoir at the upstream end and a linearly-decreasing velocity at the downstream end. To use the program for any other boundary conditions would require the user to modify the code itself and then recompile it, as will be explained further with the second program in this section.

The program will simulate the water hammer process until the time of simulation reaches TMAX. At that time the program will cease execution; it will then determine and print the maximum and minimum values of pressure head, H, and V that occurred at each node. The output from the analysis will be printed in an output file designated by the user in response to a prompt during execution.

```
  PROGRAM PROG1
*************************************************************************
*   PROGRAM NO. 1
*   APPROXIMATE-METHOD WATER HAMMER PROGRAM FOR A SINGLE STRAIGHT PIPE.
*
*   THIS PROGRAM HAS BEEN INCLUDED FOR THE CONVENIENCE OF THE READER.
*   THE AUTHOR ACCEPTS NO RESPONSIBILITY FOR ITS CORRECTNESS.
*   USERS OF THIS PROGRAM DO SO AT THEIR OWN RISK.
*************************************************************************
*                CONSTANT-HEAD RESERVOIR AT UPSTREAM END
*   VELOCITY DECREASES LINEARLY WITH TIME TO ZERO AT DOWNSTREAM VALVE
*
*                 *****  DATA DESCRIPTION  *****
*   TITLE1 =  FIRST JOB DESCRIPTION.  ANY INFORMATION, 80 COLUMNS MAXIMUM
*   TITLE2 = SECOND JOB DESCRIPTION.  ANY INFORMATION, 80 COLUMNS MAXIMUM
*   IOUT = PRINT OUTPUT INDEX. GIVES PRINTED OUTPUT EVERY IOUT-TH TIME
*          STEP. FOR EXAMPLE, IF IOUT = 3, THEN OUTPUT IS PRINTED EVERY
*          THIRD TIME STEP.
*   NPARTS = NUMBER OF PIPE SEGMENTS INTO WHICH PIPE IS DIVIDED
*   D = PIPE DIAM, IN        L = PIPE LENGTH, FT
*   F = DARCY-WEISBACH  F-VALUE OR HAZEN-WILLIAMS  C-VALUE
*   A = WAVE SPEED, FT/S     VZERO = INITIAL STEADY STATE VELOCITY, FT/S
*   HZERO = UPSTEAM RESERVOIR ELEVATION, FT
*   ELEVUP = ELEVATION OF UPSTREAM END OF PIPE, FT
*   ELEVDN = ELEVATION OF DOWNSTREAM END OF PIPE, FT
*   TMAX = MAXIMUM REAL TIME OF SIMULATION, SEC
*   TCLOSE = TIME REQUIRED FOR VALVE CLOSURE, SEC
      DIMENSION X[ALLOCATABLE](:),V[ALLOCATABLE](:),H[ALLOCATABLE](:),
     $HLOW[ALLOCATABLE](:),HHIGH[ALLOCATABLE](:),HEAD[ALLOCATABLE](:),
     $VNEW[ALLOCATABLE](:),HNEW[ALLOCATABLE](:),PIPEZ[ALLOCATABLE](:)
      REAL L,NEXP
```

Figure 9.5 An elementary computer program for the approximate method.

```
   CHARACTER TITLE1*80,TITLE2*80,ROUGH*3,CH,CH12,CH78,CH79,CHN
      CHARACTER FNAME*12
      NAMELIST /SPECS/ IOUT,NPARTS,D,L,F,A,VZERO,HZERO,ELEVUP,
     $ELEVDN,TMAX,TCLOSE
*-----------------------------------------------------------------------
      WRITE(*,791)
      READ(*,100) FNAME
      OPEN(5,FILE=FNAME)
      READ(5,100) TITLE1
      READ(5,100) TITLE2
      READ(5,SPECS)
      WRITE(*,792)
      READ(*,100) FNAME
      OPEN(6,FILE=FNAME,STATUS='NEW')
*-----------------------------------------------------------------------
      CH=CHAR(27)
      CH12=CHAR(12)
      CH78=CHAR(78)
      CH79=CHAR(79)
      CHN=CHAR(3)
      NP=NPARTS+1
      ALLOCATE (X(NP),V(NP),H(NP),HLOW(NP),HHIGH(NP),HEAD(NP),
     $VNEW(NP),HNEW(NP),PIPEZ(NP))
      WTT=L/A
      DELL=L/NPARTS
      T=0.
      NEXP=1.0
      ROUGH='F ='
      IF(F.GT.10.) NEXP=0.85
      IF(F.GT.10.) ROUGH='C ='
      DELT=DELL/A
      C=32.2/A
      INDEX=TMAX/DELT + 1
      DELEL=(ELEVDN-ELEVUP)/NPARTS
      NODES=NPARTS+1
      WRITE(6,101) CH,CH78,CHN
      WRITE(6,200)
      WRITE(6,203) TITLE1
      WRITE(6,203) TITLE2
      WRITE(6,201) IOUT,NPARTS,L,A,D,ROUGH,F,VZERO,HZERO,ELEVUP,
     $ELEVDN,WTT,TCLOSE,TMAX,DELT
      AK=12.*F*DELT/(2.0*D)
      IF(F.GT.10.) AK=12.*DELT*195./(2.0*D*(F**1.85)*(D/12.)**.17)
      IF(F.GT.10.) F=195./((F**1.85)*(VZERO**.15)*((D/12.)**.17))
      DELHF=12.*F*DELL*VZERO**2/(64.4*D)
*
      DO 300 I=1,NODES
      V(I)=VZERO
      H(I)=HZERO-(I-1)*DELHF
      HLOW(I)=H(I)
      HHIGH(I)=H(I)
      X(I)=(I-1)*DELL/L
      PIPEZ(I)=ELEVUP+(I-1)*DELEL
      HEAD(I)=H(I)-PIPEZ(I)
  300 CONTINUE
      WRITE(6,101) CH12
      WRITE(6,202)
      WRITE(6,204) T,(X(I),HEAD(I),H(I),V(I),I=1,NODES)
```

Figure 9.5, cont'd. An elementary computer program for the approximate method.

```
      DO 99 II=1,INDEX
      T=T+DELT
*  ** COMPUTE H AND V AT INTERIOR NODES **
      DO 20 I=2,NPARTS
      VNEW(I)=0.5*(V(I-1)+V(I+1)+C*(H(I-1)-H(I+1))-AK*(V(I-1)*ABS(V(I-1)
     $)**NEXP+V(I+1)*ABS(V(I+1))**NEXP))
   20 HNEW(I)=0.5*(H(I-1)+H(I+1)+(V(I-1)-V(I+1))/C-AK*(V(I-1)*ABS(V(I-1)
     $)**NEXP-V(I+1)*ABS(V(I+1))**NEXP)/C)
*  ** COMPUTE H AND V AT UPSTREAM END **
*     THIS BOUNDARY CONDITION IS FOR A CONSTANT-HEAD RESERVOIR
      HNEW(1)=HZERO
      VNEW(1)=V(2)+C*(HNEW(1)-H(2))-AK*V(2)*ABS(V(2))**NEXP
*  ** COMPUTE H AND V AT DOWNSTREAM END **
*     THIS BOUNDARY CONDITION IS FOR LINEARLY DECREASING VELOCITY
      IF(T.GT.TCLOSE) GO TO 30
      VNEW(NODES)=VZERO*(1.-T/TCLOSE)
      GO TO 31
   30 VNEW(NODES)=0.0
   31 HNEW(NODES)=H(NPARTS)+(V(NPARTS)-VNEW(NODES)-AK*V(NPARTS)*
     $ABS(V(NPARTS))**NEXP)/C
      DO 50 I=1,NODES
      IF(HNEW(I).LT.HLOW(I)) HLOW(I)=HNEW(I)
      IF(HNEW(I).GT.HHIGH(I)) HHIGH(I)=HNEW(I)
   50 HEAD(I)=HNEW(I)-PIPEZ(I)
      IF(MOD(II,IOUT).EQ.0) WRITE(6,204) T,(X(I),HEAD(I),HNEW(I),VNEW(I)
     $,I=1,NODES)
      IF(T.GT.TMAX) GO TO 400
      DO 40 I=1,NODES
      V(I)=VNEW(I)
   40 H(I)=HNEW(I)
   99 CONTINUE
*
  400 WRITE(6,101) CH12
      WRITE(6,205)
      DO 401 I=1,NODES
      HEADMX=HHIGH(I)-PIPEZ(I)
      HEADMN=HLOW(I)-PIPEZ(I)
  401 WRITE(6,206) X(I),HEADMX,HEADMN,HHIGH(I),HLOW(I)
      WRITE(6,101) CH,CH79
  100 FORMAT(A)
  101 FORMAT(3A)
  200 FORMAT(///20X,33('*')/20X,'* WATER HAMMER IN A SINGLE PIPE *'/
     $20X,33('*')//)
  201 FORMAT(//29X,'INPUT DATA'/29X,10('-')//28X,'IOUT =',I4/26X,
     $'NPARTS =',I4//31X,'L =',F7.1,' FT'/31X,'A =',F7.1,' FT/S'/
     $31X,'D =',F7.2,' IN'/31X,A,F9.4//
     $27X,'VZERO =',F7.2,' FT/S'/27X,'HZERO =',F7.1,' FT'/
     $26X,'ELEVUP =',F7.1,' FT'/26X,'ELEVDN =',F7.1,' FT'//
     $29X,'L/A =',F7.3,' SEC'//26X,'TCLOSE =',F7.2,' SEC'/
     $28X,'TMAX =',F7.2,' SEC'/28X,'DELT =',F7.3,' SEC')
  202 FORMAT(//5X,'PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS
     $OF TIME'/5X,60('-'))
  203 FORMAT(10X,A)
  204 FORMAT(//11X,2(4X,'  X   HEAD,FT  H,FT  V,FT/S ')/'T =',F6.3,
     $' SEC',2(2X,'----- ------- -----  ------  ')/(10X,2(5X,F5.3,2F6.0,
     $F7.2)))
  205 FORMAT(///18X,27('*')/18X,'* TABLE OF EXTREME VALUES *'/18X,27('*
     $')//13X,'X    MAX HEAD  MIN HEAD   MAX H   MIN H'/11X,5('-'),2X,8(
```

Figure 9.5, cont'd. An elementary computer program for the approximate method.

