CHAPTER 10

PIPE SYSTEM TRANSIENTS

A natural extension of the analysis of single-pipe systems is to more elaborate pipe systems. In practice, design situations almost always confront systems that are larger and more complex than single, straight pipelines. We now have already been introduced to most of the analysis techniques that are needed for these systems, so we can immediately begin with the simplest type, series pipe systems. We will then move on to branching pipe systems and also examine how to represent actual valve behavior in a realistic way, as opposed to the artificial linear-varying-velocity approach used in Chapter 9. This chapter will prepare us to analyze gravity-flow pipeline transient situations successfully.

10.1 SERIES PIPES

In a series pipe system each pipe (pipeline segment) in the series carries the same steady-flow discharge, but each pipe may have its own velocity, diameter, wave speed, and so on. Each segment must be straight and have constant properties and geometry. These restrictions include the very important case of a single, constant-diameter pipe which can be divided into segments, thereby creating a series pipe system, in order to analyze a pipeline with a profile containing changes in grade.

10.1.1. INTERNAL BOUNDARY CONDITIONS

The method of characteristics solution for each pipe in series proceeds as in Chapter 9. The interior nodes are treated with equations similar to Eqs. 9.64 and 9.65. Boundary conditions at the upstream and downstream ends are again represented by a suitable combination of the C^+ and C^- equations along with a reservoir, valve, or other special condition. The principal difference is the need now for *internal boundary conditions* at the series pipe junctions.

Figure 10.1 portrays a typical series pipe internal boundary condition. There are two points, P_1 and P_2, one on each side of the junction, which are very close together and

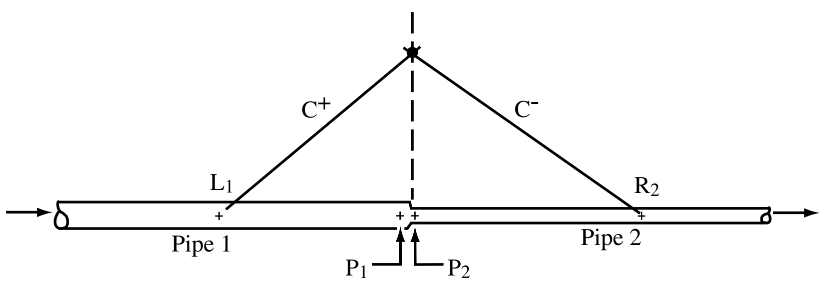

Figure 10.1 Boundary conditions at a typical series pipe junction.

represent the location of four unknown quantities, H_{P_1}, H_{P_2}, V_{P_1}, and V_{P_2}, which must be calculated. Therefore, we must find four equations to solve for these unknowns. For the upstream pipe the C^+ equation can be written, from Eq. 9.53, as

$$V_{P_1} = V_{L_1} + \frac{g}{a_1} H_{L_1} - \frac{f_1 \Delta t}{2D_1} V_{L_1} \left| V_{L_1} \right| + \frac{g}{a_1} \Delta t V_{L_1} \sin\theta_1 - \frac{g}{a_1} H_{P_1} \tag{10.1}$$

or in short form

$$V_{P_1} = C_3 - C_4 H_{P_1} \tag{10.2}$$

Similarly, for the downstream pipe the C^- equation, Eq. 9.54, yields

$$V_{P_2} = V_{R_2} - \frac{g}{a_2} H_{R_2} - \frac{f_2 \Delta t}{2D_2} V_{R_2} \left| V_{R_2} \right| - \frac{g}{a_2} \Delta t V_{R_2} \sin\theta_2 + \frac{g}{a_2} H_{P_2} \tag{10.3}$$

or in short form

$$V_{P_2} = C_1 + C_2 H_{P_2} \tag{10.4}$$

It is clear from Eqs. 10.2 and 10.4 that we have four unknowns in the two equations. The two additional required equations are obtained from the conservation of mass and work-energy principles. Assuming there is a negligible mass of fluid between points 1 and 2, conservation of mass gives

$$V_{P_1} A_1 = V_{P_2} A_2 \tag{10.5}$$

Applying the work-energy equation between these same two points 1 and 2, again with negligible mass between the points, and neglecting the difference in velocity heads and any local loss of head across the junction,

$$H_{P_1} = H_{P_2} \tag{10.6}$$

If the head loss at the junction were significant (for example, a closing valve or a pressure reducing valve), then the head loss across the valve must be included in Eq. 10.6.

Solving Eqs. 10.2, 10.4, 10.5, and 10.6 simultaneously leads to the following equations for the heads H at the junction:

$$H_{P_1} = H_{P_2} = \frac{C_3 A_1 - C_1 A_2}{C_2 A_2 + C_4 A_1} \tag{10.7}$$

Once these heads have been computed, the velocities can be found by back-substitution into Eqs. 10.2 and 10.4.

10.1.2. SELECTION OF Δt

In the previous section we presumed that our chosen C^+ and C^- characteristics intersected at the pipe junction, as Fig. 10.1 shows. This is rarely true because the slope of each characteristic line depends on the wave speed and fluid velocity in the pipe and the horizontal location of the node depends on the number of sections into which the pipe is divided. That is to say, if we extend the C^+ and C^- characteristics from adjacent nodes at a particular instant in time, they usually will not intersect at the junction. So we need a strategy to overcome this problem. If we are successful, then Eqs. 10.2, 10.4, and 10.7 will apply, as before.