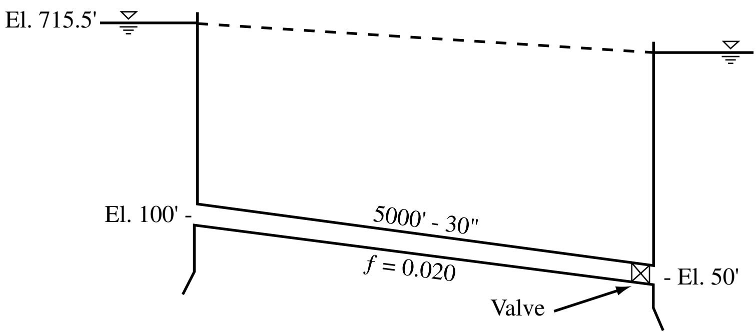
```
    $'-'),2X,8('-'),2X,6('-'),2X,6('-'))
206 FORMAT(11X,F5.3,2X,F7.0,3X,F7.0,3X,F6.0,2X,F6.0)
791 FORMAT(/' ENTER THE NAME OF YOUR INPUT DATA FILE:  '\)
792 FORMAT(/' ENTER THE NAME OF THE FILE ON WHICH THE OUTPUT IS TO BE
    $WRITTEN: '\)
     END
```

Figure 9.5, concluded. An elementary computer program for the approximate method.

The following example problem demonstrates the input data file and the output of the program.

Example Problem 9.1

The 30-in steel pipeline is 5000 ft long with a wave speed of 2500 ft/s. The steady-state velocity is 5 ft/s. The valve at the downstream end is closed in such a manner that the velocity at the valve decreases linearly to zero in 10 sec. There is initially a negligible head loss in the valve. Find the maximum and minimum pressure heads in the system during this transient, and state their location.



Using PROG1 to solve the problem, we will divide the pipe into 8 sections and set up the following data file via the text editor:

```
DEMONSTRATION OF PROGRAM NO. 1 - INPUT DATA FILE "EP91.DAT"
CONSTANT-HEAD RESERVOIR UPSTREAM & LINEARLY DECREASING VELOCITY DOWNSTREAM
&SPECS IOUT=4,NPARTS=8,D=30.,L=5000.,F=.020,A=2500.,VZERO=5.00,
       HZERO=715.5,ELEVUP=100.,ELEVDN=50.,TMAX=20.00,TCLOSE=10.00/
```

The output from the analysis is shown below. To save space only the beginning and end of the output data file are printed. A scan of the Table of Extreme Values shows that a maximum pressure head of 810 ft occurs at the valve, and the minimum pressure head of 569 ft occurs at the midpoint of the line.

```
                    *********************************
                    * WATER HAMMER IN A SINGLE PIPE *
                    *********************************

DEMONSTRATION OF PROGRAM NO. 1 - INPUT DATA FILE "EP91.DAT"
CONSTANT-HEAD RESERVOIR UPSTREAM & LINEARLY DECREASING VELOCITY DOWNSTREAM
```

```
                              INPUT DATA
                              ----------

                         IOUT =   4
                       NPARTS =   8

                            L = 5000.0 FT
                            A = 2500.0 FT/S
                            D =  30.00 IN
                            F =    .0200
                        VZERO =   5.00 FT/S
                        HZERO =  715.5 FT
                       ELEVUP =  100.0 FT
                       ELEVDN =   50.0 FT
                          L/A =  2.000 SEC
                       TCLOSE =  10.00 SEC
                         TMAX =  20.00 SEC
                         DELT =   .250 SEC


     PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS OF TIME
     ------------------------------------------------------------

                    X   HEAD,FT  H,FT  V,FT/S     X   HEAD,FT  H,FT   V,FT/S
T =  .000 SEC   ----- ------- -----  ------    ----- ------- -----  ------
                 .000   616.   716.   5.00      .125   620.   714.   5.00
                 .250   624.   712.   5.00      .375   628.   710.   5.00
                 .500   633.   708.   5.00      .625   637.   706.   5.00
                 .750   641.   704.   5.00      .875   646.   702.   5.00
                1.000   650.   700.   5.00


                    X   HEAD,FT  H,FT  V,FT/S     X   HEAD,FT  H,FT   V,FT/S
T = 1.000 SEC   ----- ------- -----  ------    ----- ------- -----  ------
                 .000   616.   716.   5.00      .125   620.   714.   5.00
                 .250   624.   712.   5.00      .375   628.   710.   5.00
                 .500   633.   708.   5.00      .625   647.   715.   4.88
                 .750   661.   723.   4.75      .875   675.   731.   4.63
                1.000   689.   739.   4.50

                   .     .      .      .         .     .      .       .

                   .     .      .      .         .     .      .       .

                   .     .      .      .         .     .      .       .

                    X   HEAD,FT  H,FT  V,FT/S     X   HEAD,FT  H,FT   V,FT/S
T =20.000 SEC   ----- ------- -----  ------    ----- ------- -----  ------
                 .000   616.   716.   -.95      .125   630.   724.   -.82
                 .250   645.   733.   -.70      .375   660.   742.   -.58
                 .500   675.   750.   -.46      .625   690.   759.   -.34
                 .750   705.   767.   -.23      .875   720.   776.   -.11
                1.000   735.   785.    .00

                   ***************************
                   * TABLE OF EXTREME VALUES *
                   ***************************
```

X	MAX HEAD	MIN HEAD	MAX H	MIN H
.000	616.	616.	716.	716.
.125	641.	603.	735.	697.
.250	666.	592.	753.	679.
.375	691.	580.	772.	661.
.500	715.	569.	790.	644.
.625	740.	575.	808.	644.
.750	763.	581.	826.	643.
.875	787.	586.	843.	642.
1.000	810.	592.	860.	642.

*　　　　*　　　　*

The second elementary program PROG1P is a modification of the first program which contains the boundary conditions for a constant-speed pump at the upstream end of the line. In addition, the valve at the downstream end has been altered to permit the velocity to change linearly from the initial steady-state value to a prescribed final steady-state value (including zero). To accommodate the pump at the upstream end, the constant-head reservoir boundary conditions are replaced with those of Eqs. 9.41 and 9.42. The downstream boundary conditions are adjusted to permit the velocity to decrease to any lower constant value. The input data file must be expanded to provide the three pump curve coefficients $A_p^{'}$, $B_p^{'}$, and $C_p^{'}$ and the final velocity at the downstream valve. Shown below are selected portions of the new code that are required in PROG1 to implement these changes.