We begin by rewriting Eq. 9.67 for deriving the value of Δt:

$$\Delta t = \frac{\Delta s}{\max|V+a|} = \frac{\Delta s}{(V+a)} = \frac{L}{N(V+a)} \tag{10.8}$$

For a given N it is clear from Eq. 10.8 that we will usually find a different Δt for each pipe in a series. Figure 10.2 illustrates this situation for two typical pipes in series. At a

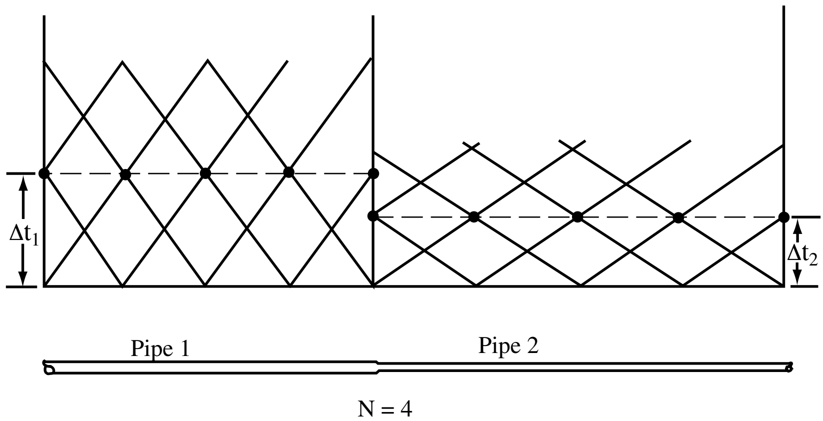

Figure 10.2 The s-t plane for a two-pipe system with equal N's.

pipe junction the result is a pair of characteristic lines that do not meet at the common end of the two pipes. However, we can bring the two characteristics closer to meeting at the junction by choosing a different N for each pipe; however, because N must be an integer, we cannot guarantee this will work. In fact, the chance of success is so small that we must discard this approach.

Another approach which shows more promise is demonstrated in Fig. 10.3. We reduce Δt for all pipes to that value for the pipe with the smallest Δt, called the "controlling"

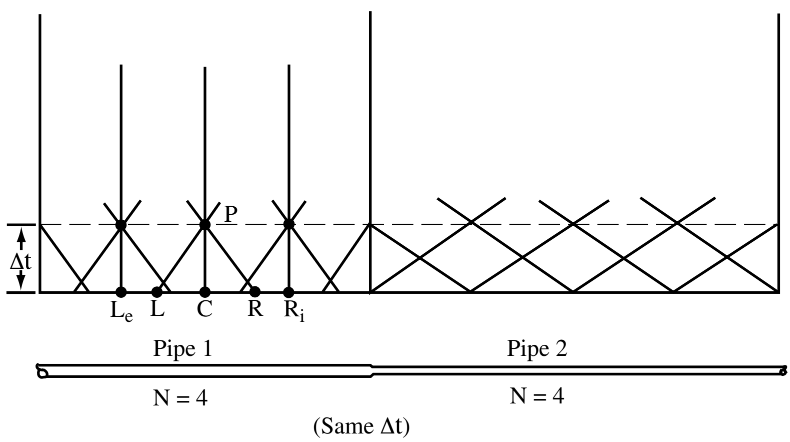

Figure 10.3 The s-t plane with equal Δt's and large interpolations.

pipe. This will force the characteristics to meet at the pipe junction. However, the other end of each characteristic line will no longer intersect at the rectangular grid points. To overcome this problem, we adopt the interpolation procedures developed in Chapter 9. Unfortunately, as can be seen from Fig. 10.3, we may then need some very large interpolations, far too large to assure the accuracy of the numerical analysis.

A technique does exist to reduce this interpolation error while still causing the characteristics to meet at the junction. Here we increase N in all pipes which initially had Δt's larger than that of the "controlling" pipe. As we increase each N, the interpolation gets successively smaller. Eventually, as shown in Fig. 10.4, we obtain a situation in each pipe where a further increase in N will cause the characteristic lines to intersect

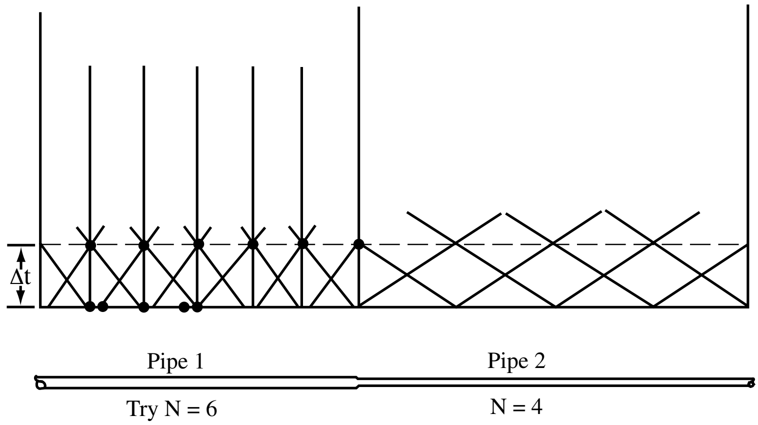

Figure 10.4 The s-t plane with variable N values and minimal interpolation.

outside the rectangular grid point. We stop increasing N just before this happens because this represents the optimum interpolation situation. We now have each of the series pipes divided into a different number of sections with minimum interpolation, but all pipes now have a common Δt.

These values N_i for each pipe i are computed by the recipe

$$N_i = \frac{L_i}{\Delta t_{\min}\left(V_i + a_i\right)} \tag{10.9}$$

where Δt_{min} is associated with the controlling pipe. Integer truncation in the computer will give the proper maximum N_i for each of the pipes in series. For the controlling pipe the original N is retained.

Because the minimization of interpolation error is so significant in the preservation of accuracy, we actually compute the amount of interpolation in the computer program for each series pipe. If the amount of interpolation remains too large, there are ways to reduce it. The easiest way is to increase the base value of N because this will cause all pipes to be divided into more parts and may lead to less interpolation. The only disadvantage is a substantial increase in the amount of computation, which is caused by the larger number of grid points.

Probably it is best to pursue some preliminary computations to determine which pipes in the series system produce the most trouble. Then move a few internal junctions (change the length of individual pipe sections without changing the total length of the system) to generate segment lengths which cause less interpolation error. Another popular technique is to adjust the wave speed to reduce interpolation error. The rationale behind this approach is that the wave speed cannot be determined very precisely anyway, so why not use this uncertainty to improve the numerical simulation accuracy? Karney and Ghidaoui (1997) recently reviewed the interpolation issue and propose a new, more flexible technique. However, they also state that no one method is best in all cases. Whichever technique minimizes interpolation error in a particular application should be used.