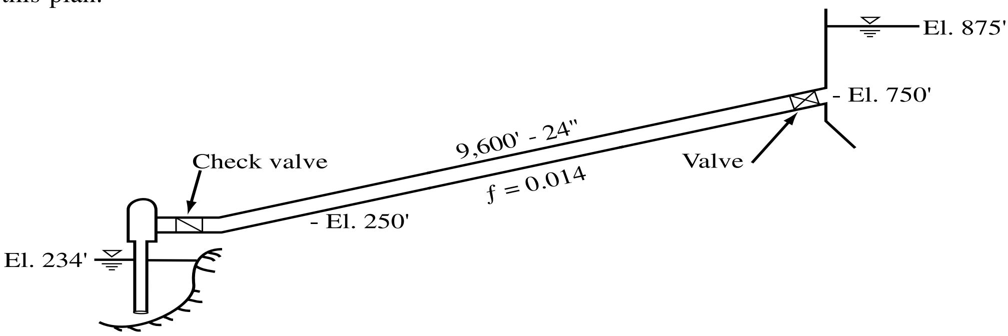
```
   .    .     .     .     .     .     .     .
   .    .     .     .     .     .     .     .
*   HSUMP = PUMP SUMP ELEVATION, FT
*   ELEVUP = ELEVATION OF UPSTREAM END OF PIPE, FT
*   ELEVDN = ELEVATION OF DOWNSTREAM END OF PIPE, FT
*   TMAX = MAXIMUM REAL TIME OF SIMULATION, SEC
*   TCLOSE = TIME REQUIRED FOR VALVE CLOSURE, SEC
*   VFINAL = FINAL VELOCITY, FT/S
*   THE VALUES OF APRIME, BPRIME, AND CPRIME ARE COMPUTED WITH THE
*      DISCHARGE IN GAL/MIN AND TOTAL HEAD FOR ALL STAGES IN FT
*      APRIME = FIRST COEFFICIENT IN PUMP CHARACTERISTIC EQUATION
*      BPRIME = SECOND COEFFICIENT IN PUMP CHARACTERISTIC EQUATION
*      CPRIME = THIRD COEFFICIENT IN PUMP CHARACTERISTIC EQUATION
   .    .     .     .     .     .     .     .
   .    .     .     .     .     .     .     .
   .    .     .     .     .     .     .     .
      NAMELIST /SPECS/ IOUT,NPARTS,D,L,F,A,VZERO,HSUMP,ELEVUP,
     $ELEVDN,TMAX,TCLOSE,VFINAL,APRIME,BPRIME,CPRIME
   .    .     .     .     .     .     .     .
   .    .     .     .     .     .     .     .
   .    .     .     .     .     .     .     .
      AREA=0.7854*D*D/144.
      APRIME=APRIME*(AREA*449.)**2
      BPRIME=BPRIME*AREA*449.
      CPRIME=CPRIME+HSUMP
      HPUMP=APRIME*VZERO*VZERO+BPRIME*VZERO+CPRIME
```

```
  .    .      .      .      .      .      .      .
  .    .      .      .      .      .      .      .
  .    .      .      .      .      .      .      .
*  ** COMPUTE H AND V AT UPSTREAM END **
*     THIS BOUNDARY CONDITION IS FOR A CONSTANT-HEAD PUMP
      C1=V(2)-C*H(2)-AK*V(2)*ABS(V(2))
      C3=(BPRIME-1.0/C)/APRIME
      C4=(CPRIME+C1/C)/APRIME
      CHEK=4.*C4/(C3*C3)
      IF(CHEK.GT.0.) GO TO 25
      VNEW(1)=0.5*C3*(-1.+SQRT(1.0-CHEK))
      GO TO 26
   25 VNEW(1)=0.
   26 HNEW(1)=(VNEW(1)-C1)/C
*  ** COMPUTE H AND V AT DOWNSTREAM END **
*     THIS BOUNDARY CONDITION IS FOR LINEARLY DECREASING VELOCITY
      IF(T.GT.TCLOSE) GO TO 30
      VNEW(NODES)=VZERO-(T/TCLOSE)*(VZERO-VFINAL)
      GO TO 31
   30 VNEW(NODES)=VFINAL
   31 HNEW(NODES)=H(NPARTS)+(V(NPARTS)-VNEW(NODES)-AK*V(NPARTS)*
     $ABS(V(NPARTS))**NEXP)/C
  .    .      .      .      .      .      .      .
  .    .      .      .      .      .      .      .
```

Example Problem 9.2

A four-stage Johnston 20 CC turbine pump (the pump characteristic diagram is in Appendix B) with 15 3/4 in impellers is used to pump water from a river to an elevated storage reservoir. The welded-steel pipeline is 9600 ft long, 24 in inside diameter, with a wall thickness of 0.1875 in and a friction factor of 0.014. A special valve is located at the downstream end of the line to cause the velocity at the valve to vary linearly with time.

The design engineer would like to close the valve in 30 sec before shutting down the pump. Determine the maximum and minimum pressure heads which would occur under this plan.



We will use PROG1P and divide the pipeline into 10 parts to solve the problem. But first we must calculate the wave speed and the coefficients of the parabolic equation which will model the pump characteristics.

The pipe has a *D/e* ratio of 128, so we can use Eq. 8.33 for thin-walled pipes to compute the wave speed. Using Case (b) restraint because it gives the highest wave speed, we find

$$a = \frac{4720}{\sqrt{1 + \frac{3\times10^5}{30\times10^6}\frac{24}{0.1875}\left(1-0.3^2\right)}} = 3200 \quad \text{ft/s}$$

To determine the parabolic equation coefficients, we select three points on the pump characteristic diagram, write three equations with the unknown coefficients in them, and then solve the equations. We will use the following points:

Q = 0, 4000, and 7000 gal/min and h = 254, 200, and 137 ft/stage

The coefficients $A_p' = -1.071\times10^{-6}$, $B_p' = -9.215\times10^{-3}$, and $C_p' = 254$ are the result. However, this is a four-stage pump, so each of the coefficients must be multiplied by four before they are inserted into the program. The data file created by the text editor follows:

```
SOLUTION FOR EXAMPLE PROBLEM 9.2 - INPUT DATA FILE "EP92.DAT"
JOHNSTON 20 CC PUMP UPSTREAM, VALVE DOWNSTREAM CLOSING IN 30 SEC
&SPECS IOUT=1000, NPARTS=10, D=24.00, L=9600., F=0.014, A=3200.,
     VZERO=4.11, HSUMP=234., ELEVUP=250., ELEVDN=750., TCLOSE=30.,
     TMAX=60., VFINAL=0., APRIME=-4.28E-6, BPRIME=-0.03686, CPRIME=1016./
```

The results of the analysis are shown below; they reveal that the maximum pressure head of 1029 ft occurs at the pump, while the minimum pressure head of 125 ft occurs at the valve.

```
         **********************************
                    * WATER HAMMER IN A SINGLE PIPE *
                    **********************************

SOLUTION FOR EXAMPLE PROBLEM 9.2 - INPUT DATA FILE "EP92.DAT"
JOHNSTON 20 CC PUMP UPSTREAM, VALVE DOWNSTREAM CLOSING IN 30 SEC


                              INPUT DATA
                              ----------

                             IOUT =1000
                           NPARTS =   10

                                L = 9600.0 FT
                                A = 3200.0 FT/S
                                D =   24.00 IN
                                F =     .0140

                           VFINAL =     .00 FT/S
                            VZERO =    4.11 FT/S
                            HSUMP =   234.0 FT
                           ELEVUP =   250.0 FT
                           ELEVDN =   750.0 FT
```

```
                         L/A =   3.000 SEC

                      TCLOSE =   30.00 SEC
                        TMAX =   60.00 SEC
                        DELT =    .300 SEC
                      APRIME =-.4280E-05
                      BPRIME =-.3686E-01
                      CPRIME = .1016E+04

     PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS OF TIME
     ------------------------------------------------------------

                  X    HEAD,FT  H,FT  V,FT/S       X    HEAD,FT  H,FT  V,FT/S
T =   .000 SEC  ----- ------- -----  ------      ----- ------- -----  ------
                 .000   642.   892.   4.11        .100   591.   891.   4.11
                 .200   539.   889.   4.11        .300   487.   887.   4.11
                 .400   435.   885.   4.11        .500   384.   884.   4.11
                 .600   332.   882.   4.11        .700   280.   880.   4.11
                 .800   228.   878.   4.11        .900   177.   877.   4.11
                1.000   125.   875.   4.11

                   ***************************
                   * TABLE OF EXTREME VALUES *
                   ***************************

              X     MAX HEAD  MIN HEAD   MAX H   MIN H
            -----   --------  --------  ------  ------
             .000    1029.       642.    1279.    892.
             .100     977.       591.    1277.    891.
             .200     926.       539.    1276.    889.
             .300     874.       487.    1274.    887.
             .400     822.       435.    1272.    885.
             .500     770.       384.    1270.    884.
             .600     722.       332.    1272.    882.
             .700     674.       280.    1274.    880.
             .800     626.       228.    1276.    878.
             .900     577.       177.    1277.    877.
            1.000     529.       125.    1279.    875.
```

*   *   *

9.2 COMPLETE METHOD OF CHARACTERISTICS

In solving the complete equations we can proceed in a manner similar to that for the approximate method. However, in this case, we will use the original Eqs. 8.57 and 8.58.

9.2.1. THE COMPLETE EQUATIONS

We once again use the linear constant multiplier *l* to combine the Euler and conservation of mass equations. Multiplying Eq. 8.57 by *l* and adding the result to Eq. 8.58 gives

$$\lambda \frac{dV}{dt} + \frac{\lambda}{\rho}\frac{\partial p}{\partial s} + \lambda g \frac{dz}{ds} + \lambda \frac{f}{2D} V|V| + a^2 \frac{\partial V}{\partial s} + \frac{1}{\rho}\frac{dp}{dt} = 0 \qquad (9.43)$$

To repeat the same procedure, we must separate dV/dt and dp/dt into their component parts. The result is

$$\left(\lambda\frac{\partial V}{\partial t}+\lambda V\frac{\partial V}{\partial s}\right)+\frac{\lambda}{\rho}\frac{\partial p}{\partial s}+\lambda g\frac{dz}{ds}+\lambda\frac{f}{2D}V|V|+a^2\frac{\partial V}{\partial s}+\left(\frac{1}{\rho}\frac{\partial p}{\partial t}+\frac{V}{\rho}\frac{\partial p}{\partial s}\right)=0 \quad (9.44)$$

Regrouping the terms in the equation gives

$$\left[\lambda\frac{\partial V}{\partial t}+\left(\lambda V+a^2\right)\frac{\partial V}{\partial s}\right]+\left[\frac{1}{\rho}\frac{\partial p}{\partial t}+\left(\frac{\lambda}{\rho}+\frac{V}{\rho}\right)\frac{\partial p}{\partial s}\right]+\lambda g\frac{dz}{ds}+\lambda\frac{f}{2D}V|V|=0 \quad (9.45)$$

As before,

$$\left[\lambda\frac{\partial V}{\partial t}+\left(\lambda V+a^2\right)\frac{\partial V}{\partial s}\right]=\lambda\frac{dV}{dt} \quad \text{if} \quad \lambda\frac{ds}{dt}=\lambda V+a^2 \quad (9.46)$$

and

$$\left[\frac{1}{\rho}\frac{\partial p}{\partial t}+\left(\frac{\lambda}{\rho}+\frac{V}{\rho}\right)\frac{\partial p}{\partial s}\right]=\frac{1}{\rho}\frac{dp}{dt} \quad \text{if} \quad \frac{1}{\rho}\frac{ds}{st}=\frac{\lambda}{\rho}+\frac{V}{\rho} \quad (9.47)$$

Thus we require for ds/dt that

$$\frac{ds}{dt}=V+\frac{a^2}{\lambda} \quad \text{and} \quad \frac{ds}{dt}=\lambda+V \quad (9.48)$$

Equating these two expressions to eliminate ds/dt and then solving for l leads to

$$\lambda = \pm a \quad (9.49)$$

With l again equal to the wave speed, we find that the equations for the characteristics are

$$\frac{ds}{dt}=V+a \quad \text{and} \quad \frac{ds}{dt}=V-a \quad (9.50)$$

Finally we replace the pressure in favor of total head using $p=\gamma(H-z)$.

The final set of equations, which are the analogs of Eqs. 9.8 and 9.9, is

$$C^+: \quad \frac{dV}{dt}+\frac{g}{a}\frac{dH}{dt}-\frac{g}{a}V\frac{dz}{ds}+\frac{f}{2D}V|V|=0 \quad \text{only when} \quad \frac{ds}{dt}=V+a \quad (9.51)$$

$$C^-: \quad \frac{dV}{dt}-\frac{g}{a}\frac{dH}{dt}+\frac{g}{a}V\frac{dz}{ds}+\frac{f}{2D}V|V|=0 \quad \text{only when} \quad \frac{ds}{dt}=V-a \quad (9.52)$$

These ordinary differential equations are quite similar to those for the approximate case. However, the characteristic lines in the s-t plane, which were of constant slope for the approximate method, are now *curved*, their slope a function of $V(s,t)$. This is an important distinction because it introduces some complications into the numerical solution procedure which we must address.

9.2.2. THE NUMERICAL SOLUTION

We first assume that the characteristic curves can be approximated as straight lines over each single Δt interval. This assumption is attractive because (1) Δt may be made as small as one wishes, and (2) usually $a>>V$, causing ds/dt to be nearly constant. But the slopes of the C^+ and C^- characteristic lines are no longer the same in magnitude.

The problem this creates in the finite difference approximations to the differential equations can be seen in Fig. 9.6. Assume now that the grid intervals Δs and Δt have been chosen (we will see shortly how this is done), and once again we seek to find the values of H and V at P. The curved characteristics intersecting at P are approximated by straight lines, whose slopes have been determined by the known values of velocity at the earlier time. We now see in Fig. 9.6 that the characteristics intersecting at P no longer pass through the grid points Le and Ri, but instead they pass through the $t =$ constant line at points identified as L and R somewhere between Le and Ri.

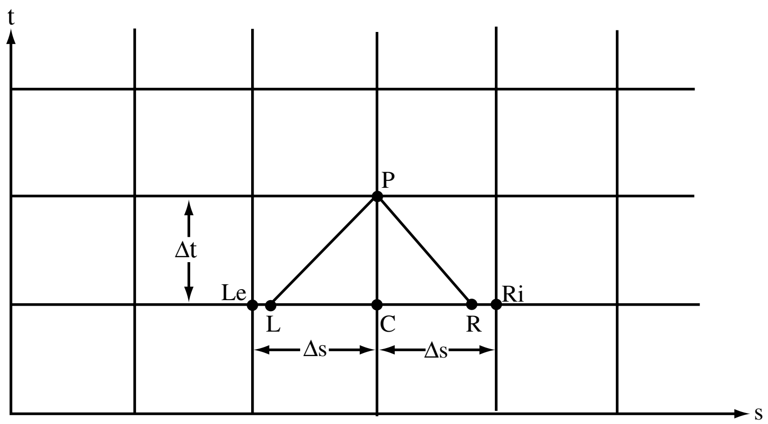


Figure 9.6 Interpolation of H and V values on the Δs-Δt grid.

For this situation the finite difference approximations to Eqs. 9.51 and 9.52 become

$$\frac{V_P - V_L}{\Delta t} + \frac{g}{a}\frac{H_P - H_L}{\Delta t} - \frac{g}{a} V_L \frac{dz}{ds} + \frac{f}{2D} V_L |V_L| = 0 \tag{9.53}$$

$$\frac{V_P - V_R}{\Delta t} - \frac{g}{a}\frac{H_P - H_R}{\Delta t} + \frac{g}{a} V_R \frac{dz}{ds} + \frac{f}{2D} V_R |V_R| = 0 \tag{9.54}$$

The new difficulty here is that the values of V_L, H_L, V_R, and H_R are not known, thereby causing Eqs. 9.53 and 9.54 to include six unknowns rather than the two unknowns which occurred in the approximate method. We overcome this problem by choosing Δt so that point L is near Le and R is near Ri; now linear interpolation becomes an accurate way to evaluate the values of H and V at points L and R. Figure 9.7 defines the parameters needed in the interpolation procedure.

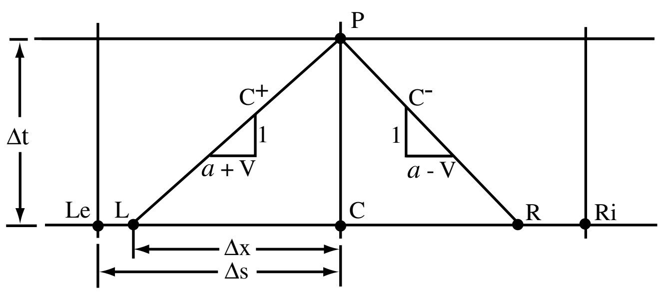


Figure 9.7 Parameters in the interpolation procedure.

Along the C^+ characteristic,

$$\frac{\Delta x}{\Delta s} = \frac{V_L - V_C}{V_{Le} - V_C} = \frac{H_L - H_C}{H_{Le} - H_C} \tag{9.55}$$

with

$$\frac{\Delta x}{\Delta t} = \frac{a + V_L}{1} \tag{9.56}$$

Solving these two equations for V_L and H_L yields

$$V_L = \left(V_{Le} - V_C\right)\frac{\Delta x}{\Delta s} + V_C \quad \text{and} \quad H_L = \left(H_{Le} - H_C\right)\frac{\Delta x}{\Delta s} + H_C \tag{9.57}$$

Replacing Δx in these equations using the same relation for $\Delta x/\Delta t$ now produces

$$V_L = \frac{V_C + a\dfrac{\Delta t}{\Delta s}\left(V_{Le} - V_C\right)}{1 - \dfrac{\Delta t}{\Delta s}\left(V_{Le} - V_C\right)} \tag{9.58}$$

and

$$H_L = H_C + \frac{\Delta t}{\Delta s}\left(H_{Le} - H_C\right)\left(a + V_L\right) \tag{9.59}$$

A similar analysis along the C^- characteristic gives

$$V_R = \frac{V_C + a\dfrac{\Delta t}{\Delta s}\left(V_{Ri} - V_C\right)}{1 - \dfrac{\Delta t}{\Delta s}\left(V_{Ri} - V_C\right)} \tag{9.60}$$

and

$$H_R = H_C + \frac{\Delta t}{\Delta s}\left(H_{Ri} - H_C\right)\left(a - V_R\right) \tag{9.61}$$

Because $\frac{\Delta t}{\Delta s}\left(V_{Le} - V_C\right)$ is on the order of $\frac{V}{a+V}$, which is very small compared to 1, it is a good approximation to neglect the second terms in the denominators of Eqs. 9.58 and 9.60. The results are

$$V_L = V_C + a\frac{\Delta t}{\Delta s}\left(V_{Le} - V_C\right) \tag{9.62}$$

and

$$V_R = V_C + a\frac{\Delta t}{\Delta s}\left(V_{Ri} - V_C\right) \tag{9.63}$$

Since we now have known values for V_L, H_L, V_R, and H_R, we can solve Eqs. 9.53 and 9.54 simultaneously for V_P and H_P. The solutions are

$$V_P = \frac{1}{2}\left[\left(V_L + V_R\right) + \frac{g}{a}\left(H_L - H_R\right) + \frac{g}{a}\Delta t\left(V_L - V_R\right)\sin\theta - \frac{f\Delta t}{2D}\left(V_L\left|V_L\right| + V_R\left|V_R\right|\right)\right] \tag{9.64}$$

$$H_P = \frac{1}{2}\left[\frac{a}{g}\left(V_L - V_R\right) + \left(H_L + H_R\right) + \Delta t\left(V_L + V_R\right)\sin\theta - \frac{a}{g}\frac{f\Delta t}{2D}\left(V_L\left|V_L\right| - V_R\left|V_R\right|\right)\right] \quad (9.65)$$

in which $\sin\theta = dz/ds$ is positive for pipes sloping upward in the downstream direction. Our next step is to determine how the Δs-Δt grid is established.

9.2.3. THE Δs-Δt GRID

The non-constant slope of the curving characteristics and the decision to approximate each as a straight line over a small time interval now force us to face the problem of finding appropriate values of Δs and Δt which will yield accurate and numerically stable solutions. Actually it is always possible to find a pair of values for Δs and Δt which will require no interpolations *for a given section* of the characteristic lines. However, seeking these values would lead to a confusing array of Δs's and Δt's which would make it impossible to keep track of where and when things are happening.