Another "trick" which may work well in conjunction with the previous technique is to "simplify" the pipeline profile. Often a pipeline has literally dozens of changes in grade, and following these grades precisely would require a series pipe analysis with dozens of pipe segments. This problem formulation, along with the minimization of interpolation error, could lead to prohibitive computation times. However, since experience has shown that pipeline slope has little effect on water hammer pressures, the actual pipeline profile can be replaced with a model containing only a few segments and a simplified profile without seriously affecting the results of the analysis. This feature permits short pipe segments to be combined with longer segments and/or several short segments to be combined into a single long segment. This can be done with concurrent attention to opportunities to minimize interpolation error. Care should be taken, however, to attempt to include the high and low points along the pipeline as junctions, since they tend to be the critical pressure points. With some experience and care in approximating a pipeline profile, the user will be able to analyze a system accurately with a minimum of computation.

10.1.3. THE COMPUTER PROGRAM

The computer program PROG2 for the solution of series pipe problems is included on the CD. It is an extension of PROG1 which accounts for the differing properties in the series pipes. The input parameters are again defined with COMMENT statements in the program listing, and the user must develop the steady-state input conditions outside the program.

In this program a double subscripting of variables is required since parameters now vary from pipe to pipe. Some caution is appropriate in choosing a base value for N. If a relatively short pipe occurs in the system, it will probably be the controlling pipe and produce the minimum Δt. If other considerably longer pipe segments exist in the system, they will have a large number of sections. A short preliminary computer run ($TMAX = 2\Delta t$) with a small N will permit you to examine the effect of parameters on the analysis and see how much interpolation, which is displayed, is required.

This program assumes a reservoir at the upstream end and a closing valve and reservoir at the downstream end. Head loss coefficients must be entered for the valve which can be closed at two different rates. The program also provides a printer plot of maximum and minimum pressure heads along the pipeline, a printer plot of pressure head vs. time for up to four points along the pipeline, a table of pressure head and velocity vs. time for the same four points, and a data file which can be read by an external graphing program to make traditional graphs of pressure head vs. time. The subroutines PGRAPH and PROFILE are used to accomplish these tasks; input data requirements and input parameter descriptions are included as COMMENT statements in the subroutine source listings.

To demonstrate the use of this program and its various features, we now look at a simple example. Though we have not yet discussed how real valve input data is developed, we include such a valve treatment in this example; the details of modeling the valve head loss are included later in this chapter.

Example Problem 10.1

The series pipe system shown on the next page conveys 800 gal/min and has a valve at the downstream end which closes at a uniform rate until it is fully closed after 5 sec. Find the maximum and minimum pressures in the system and their points and times of occurrence. Also provide printer plots of the extreme pressure heads along the pipeline and pressure head vs. time at the valve and at the pipe junction, tables of pressure head vs. time at these two points, and a traditional plot of pressure head vs. time at the two points.

The input data file to accomplish these tasks follows:

```
DEMONSTRATION OF PROGRAM NO. 2 - INPUT DATA FILE "EP101.DAT"
RESERVOIR UPSTREAM, VALVE CLOSING LINEARLY IN 5 SEC DOWNSTREAM
&SPECS NPIPES=2,NPARTS=5,IOUT=10,QZERO=800.,HZERO=1780.,ZEND=1260.,
       HATM=32.,TMAX=15.00,DTNEW=0.,TC1=0.,TC2=5.00,PC1=100.,
       PFILE=T,HVPRNT=T,PPLOT=T,GRAPH=T,RERUN=F/
0.   .0167  .0313  .0556  .100  .1787  .3333  .625  1.25  2.50  5.27
12.00  3000.  .015  3000.  1280.
 8.00  2000.  .018  2800.  1210.
&GRAF NSAVE=2,IOUTSA=2,PIPE=2,2,0,0,NODE=1,999,0,0/
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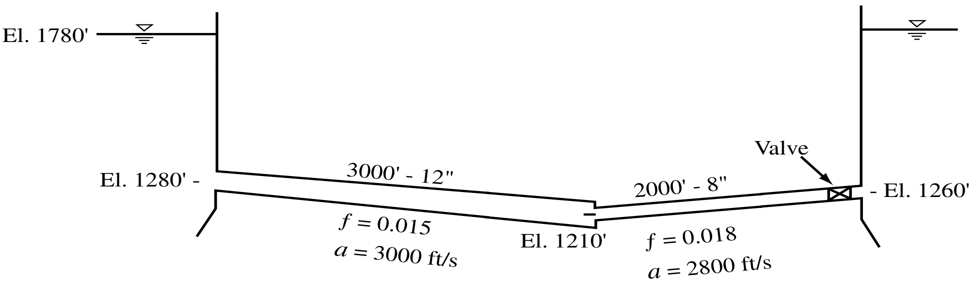


The initial portion of the output file follows. The printout is too lengthy to reproduce in its entirety but is contained on the CD as OUT101 and may be reviewed with the text editor. The page accompanying the abbreviated printout shows the traditional graph of pressure head vs. time as produced by Axum software from Trimetrix. This software reads the plot file created at the end of execution when GRAPH=T is in the input data (see above data file).