One solution for this problem is to select a uniform rectangular grid on the s-t plane where Δs and Δt are fixed for all time at values which minimize the interpolation and simplify the programming. This method is called the *rectangular grid* method. To establish the grid dimensions, we proceed in the same manner as with the approximate method. We decide how many parts in which to divide the pipeline, thereby fixing Δs. Then the integrated characteristic equations are used to select Δt. The resulting integrated characteristic equations (assuming constant V) are

$$\begin{aligned} \Delta t &= \frac{\Delta s}{V + a} \quad \text{for the } C^{+} \text{ characteristic} \\ \Delta t &= \frac{\Delta s}{V - a} \quad \text{for the } C^{-} \text{ characteristic} \end{aligned} \quad (9.66)$$

Because this interpolation procedure implies that the points L and R are between points Le and Ri, we must limit Δt to assure this is always so. The preceding equations suggest the criterion

$$\Delta t \le \frac{\Delta s}{\max\left|a + V\right|} \quad (9.67)$$

in which $\max\left|a + V\right|$ is the maximum expected absolute value of the sum of the wave speed and velocity. If the location of points L and R fall "outside" the grid points Le and Ri, numerical stability and accuracy problems will develop, as we demonstrate later. These problems are related to the earlier discussion of how "messages" are transmitted along the pipeline. When points L and R are outside the grid points, the procedure uses information in computing H_P and V_P which hasn't physically had enough time to reach point P. This is improper numerical procedure which will lead at best to inaccurate results and at worst to numerical instability. We must guard against this happening in our computer programs.

A computer program (not included) which would incorporate the complete method of characteristics would be very similar to PROG1 for the approximate method. It is only necessary to add a few lines of code to compute the proper Δt and $\sin\theta$ and a few more lines to compute V_L, H_L, V_R, and H_R from Eqs. 9.59, 9.61, 9.62, and 9.63.

A comparison of results from the two methods when $a >> V$ shows for typical situations that the two methods produce essentially the same answers. This indicates that the basic assumption behind the approximate method is sound, namely that the time variation in V and H is more significant that the spatial variation. So one might wonder why the complete method would ever be used. It turns out that the matching of internal boundary conditions in more complex pipe systems between pipes of different sizes and wave speeds requires the interpolation procedure we have just derived. In these more complex problems we will use the complete equations because they already include the interpolation procedure.

9.3 SOME PARAMETER EFFECTS ON SOLUTION RESULTS

It is both informative and useful to examine some effects of the parameters of the problems on solution results. The results here include the effects of friction, the number of parts into which the pipe is divided, the slope of the pipe, and the effect of rate of velocity change. An example of numerical instability and inaccuracy is presented. The pipeline in Fig. 9.8 is used to demonstrate these effects.

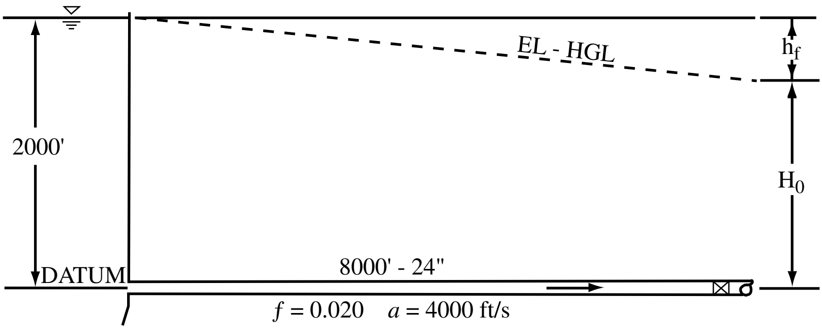


Figure 9.8 Model system for the investigation of sensitivity to system parameters.

9.3.1. THE EFFECT OF FRICTION

The effect of friction on solutions is studied by introducing initial velocities of 2.5 ft/s, 5.0 ft/s, and 10.0 ft/s into the pipeline of Fig. 9.8. We will do this for sudden valve closure. The results are listed in Table 9.1. The *increase* in pressure at the valve seems to

Table 9.1
The Effects of Friction on the Maximum Water Hammer Pressure at the Valve for Sudden Valve Closure (N = 6)

Steady Velocity ft/s	Computed H_{max} ft	h_f ft	H_0 ft	$\Delta H = -a\,\Delta V/g$ ft	$h_f + H_0 + \Delta H$ ft
2.5	2311	8	1992	311	2311
5.0	2621	31	1969	621	2621
10.0	3242	124	1876	1242	3242

be the sum of the friction loss in the pipe and ΔH from Eq. 8.8. In fact it appears that the maximum pressure occurring at the valve may be estimated by the formula

$$H_{max} \approx H_0 + h_f + \Delta H \tag{9.68}$$

Keep in mind this is an approximation which seems to work in this case but should be applied with caution to other situations.

To develop a grasp of how friction affects the results in a water hammer situation, we show in Fig. 9.9 the position of the *EL-HGL* at successive times as the wave propagates through the pipe. The pipeline of Fig. 9.8 will be used with an initial steady velocity of 10 ft/s and sudden valve closure. The increase in head ΔH propagates upstream at approximately the wave velocity, increasing the steady state head at each point by an amount ΔH. It might seem after a time L/a that the *EL-HGL* would be a line parallel to the original steady state *EL-HGL* but positioned ΔH above it. However, a chain of events occurs, beginning at the first time step, which causes the pressure head at each point in the pipeline to continue to creep upward even though the pressure wave has already passed. This happens because the fluid in the pipe is not in equilibrium. Even though the velocity is zero, there is a pressure gradient caused by the sloping *EL-HGL*. As a consequence, there is a small downstream velocity which develops to eliminate the pressure gradient and bring the system into equilibrium. This process of upward adjustment continues until it is interrupted by the returning pressure wave. At the valve the

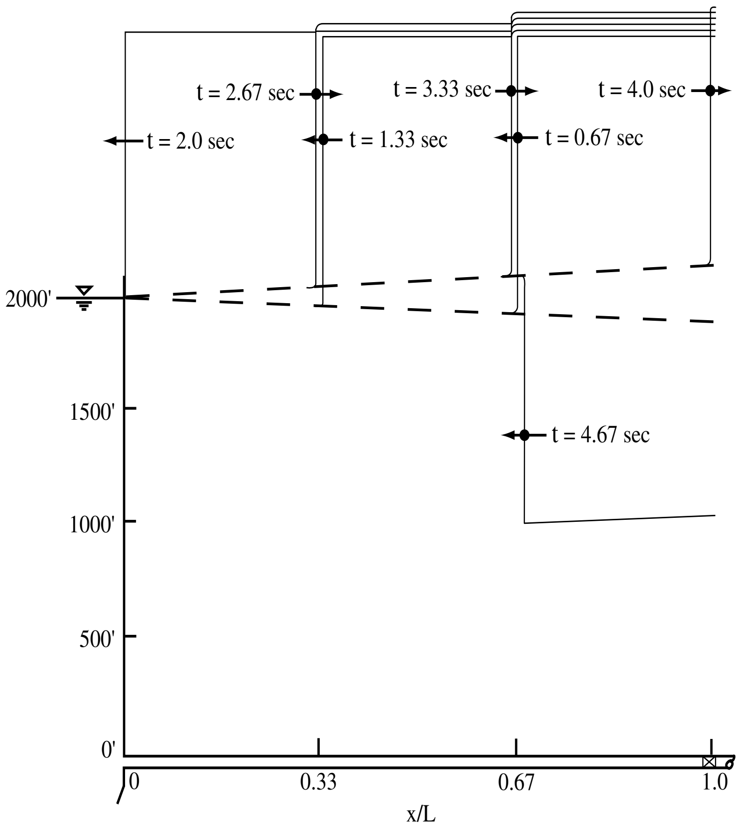


Figure 9.9 The progression of frictional effects in a single pipe with sudden valve closure.

cumulative upward pressure adjustment is equal in amount to the original steady state friction loss in the pipeline. This adjustment explains why Eq. 9.68 works in this case. Keep in mind that the purpose of this example is to create for the reader a physical feeling for the process; it is not intended to be a substitute for an analysis. It does, however, illustrate the value of a computer program in determining accurately the effects of friction.

9.3.2. THE EFFECT OF THE SIZE OF *N*

It seems reasonable to expect the accuracy of results to increase as the number of pipeline segments N increases since $\Delta s = L/N$. It is surprising then to discover that the choice of N has relatively little effect on the solution. For example, for sudden valve closure with an initial velocity of 5 ft/s in Fig. 9.8, the maximum and minimum values of H differ by less than 10 ft between solutions for $N = 3$ and $N = 18$, as shown in Table 9.2. For lower initial velocities, the difference is even less significant.

There are two points to be made here. First, except in the case of a rapidly-varying velocity, there is little to be gained by using a larger N than is necessary. Second, for a given simulation time the number of grid points and the subsequent computer execution time varies as N^2. However, we must also select Δt sufficiently small to capture accurately such time-varying boundary conditions as the movement of a valve, and smaller values for Δt are directly linked to larger values of N.

Table 9.2
Effects of N-value on Pressure Head (ft) at the Valve for Rapid and Slow Velocity Change (ΔV = 5 ft/s)

	Sudden Valve Closure		Valve Closure in $4L/a$ sec	
N	H_{max}	H_{min}	H_{max}	H_{min}
3	2611	1417	2288	1969
6	2616	1412	2290	1969
18	2619	1409	2292	1969

9.3.3. THE EFFECT OF PIPE SLOPE

Flows in pipelines ranging in slope over $\pm 25\%$ were simulated to determine the effect of slope. Results were nearly identical for both extremes of slope. While the slope of the pipe should not be ignored (it is needed in computing the pressure head, anyway), the reader should be comfortable when "smoothing" a pipeline system profile to reduce the number of series pipes to a manageable number of constant-slope pipes. By this means we can use the series pipe program of Chapter 10 to obtain accurate estimates of pressure head along the pipeline while still exercising control over Δs and Δt.

9.3.4. NUMERICAL INSTABILITY AND ACCURACY

Earlier it was stated that Δt should be limited in size to insure that points L and R in Fig. 9.6 remain between the grid points Le and Ri at all times. If not, numerical instability was presumed to occur as well as inaccuracy in the computed results.

To demonstrate these two problems, we again use the pipeline of Fig. 9.8 with sudden valve closure. While Δs was computed and held constant, Δt was assigned four different values to illustrate the two problems. The first Δt was chosen so that L and Le coincide, a case requiring minimum interpolation and leading to maximum accuracy. We will refer to this value as Δt_0. Higher values of Δt would cause L to move outside Le, leading to numerical instability. Lower values of Δt would lead to poorer linear interpolations, resulting in inaccurate results. The four computer simulations are identified in the legend in Fig. 9.10.

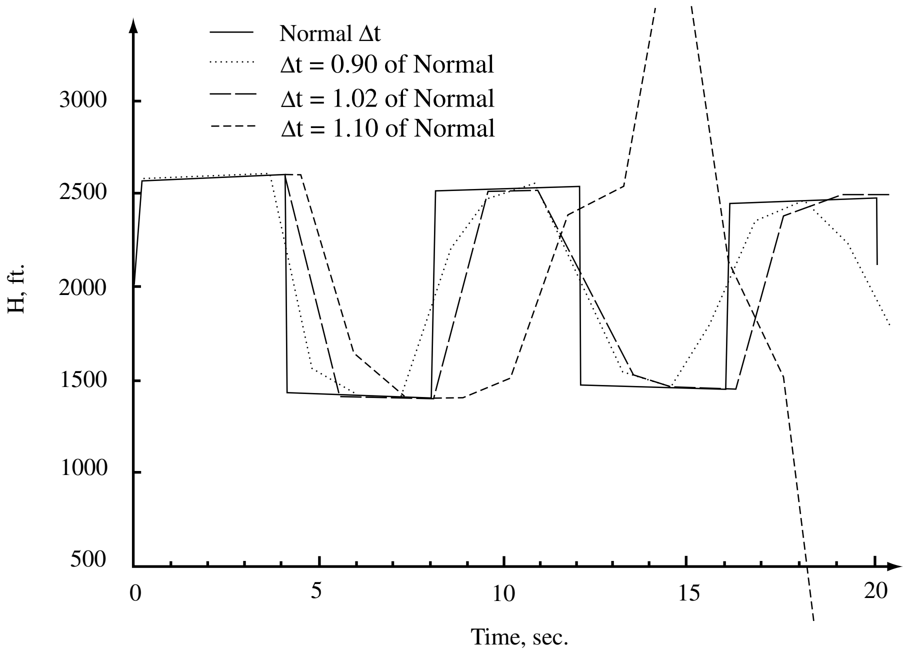


Figure 9.10 Numerical instability and inaccuracy in a single pipe with sudden valve closure.

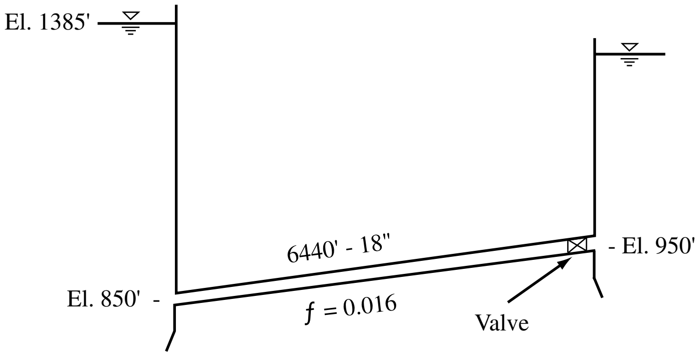
For the normal value of Δt, the plot shows the typical nearly-square wave from Chapter 7. When Δt is reduced to 90% of normal, the effect is to round off the sharp corners and distort the timing of events, a diffusive process. At 102% of the normal Δt, we see a gradual deterioration of the simulation with time. Finally, at 110% of the normal Δt, the resulting numerical stability is strikingly evident.

In summary, it is important to locate the points L and R inside of and as close to the grid intersections as possible. This minimizes interpolation errors and retains the numerical accuracy of the simulation. It is even more important to insure that the points L and R never are outside the grid points. This care will prevent inaccuracies associated with numerical instability which could be insidiously present in calculations, especially where the duration of the simulation is not so long that the more dramatic effects of instability can become obvious.

9.4 PROBLEMS

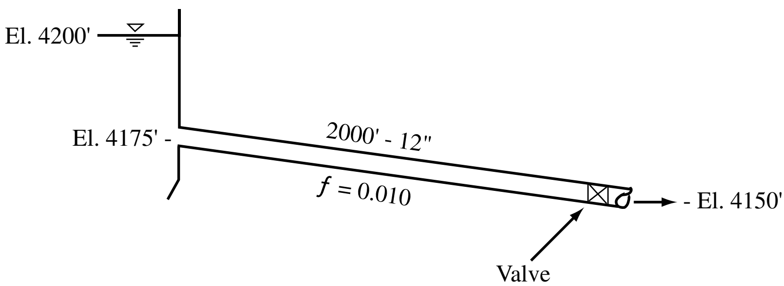
9.1 The ductile iron pipeline shown below is 6440 ft long, 18 inches inside diameter, with a wave speed of 3220 ft/s. It carries 3970 gal/min of water with $f = 0.016$. The valve at the downstream end can cause the velocity to vary linearly with time.

Calculate the maximum and minimum pressure heads and their locations for closure times of 0, 4, 8, and 12 sec. Use PROG1 with $NPARTS = 10$ in your analysis.



9.2 A 12-inch PVC pipeline with an inside diameter of 12.091 in, a wall thickness of 0.311 in, and an f of 0.010 carries 2150 gal/min of irrigation water from a supply reservoir to an irrigation network. A valve which can vary the velocity linearly with time is at the downstream end of the pipeline.

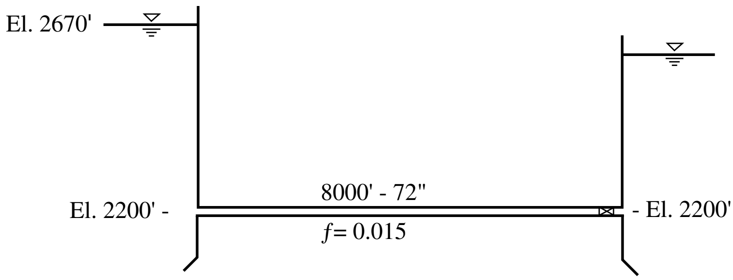
Find the minimum time of valve closure that can be used if no negative pressure can be permitted in the line. Neglect any effect of the system downstream of the valve. Use PROG1 with Case (c) restraint for the pipeline, and divide it into 10 parts for the analysis.



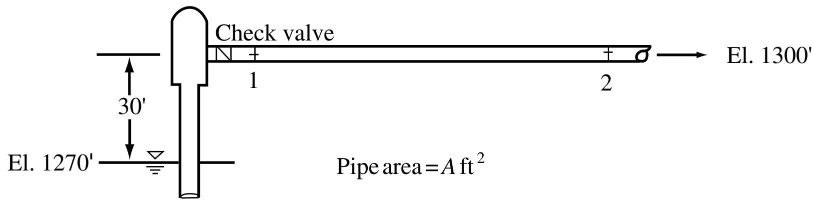
9.3 The pipeline of Problem 8.7 has $f = 0.015$ and carries water between two reservoirs, as shown below. The valve at the downstream end of the pipe is capable of varying the velocity linearly with time.

(a) Can the valve be closed suddenly without causing a negative pressure in the pipeline?
(b) What would the maximum and minimum pressure heads in the line be if the valve were closed in $4L/a$ seconds?

Use PROG1 with $NPARTS = 10$ in your analysis.



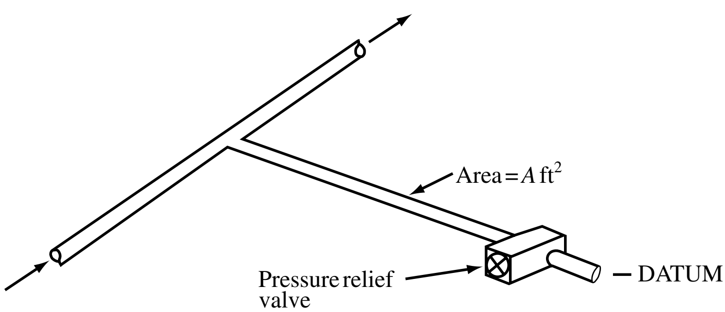
9.4 The characteristics of the pump below can be approximated by $h_p = A_p Q + B_p$. Using the C^- equation in the form $H_P = C_1 + C_2 V_P$, write the boundary conditions for the pump, using a datum of sea level, so that you provide the equations needed to determine H_P and V_P.