```
DEMONSTRATION OF PROGRAM NO. 2 - INPUT DATA FILE "IP101.DAT"
RESERVOIR UPSTREAM, VALVE CLOSING LINEARLY IN 5 SEC DOWNSTREAM

                          INPUT DATA
                          ----------

                         IOUT =  10
                       NPARTS =   5
                       NPIPES =   2

                        QZERO =  800.0 GPM
                        HZERO = 1780.0 FT

                         ZEND = 1260.0 FT
                         HATM =   32.0 FT

                         TMAX =  15.00 SEC
                         DELT =   .143 SEC
                          TC1 =    .00 SEC
                          PC1 = 100.00 PERCENT OPEN
                          TC2 =   5.00 SEC, VALVE IS CLOSED
```

VALVE LOSS COEFFICIENTS

% OPEN	1.0/KL
0.	.000E+00
10.	.167E-01
20.	.313E-01
30.	.556E-01
40.	.100E+00
50.	.179E+00
60.	.333E+00
70.	.625E+00
80.	.125E+01
90.	.250E+01
100.	.527E+01

PIPE INPUT DATA

PIPE	DIAM,IN	LENGTH,FT	WAVE SPD,FT/S	PIPEZ,FT	F	VEL,FT/S
1	12.00	3000.0	3000.	1280.	.0150	2.27
2	8.00	2000.0	2800.	1210.	.0180	5.10

PIPE	DELT,SEC	PARTS	SINE	L/A,SEC	INTERPOLATION
1	.200	7	-.02333	1.00	.002
2	.143	5	.02500	.71	.002

PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS OF TIME

T = .000 SEC

	X	HEAD,FT	H,FT	V,FT/S	X	HEAD,FT	H,FT	V,FT/S
PIPE 1	.000	500.	1780.	2.27	.143	509.	1779.	2.27
	.286	519.	1779.	2.27	.429	528.	1778.	2.27
	.571	538.	1778.	2.27	.714	547.	1777.	2.27
	.857	557.	1777.	2.27	1.000	566.	1776.	2.27
PIPE 2	.000	566.	1776.	5.10	.200	552.	1772.	5.10
	.400	538.	1768.	5.10	.600	523.	1763.	5.10
	.800	509.	1759	5.10	1.000	495.	1755.	5.10

T = 1.426 SEC

	X	HEAD,FT	H,FT	V,FT/S	X	HEAD,FT	H,FT	V,FT/S
PIPE 1	.000	500.	1780.	2.27	.143	509.	1779.	2.27
	.286	519.	1779.	2.27	.429	528.	1778.	2.27
	.571	538.	1778.	2.27	.714	547.	1777.	2.27
	.857	557.	1777.	2.27	1.000	566.	1776.	2.27
PIPE 2	.000	566.	1776.	5.10	.200	552.	1772.	5.10
	.400	538.	1768.	5.10	.600	524.	1764.	5.10
	.800	509.	1759.	5.10	1.000	495.	1755.	5.10

```
                        **************************
                        * TABLE OF EXTREME VALUES *
                        **************************

           X     MAX HEAD  TIME    MIN HEAD  TIME    MAX H    MIN H
         -----   --------  ----    --------  ----   ------   ------
PIPE 1
          .000     500.0   15.1      500.0   15.1    1780.    1780.
          .143     746.9    6.7      346.4   10.1    2017.    1616.
          .286     777.7    6.6      341.1   10.1    2038.    1601.
          .429     794.1   11.8      332.1   15.1    2044.    1582.
          .571     820.4   11.8      280.1   15.1    2060.    1520.
          .714     834.2   11.8      289.1   15.1    2064.    1519.
          .857     858.7   11.4      256.7   14.8    2079.    1477.
         1.000     878.4   11.4      248.0   14.8    2088.    1458.
           X     MAX HEAD  TIME    MIN HEAD  TIME    MAX H    MIN H
         -----   --------  ----    --------  ----   ------   ------
PIPE 2
          .000     878.4   11.4      248.0   14.8    2088.    1458.
          .200     961.5    5.7       92.7    9.1    2182.    1313.
          .400     962.4    5.6       60.4    9.1    2192.    1290.
          .600     955.5    5.6       47.3    9.1    2196.    1287.
          .800     946.0    5.7       34.2    9.4    2196.    1284.
         1.000     936.3    5.6       15.6    9.4    2196.    1276.