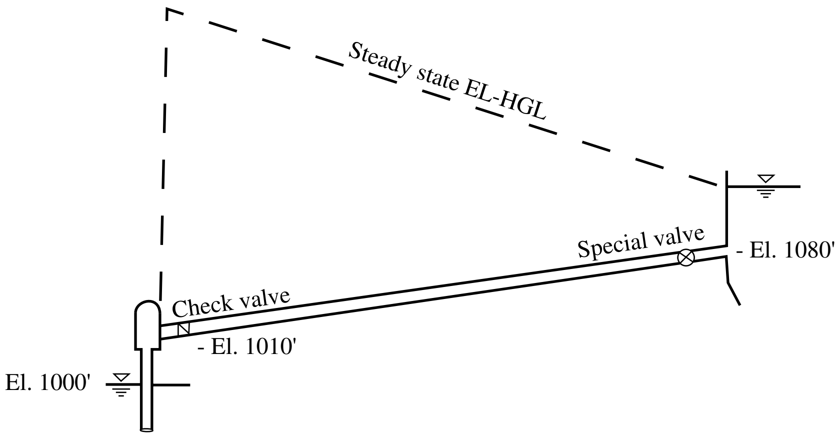
9.5 The lateral line extending from the main pipeline in the sketch below ends in a pressure relief valve which will open if the pressure exceeds a maximum value p_{max}. When the valve is open, the discharge from the valve into the atmosphere is

$$Q\left(\text{ft}^3/\text{s}\right) = K\sqrt{\text{pressure head (ft) in pipe just upstream of valve}}$$

A water hammer analysis requires that the boundary condition at the valve be found. Your task is to combine the above equation with the C^+ equation $V_P = C_1 - C_2 H_P$ to develop an equation for V_P at the upstream side of the valve.



9.6 An eight-stage Johnston 14 BC turbine pump (the pump characteristic diagram is in Appendix B) with 11-inch impellers pumps 850 gal/min through the pipeline below. The welded steel pipeline is one mile long, 6 in inside diameter with a wall thickness of 0.135 in. Case (b) restraint applies and $f = 0.016$ may be used. A special valve at the downstream end of the line reduces the discharge from 850 gal/min to 250 gal/min linearly with time.



The engineer in charge is considering four separate closure times of 3, 6, 9, and 12 seconds and wants to know the maximum and minimum pressures occurring in each case. You are to complete the following tasks:

(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A_p' Q^2 + B_p' Q + C_p'$$

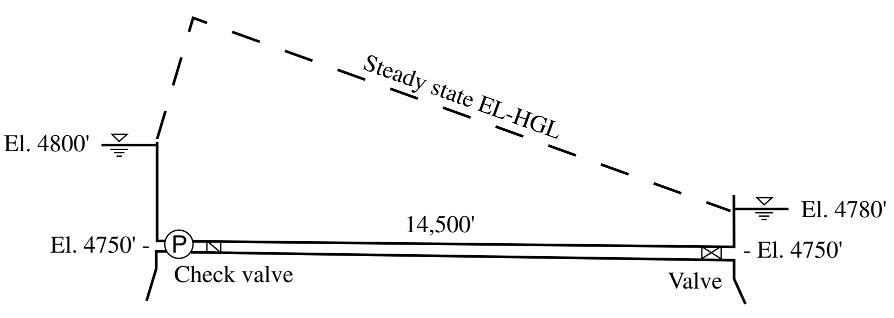
(b) Plot this equation on the pump characteristic diagram to demonstrate how well it fits over the range from 0 to 1200 gal/min.

(c) Calculate the wave speed in the pipe.

(d) For the four closure times, find the maximum and minimum pressures at the valve and the check valve and the times they occur.

(e) Plot pressure head as a function of time at both locations for a time at least $4L/a$ seconds past the last valve movement. Use PROG1P with *NPARTS* = 5 in your analysis.

9.7 The T-30 Transite pipe of Problem 8.3 is installed in a pumping system. To determine f for the pipe, assume it is hydraulically smooth. The pipeline configuration is shown below. The pump is a Johnston 20 CC single-stage turbine pump (see Appendix B) with a 15 3/4-in impeller. The pump runs at constant speed while the downstream valve is closed to decrease the velocity at the valve linearly with time to 2 ft/s in 15 sec.



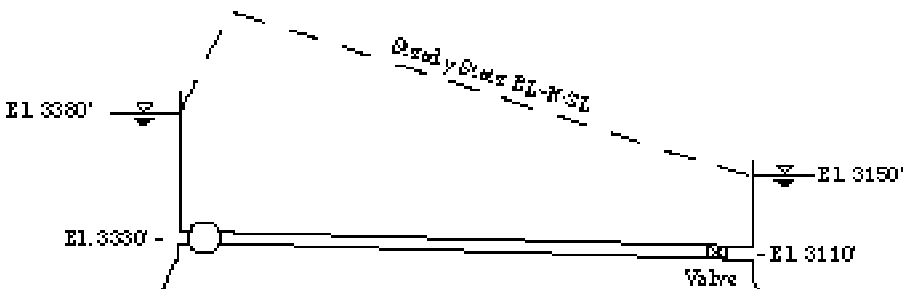
Complete the following tasks:

(a) Develop with program PUMPC a parabolic equation for the pump characteristic of the form

$$h_p = A_p' Q^2 + B_p' Q + C_p'$$

(b) Plot this equation on the pump characteristic diagram to demonstrate how well it fits over the range from 0 to 7000 gal/min.
(c) Calculate the wave speed in the pipe.
(d) Find the maximum and minimum pressures at the valve and the check valve and the times they occur.
(e) Plot pressure head as a function of time at both locations for a time at least $4L/a$ seconds after the last valve movement. Use PROG1P with *NPARTS* = 5.

9.8 A single-stage Johnston 18 DC turbine pump (see Appendix B) with a $13\frac{3}{16}$-in impeller is used to increase the discharge in the gravity flow pipeline below. The welded steel line is 12 in inside diameter and three miles long with $f = 0.014$. The wall



thickness is 0.179 in and Case (b) restraint applies. While the pump runs at constant speed, the downstream valve closes in a manner that causes the discharge in the pipe to decrease linearly with time to zero in 30 sec. If the pressure head at the pump exceeds the shutoff head during the shutdown procedure, flow backward through the pump will occur according to the relation

$$Q(\text{gal/min}) = 6.0\left(H_{P_1} - H_{shutoff}\right)$$

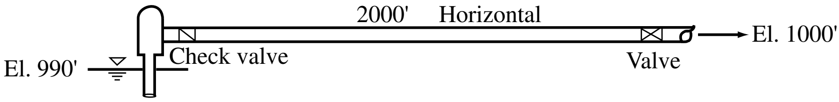
Your tasks:

(a) Calculate the steady state discharge and velocity in the pipeline.
(b) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A'_p Q^2 + B'_p Q + C'_p$$

(c) Plot this equation on the pump characteristic diagram to demonstrate how well it fits over the range from 0 to 6000 gal/min.
(d) Calculate the wave speed in the pipe.
(e) Formulate the new boundary conditions at the pump.
(f) Modify and recompile PROG1P with the revised boundary conditions.
(g) Find the maximum and minimum pressures at the valve and the pump and the times they occur.
(f) Plot pressure head versus time at both locations for a time extending at least $4L/a$ seconds after the last valve movement. Use PROG1PR with *NPARTS* = 5 in your analysis.

9.9 The pump in the horizontal pipeline below is a two-stage Johnston 18 DC pump (see Appendix B) with $13\frac{3}{4}$-in impellers that provides a flow of 4600 gal/min in the line. The valve at the downstream end is closed rapidly so that the velocity decreases linearly with time to 20% of its steady state value in 2 sec. The pipe is 24 in inside diameter with $f = 0.022$ and a wave speed of 3500 ft/s.



Your tasks:

(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

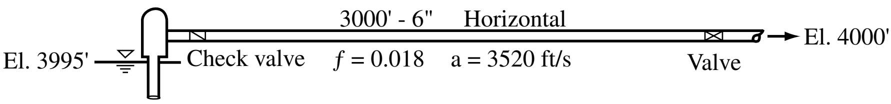
$$h_p = A'_p Q^2 + B'_p Q + C'_p$$

(b) Plot this equation on the pump characteristic diagram to how it fits over the range from 0 to 6000 gal/min.

(c) Find the maximum and minimum pressures at the valve and the check valve and the times they occur.

(d) Plot pressure head as a function of time at both locations for at least $4L/a$ seconds past the end of valve movement. Use program PROG1P with $NPARTS = 4$ in your analysis.

9.10 The pump in this pipeline is a four-stage Johnston 14 BC pump (see Appendix B)



with an 11-in impeller. Under steady flow conditions with the valve open, the discharge is 850 gal/min. The valve will be closed so that the velocity at the valve decreases linearly with time to zero.

The engineer in charge is considering three separate closure times of 3, 5, and 7 seconds and wants to know the maximum and minimum pressures occurring in each case. Your assignment is to complete these tasks:

(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

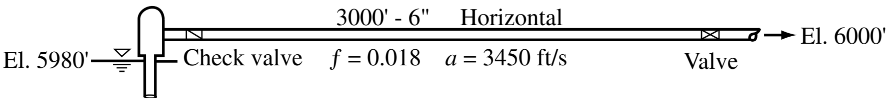
$$h_p = A'_p Q^2 + B'_p Q + C'_p$$

(b) Plot this equation on the pump characteristic diagram to see how it fits over the range from 0 to 1200 gal/min.

(c) For the three closure times find the maximum and minimum pressures at the valve and the check valve and the times they occur.

(d) Plot pressure head as a function of time at both locations at least $4L/a$ seconds past the ending of valve movement. Use PROG1P with $NPARTS = 5$.