MAX HEAD = 962.4 FT (416.6 PSI) IN PIPE 2 AT X = .400 AT T = 5.56 SEC

MIN HEAD =  15.6 FT (  6.7 PSI) IN PIPE 2 AT X =1.000 AT T = 9.41 SEC
```

* * *

10.2 BRANCHING PIPES

A new feature, not occurring in direct series pipes, is found when three or more pipes join at a junction. We will call these branching pipe systems. As a practical measure, we will consider only three-pipe and four-pipe junctions.

10.2.1. THREE-PIPE JUNCTIONS

The typical three-pipe junction is shown in Fig. 10.5, with the initial flow directions indicated by arrows whose directions are established by the steady state conditions. That is, the signs of terms in the equations to be written and the characteristic lines to be followed will be determined by the steady flow behavior (Since the direction of flow was readily apparent in our earlier problems, there was no previous need to emphasize its determination).

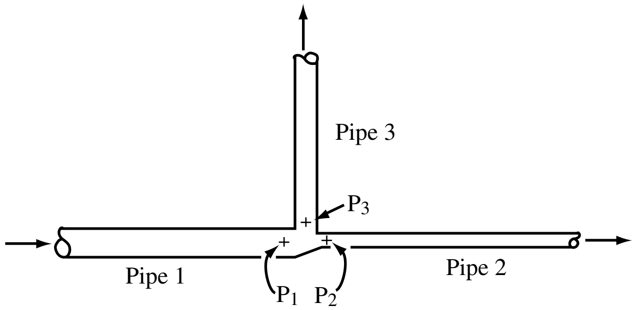
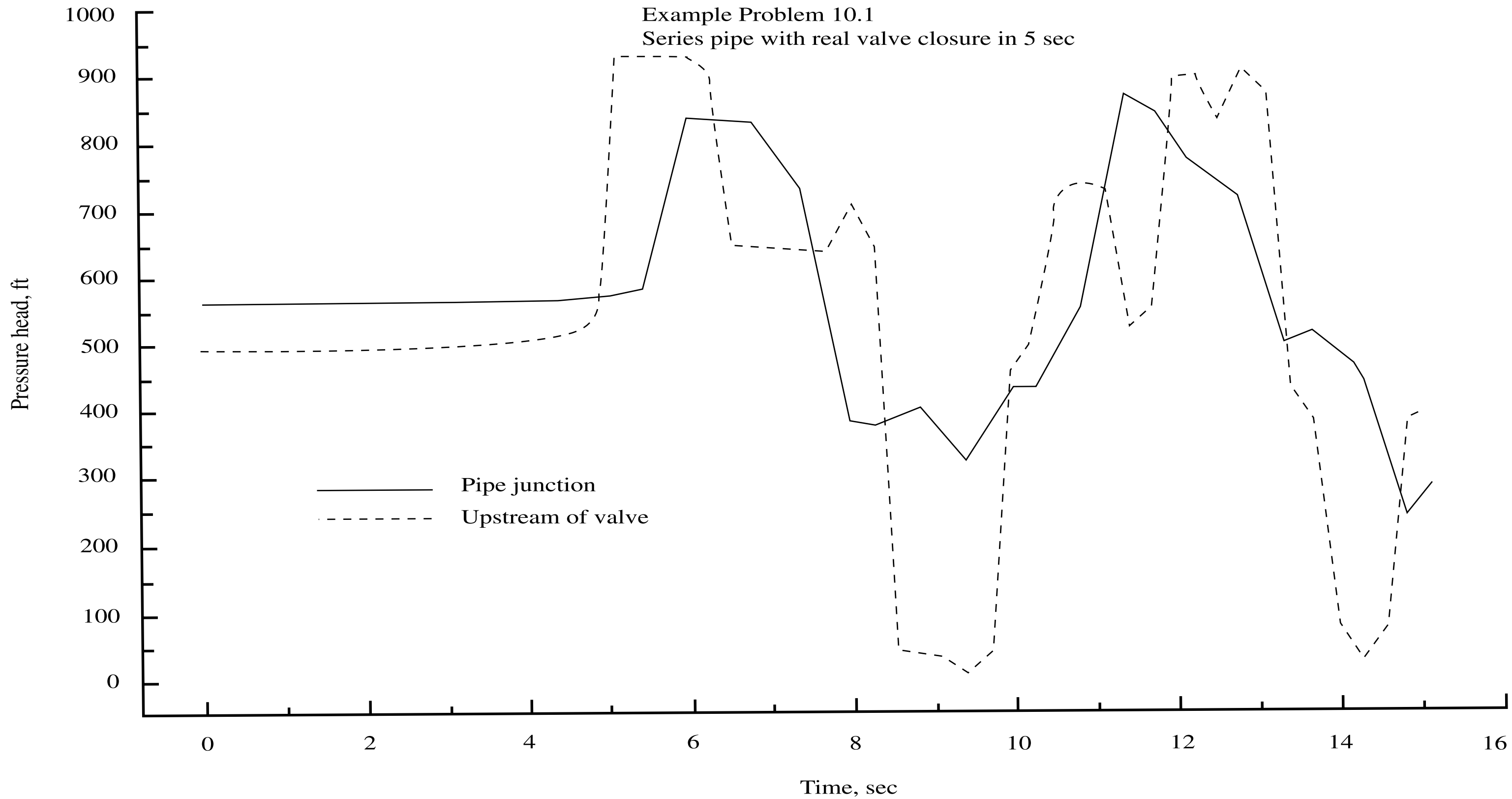


Figure 10.5 The one-in, two-out, three-pipe junction.


Example Problem 10.1
Series pipe with real valve closure in 5 sec
Pressure head, ft
1000
900
800
700
600
500
400
300
200
100
0
Pipe junction
Upstream of valve
0
2
4
6
8
10
12
14
16
Time, sec

For the pipe junction with one inflow and two outflows, the following equations describe the relations between the six unknowns

Pipe 1, C^+ $$V_{P_1} = C_1 - C_2 H_{P_1} \tag{10.10}$$

Pipe 2, C^- $$V_{P_2} = C_3 + C_4 H_{P_2} \tag{10.11}$$

Pipe 3, C^- $$V_{P_3} = C_5 + C_6 H_{P_3} \tag{10.12}$$

Conservation of mass $$V_{P_1} A_1 = V_{P_2} A_2 + V_{P_3} A_3 \tag{10.13}$$

Work-energy, neglecting local loss $$H_{P_1} = H_{P_2} = H_{P_3} \tag{10.14}$$

Here the subscripts indicate the unknown velocities and heads at the pipe junction. Equation 10.14 is actually two independent equations.

Solving this linear set of equations leads to

$$H_{P_1} = H_{P_2} = H_{P_3} = \frac{C_1 A_1 - C_3 A_2 - C_5 A_3}{C_2 A_1 + C_4 A_2 + C_6 A_3} \tag{10.15}$$

Back substitution of these heads into Eqs. 10.10, 10.11, and 10.12 yields the velocities.

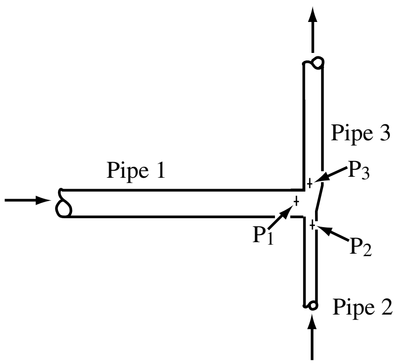


Figure 10.6 The two-in, one-out, three-pipe junction.

If instead we have a three-pipe junction with two inflows and one outflow, as shown in Fig. 10.6, an similar analysis would lead to the following equations for finding the unknown velocities and heads:

$$H_{P_1} = H_{P_2} = H_{P_3} = \frac{C_1 A_1 + C_3 A_2 - C_5 A_3}{C_2 A_1 + C_4 A_2 + C_6 A_3} \tag{10.16}$$

$$V_{P_1} = C_1 - C_2 H_{P_1} \tag{10.17}$$

$$V_{P_2} = C_3 - C_4 H_{P_2} \tag{10.18}$$

$$V_{P_3} = C_5 + C_6 H_{P_3} \tag{10.19}$$

All three-pipe junctions fit into one of the two categories above unless there is an external demand at the junction. Figure 10.7 shows such a situation. The only effect of this external demand is to modify the mass conservation relation. The energy and C^+ and C^- equations remain the same. The modified mass conservation equation is

$$V_{P_1} A_1 = V_{P_2} A_2 + V_{P_3} A_3 + Q \tag{10.20}$$

The impact on the solution is the addition of one term in the head equations:

$$H_{P_1} = H_{P_2} = H_{P_3} = \frac{C_1 A_1 - C_3 A_2 - C_5 A_3 - Q}{C_2 A_1 + C_4 A_2 + C_6 A_3} \tag{10.21}$$

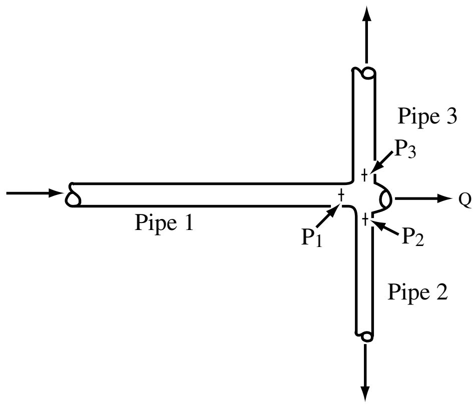


Figure 10.7 The three-pipe junction with a constant demand outflow.

Examination of the above sets of equations for H_P for the various junctions reveals a consistency in form which would allow the reader, with some experience, to write directly the equation sets for H_P by inspection.

10.2.2. FOUR-PIPE JUNCTIONS

Four-pipe junctions are analyzed with the same techniques as were the three-pipe junctions. We will examine only one application and let the reader practice on others.

Figure 10.8 shows a one-in, three-out, four-pipe junction with, in addition, a constant

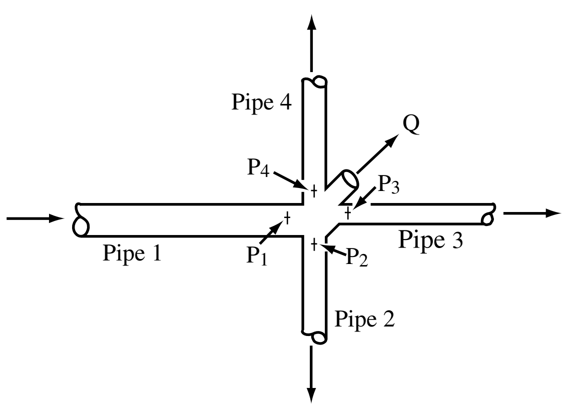


Figure 10.8 The one-in, three-out, four-pipe junction.

demand Q at the junction. The equations of the four characteristic lines are

Pipe 1, C^+ $$V_{P_1} = C_1 - C_2 H_{P_1} \tag{10.22}$$

Pipe 2, C^- $$V_{P_2} = C_3 + C_4 H_{P_2} \tag{10.23}$$

Pipe 3, C^- $$V_{P_3} = C_5 + C_6 H_{P_3} \tag{10.24}$$

Pipe 4, C^- $$V_{P_4} = C_7 + C_8 H_{P_4} \tag{10.25}$$

Conservation of mass $$V_{P_1} A_1 = V_{P_2} A_2 + V_{P_3} A_3 + V_{P_4} A_4 + Q \tag{10.26}$$

Work-energy, neglecting local loss $$H_{P_1} = H_{P_2} = H_{P_3} = H_{P_4} \tag{10.27}$$