9.11 The pump in the pipeline below is a five-stage Johnston 14 BC pump (see Appendix B) with an 11-inch diameter impeller. The steady state discharge in the system



is 850 gal/min. The valve at the downstream end of the line moves so the velocity at the valve decreases linearly to 4.0 ft/s in 4 sec.

Your tasks:

(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A_p' Q^2 + B_p' Q + C_p'$$

(b) Plot this equation on the pump characteristic diagram to show how it fits over the range from 0 to 1200 gal/min.

(c) Find the maximum and minimum pressures at the valve and the check valve and the times they occur.

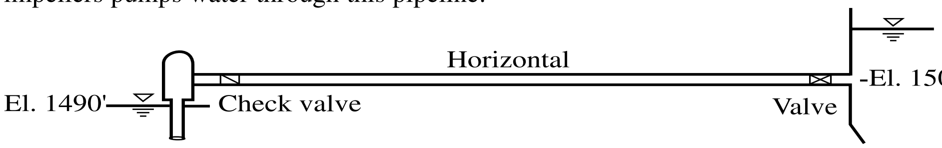
(d) Plot pressure head as a function of time at both locations for at least $4L/a$ seconds after the ending of valve movement. Use program PROG1P with $NPARTS = 5$.

9.12 Solve Problem 9.11 but

(a) only reduce the velocity to 5.0 ft/s;

(b) use a wave speed of 3750 ft/s.

9.13 A five-stage Johnston 12 ES turbine pump (see Appendix B) with $7\frac{13}{16}$-in impellers pumps water through this pipeline:



With steady flow and the valve open, the flow is 1600 gal/min. The welded steel line has $f = 0.017$, is 5000 ft long and has a 10-in outside diameter with a wall thickness of 0.135 in. The pipe is installed so that Case (b) restraint most nearly applies. The special valve at the downstream end permits the velocity to be varied linearly with time. Now the engineer wishes to reduce the steady state flow from 1600 gal/min to 250 gal/min. Two closure times of 3 sec and 6 sec are under consideration. To choose between the two, it is desired to know the extreme pressures developed under each closure time.

Your tasks are as follows:

(a) Develop with program PUMPC a parabolic equation for the pump characteristics of the form

$$h_p = A_p' Q^2 + B_p' Q + C_p'$$

(b) Plot this equation on the pump characteristic diagram to see how it fits over the range from 0 to 2000 gal/min.

(c) Calculate the wave speed in the pipe.

(d) Find the maximum and minimum pressures at the valve and the check valve and the times they occur for both closure times.

(e) Plot pressure head as a function of time at both locations for at least $4L/a$ seconds past the last valve movement. Use program PROG1P with $NPARTS = 5$.

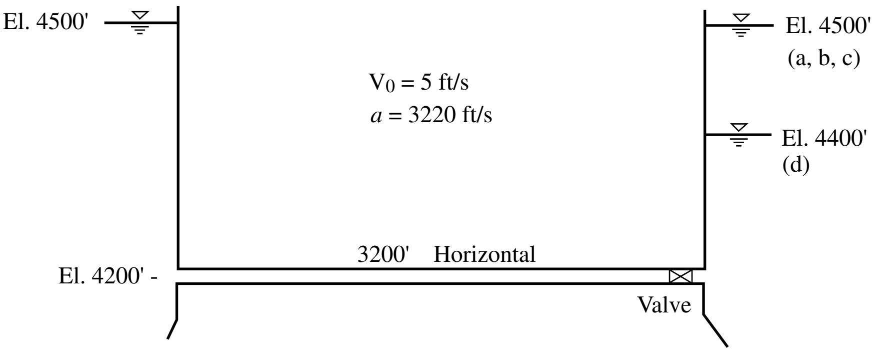
9.14 Answer the first three questions *neglecting friction* in the pipeline shown below.

(a) What is the maximum pressure head which will result from sudden valve closure?

(b) What is the minimum pressure head which will result from sudden valve closure?

(c) Sketch pressure head vs. time at the center of the 3220-ft pipeline for the first $4L/a$ seconds.

(d) If there is a 100-ft *friction loss* in the pipeline, what would be the maximum pressure head resulting from sudden valve closure?

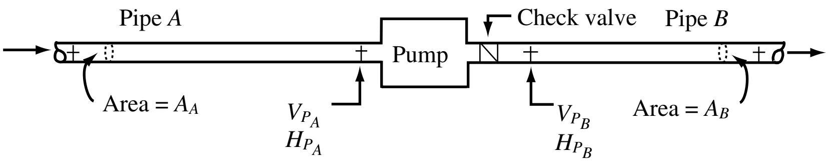


9.15 The booster pump in the pipeline below provides a head increase h_p which can be described by

$$h_p = A_p^{'} Q^2 + B_p^{'} Q + C_p^{'}$$

Develop a set of boundary condition equations which can be solved for V_{P_A}, V_{P_B}, H_{P_A}, and H_{P_B} under all water hammer conditions. Then solve this set of equations to give individual formulas for the calculation of each of these variables under any condition. Assume no negative pressures will occur.

$$C^+ : \ V_{P_A} = C_1 - C_2 H_{P_A} \qquad C^- : \ V_{P_B} = C_3 + C_4 H_{P_B}$$

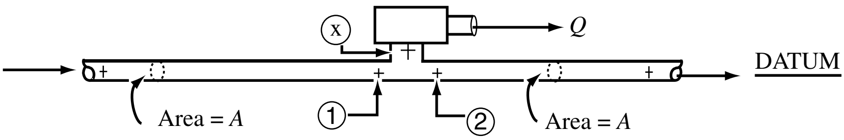


9.16 The surge relief valve in the pipeline shown atop the next page is designed to open when the pressure at the valve exceeds a specified value. When this occurs, the valve opens suddenly and discharges water into the atmosphere according to the equation

$$Q\left(\text{ft}^3/\text{s}\right) = C\sqrt{\text{Pressure head at } x}$$

Using the centerline of the horizontal pipe as the datum, develop an equation for H_P $\left(H_P = H_{P_1} = H_{P_2} = H_{P_x}\right)$ when the surge valve is open. The C^+ and C^- equations are

$$C^+ : \ V_P = C_3 - C_4 H_P \qquad C^- : \ V_P = C_1 + C_2 H_P$$



9.17 The pipeline shown below discharges into the atmosphere just downstream of the gate valve. The surge relief valve just upstream of the gate valve will open when the pressure head in the pipe exceeds H_{max}. When this occurs, the relief valve opens suddenly, and the discharge is then given by

$$Q\left(\mathrm{ft}^3/\mathrm{s}\right) = K\sqrt{\text{Pressure head in pipe at surge valve}}$$

Your tasks:

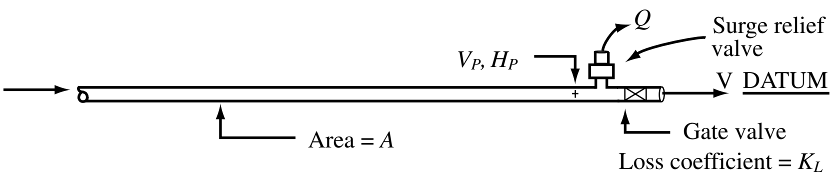
(a) Write a set of equations which can be used to solve for all the variables if
  (1) the surge valve is not open,
  (2) both the surge valve and the gate valve are open,
  (3) the surge valve is open and the gate valve is closed.

(b) Arrange the equations for each condition so they involve H_P only.

(c) For condition (2) above, solve the equation for H_P.

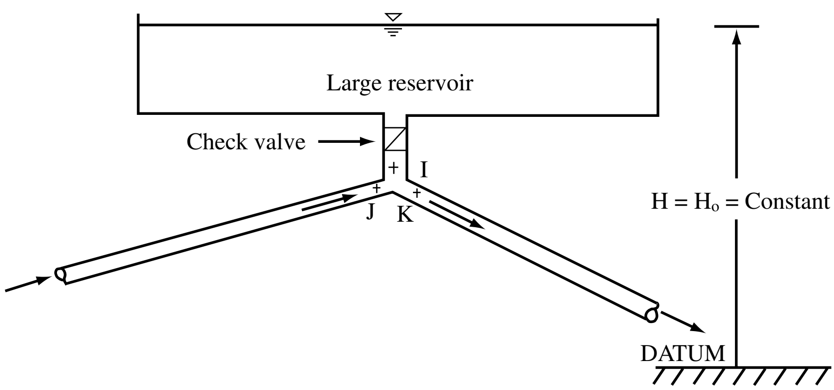
(d) Explain how you would decide which of the conditions would apply.

The C+ equations is $C^+ :\ V_P = C_3 - C_4 H_P$



9.18 The large reservoir is a one-way surge tank connected to the pipeline through a very short pipe I with a check valve. The check valve in the short pipe prevents flow from the pipeline into the reservoir. When H in the pipeline at the junction drops below the value of H_0 in the reservoir, the check valve opens and flow enters the pipeline from the reservoir. The equation for discharge from the reservoir into the pipeline is

$$Q = KA\sqrt{2g\left(H_0 - H_{P_I}\right)}$$



(a) How many unknowns exist at the junction? List them.
(b) Write down the independent equations that contain these unknowns.
(c) From these equations develop an equation containing only H_P as an unknown.
(d) Solve the equation for H_P, explaining how you would select the proper sign in the quadratic solution. Also explain how you would know whether the check valve is open.

All pipes have the same area A. All pipes have the same wave speed a. The C^+ and C^- equations are C^+: $V_P = C_3 - C_4 H_P$ $\quad$ C^-: $V_P = C_1 + C_2 H_P$.