Solving these equations for the head values at the junction,

$$H_{P_1} = H_{P_2} = H_{P_3} = H_{P_4} = \frac{C_1 A_1 - C_3 A_2 - C_5 A_3 - C_7 A_4 - Q}{C_2 A_1 + C_4 A_2 + C_6 A_3 + C_8 A_4} \tag{10.28}$$

As before, back substitution into the C^+ and C^- equations will give the velocities.

As we will see when pipe networks are discussed, a computer program can be written which will examine the pipes at each junction and automatically classify them according to number of pipes and junction configuration. The approach to the analysis is to employ the pattern we have observed in the previous examples.

10.3 INTERIOR MAJOR LOSSES

Occasionally a device is located in the interior of a pipeline which causes a significant loss in the system, either by choice (a pressure-reducing valve) or by necessity (e.g., a constriction, a meter, a partially-closed valve). Whatever the cause, if the loss is significant in comparison with other frictional losses, it must be included in the analysis.

Possible approaches to the treatment of this loss are to distribute it uniformly along the pipe by increasing the roughness of the pipe, to lump it into the boundary condition at one of the junctions, or to analyze it at its actual pipe location. Because the latter approach most resembles the true physical situation, we will approach the problem in this way.

We assume the pipe on each side of the loss has a different size, wave speed, and length; we further assume the energy loss across the device is proportional to the square of the velocity in the downstream pipe. The approach is similar to that used for the series pipe junction (see Fig. 10.9):

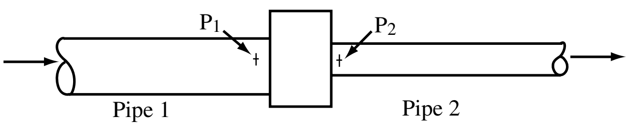


Figure 10.9 A model for interior major losses.

The first three equations apply directly and are rewritten below:

Pipe 1, C^+ $$V_{P_1} = C_1 - C_2 H_{P_1} \tag{10.29}$$

Pipe 2, C⁻ $$V_{P_2} = C_3 + C_4 H_{P_2} \tag{10.30}$$

Conservation of mass $$V_{P_1} A_1 = V_{P_2} A_2 \tag{10.31}$$

Application of the work-energy equation across the device assumes the loss would be the same as for steady flow at the instantaneous unsteady velocity:

Work-energy $$H_{P_1} = H_{P_2} + K_L \frac{V_{P_2}^2}{2g} \tag{10.32}$$

Combining Eqs. 10.29 through 10.32 provides the following equation for V_{P_2}:

$$V_{P_2}^2 + \frac{2g}{K_L}\left(\frac{1}{C_4} + \frac{A_2}{A_1}\frac{1}{C_2}\right)V_{P_2} - \frac{2g}{K_L}\left(\frac{C_3}{C_4} + \frac{C_1}{C_2}\right) = 0 \tag{10.33}$$

To solve the quadratic equation, we define

$$C_5 = \frac{2g}{K_L}\left(\frac{1}{C_4} + \frac{A_2}{A_1}\frac{1}{C_2}\right) \qquad C_6 = \frac{2g}{K_L}\left(\frac{C_3}{C_4} + \frac{C_1}{C_2}\right) \tag{10.34}$$

so that Eq. 10.33 can be written as

$$V_{P_2}^2 + C_5 V_{P_2} - C_6 = 0 \tag{10.35}$$

The quadratic formula solution is

$$V_{P_2} = \frac{1}{2}\left[-C_5 \pm \sqrt{C_5^2 + 4C_6}\right] \tag{10.36}$$

Because C_2 and C_4 are always positive, C_5 must be positive. Therefore, the (+) sign in front of the radical must be retained, for the velocity would otherwise always be negative. The final equation is

$$V_{P_2} = \frac{C_5}{2}\left[-1 + \sqrt{1 + \frac{4C_6}{C_5^2}}\right] \tag{10.37}$$

The upstream velocity and the two heads can be found by back-substitution into Eqs. 10.29 through 10.31.

One additional problem is created by this type of loss. If V_{P_2} should become negative, the work-energy equation would not be valid in its present form. For reverse flow the proper work-energy equation would be

$$H_{P_1} + K_{L_{rev}} \frac{V_{P_1}^2}{2g} = H_{P_2} \tag{10.38}$$

We must then reconstitute Eq. 10.33, revise the constants C_5 and C_6 and develop anew the equation for V_{P_2}.

10.4 REAL VALVES

Of all unsteady flow situations in pipes, it is likely that those caused by valve movement will be the most common. By constricting the flow, the closing valve creates an increasing head loss in the pipe system which causes the flow to decelerate. Different types of valves create head loss in different ways which are determined by not only the structure of the valve but also the details of the closure sequence.

For steady-state hydraulics the equation for head loss through a valve has traditionally been based on a form that has its roots in dimensional analysis:

$$h_L = K_L \frac{V^2}{2g} \tag{10.39}$$

Here h_L is the head loss, V is the velocity in the pipe (not in the valve), and K_L is the valve loss coefficient (see Chapter 2). Again we assume the steady-flow equation can be used in the unsteady situation to predict the head loss at the instantaneous velocity.

In many instances the user may have information which quantifies the valve loss coefficient for only two or three valve settings. It is then important to create a continuous variation in K_L with valve position so a transient analysis can be performed with some confidence that the results will be reasonably accurate.

Loss coefficients for a given valve at different actuator positions are determined by direct laboratory measurements. That is, K_L is a function of actuator position. If we know how the valve actuator moves with time, then we can determine a numerical value for K_L at that time. We then insert that value into Eq. 10.32 and solve for the values of the other variables. We will now demonstrate the application of this technique for valves at the interior of a pipeline and at the downstream end.

10.4.1. VALVE IN THE INTERIOR OF A PIPELINE

We presume the valve in the interior of the pipeline is scheduled to close (or open) according to some prescribed timetable. The equations describing this internal boundary condition are given by Eqs. 10.29 through 10.32 with equal pipe areas on both sides of the valve. Figure 10.10 illustrates the situation:

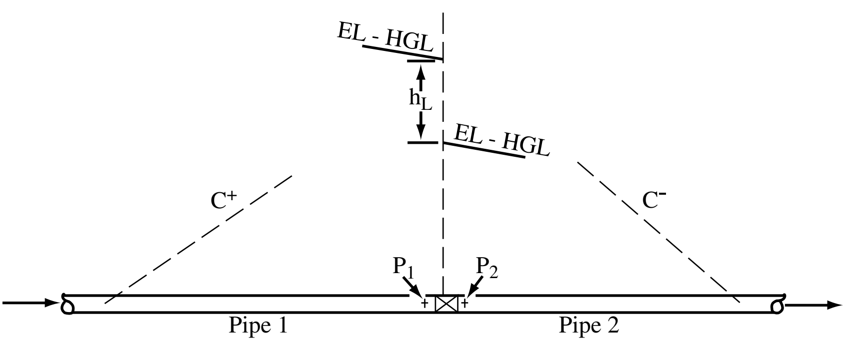


Figure 10.10 Valve in a pipeline of constant diameter.

We assemble the applicable equations in their modified form as

Pipe 1, C^+ $$V_{P_1} = C_3 - C_4 H_{P_1} \tag{10.40}$$

Pipe 2, C^- $$V_{P_2} = C_1 + C_2 H_{P_2} \tag{10.41}$$

Conservation of mass $$V_{P_1} = V_{P_2} \tag{10.42}$$

Work-energy $$H_{P_1} = H_{P_2} + K_L \frac{V_{P_2}^2}{2g} \tag{10.43}$$

The equation obtained by combining Eqs. 10.40 through 10.43 is

$$V_{P_2}^2 + \frac{2g}{K_L}\left(\frac{1}{C_4} + \frac{1}{C_2}\right) V_{P_2} - \frac{2g}{K_L}\left(\frac{C_3}{C_4} + \frac{C_1}{C_2}\right) = 0 \tag{10.44}$$

While keeping K_L separate, definition of the coefficients

$$C_5 = 2g\left(\frac{1}{C_4} + \frac{1}{C_2}\right) \qquad C_6 = 2g\left(\frac{C_3}{C_4} + \frac{C_1}{C_2}\right) \tag{10.45}$$

leads to the velocity expression

$$V_{P_1} = V_{P_2} = \frac{C_5}{2K_L}\left[-1 + \sqrt{1 + \frac{4C_6 K_L}{C_5^2}}\right] \tag{10.46}$$

This equation is correct so long as the flow is in the original downstream direction. If the flow reverses, then we must modify Eq. 10.43 and re-solve the set of equations to obtain

$$V_{P_1} = V_{P_2} = \frac{C_5}{2K_{L_{rev}}}\left[1 - \sqrt{1 - \frac{4C_6 K_{L_{rev}}}{C_5^2}}\right] \tag{10.47}$$

We should recall here that the C^+ and C^- characteristics and the equations associated with them are determined by the original flow direction and need not be redefined as the result of a temporary flow reversal. However, the reverse-flow head loss characteristics of valves are generally different from those for forward flow and are often not available. Under these circumstances the analyst may be forced to use the forward-flow characteristics for flow in both directions. While this assumption may well be acceptable for gate, ball, cone, and plug valves, it is questionable for globe, angle, and butterfly valves.

10.4.2. VALVE AT DOWNSTREAM END OF PIPE AT RESERVOIR

Valves quite often are located at the downstream ends of pipelines, so we will consider this common case to see what modifications of the previous equations are needed to effect a solution. Now we are no longer talking about an interior boundary condition, but the approach is the same. We consider the valve to be positioned just before a reservoir so the pressure head downstream of the valve is fixed at the reservoir elevation (see Fig. 10.11). Other downstream conditions may occur; e.g., a free discharge from the valve into the atmosphere would fix the pressure downstream of the valve at zero gage.

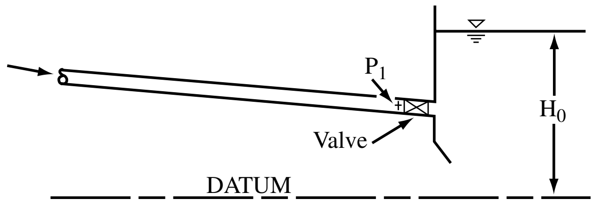


Figure 10.11 Valve and reservoir at the downstream end of a pipeline.

In Fig. 10.11 we now see only two unknowns at the valve, so the required equations are

C+
$$V_{P_1} = C_3 - C_4 H_{P_1} \tag{10.48}$$

Work-energy
$$H_{P_1} = H_0 + K_L \frac{V_{P_1}^2}{2g} \tag{10.49}$$

Combining these two equations to form the quadratic equation for V_{P_1} yields

$$V_{P_1}^2 + \frac{2g}{K_L C_4} V_{P_1} + \frac{2g}{K_L}\left(H_0 - \frac{C_3}{C_4}\right) = 0 \tag{10.50}$$

With the coefficients

$$C_5 = \frac{2g}{C_4} \qquad C_6 = 2g\left(H_0 - \frac{C_3}{C_4}\right) \tag{10.51}$$

the solution to the quadratic equation is

$$V_{P_1} = \frac{C_5}{2K_L}\left[-1 + \sqrt{1 - \frac{4C_6 K_L}{C_5^2}}\right] \tag{10.52}$$

or, for reverse flow,

$$V_{P_1} = \frac{C_5}{2K_{L_{rev}}}\left[1 - \sqrt{1 + \frac{4C_6 K_L}{C_5^2}}\right] \tag{10.53}$$

It is now clear that we can determine the impact of valve movement on pressures and velocities in the pipe system whenever we are able to express the valve loss coefficient as a function of time for the given closure (or opening) schedule. We will now see how to achieve this objective.

10.4.3. EXPRESSING K_L AS A FUNCTION OF TIME

To solve transient problems with closing or opening valves, we must learn how to enter the valve position schedule into the computer program so that the value of K_L can be found at any time. We begin by assuming that values of K_L for more than one valve position or setting are available from the manufacturer. We find it convenient to arrange the computer program so it accepts values of K_L at 11 evenly spaced positions ranging from 100% open to 0% open (closed). In other words, we wish to synthesize a K_L vs. percent-open table from available data. Soon we will see how to do this. Consequently, if the percent-open is known at a particular time from the closure schedule, then the computer program can interpolate the correct value of K_L from the percent-open table. And this value can next be entered into Eq. 10.52 or Eq. 10.53, and the solution can be completed for this time step. Next we examine the details of a technique to generate a table of K_L vs. percent-open.

Usually head loss characteristics of valves are expressed in one of three ways. Two of these employ the K_L or C_v coefficients discussed in Chapter 2. The third method relies on the nondimensional valve-closure function τ, defined as

$$\tau = \sqrt{\frac{K_{L_0}}{K_L}} \tag{10.54}$$

in which K_{L_0} is the loss coefficient when the valve is fully open. This nondimensional form of the loss coefficient has the advantage of varying between 0 and 1 and is preferred by some. Because head loss coefficients are generally provided as K_L's, we will use this form here. Several attempts have been made to present the transient pressures developed by valve closure in graphical form. A typical work by Wood and Jones (1973) briefly reviews these methods and then presents their own comprehensive graphs. As they point out, it is impossible to include all of the effects of friction and system configuration in simple graphical form. For this reason we will bypass the graphical approaches and concentrate on computerizing the representation of any valve in any pipeline configuration.

We will begin by determining the values of K_L needed to complete the K_L vs. percent-open table. In this example we will work with a gate valve (see Street et al., 1996), for which K_L values are provided for only four positions (see Table 10.1). Loss coefficients and other data for different valves are provided in Appendix C. The next step is

Table 10.1

Loss Coefficients for a Gate Valve (Street et al., 1996)

% Open	K_L	$1/K_L$
100	0.19	5.27
75	1.15	0.87
50	5.6	0.18
25	24.0	0.04

to construct a graph of K_L and $1/K_L$ as a function of percent-open by plotting the values from Table 10.1, constructing a smooth curve through the data, and extending it over the full range of percent-open from 100% to 0%. The result is shown in Fig. 10.12.

Clearly a problem exists in the plot of K_L vs. percent-open near valve closure where K_L approaches infinity as velocity goes to zero. To avoid this problem, we will develop the full-range curve only for $1/K_L$. Note in our solution for velocity that it is $1/K_L$ that appears in the equations rather than K_L.

Next we construct a table of $1/K_L$ values for uniform increments of percent-open by reading the values from Fig. 10.12. The results are listed in Table 10.2. Now, if the valve-closure schedule is known (percent-open vs. time), then we will know the percent-open at any given time. We can then interpolate the proper value of K_L from Table 10.2 and proceed with our solution. This requires the computer program to be capable of performing the interpolation process. This is accomplished by fitting a straight line (or curve) between two (or more) data points in the table and interpolating. In such problems linear (straight-line) interpolation is often adequate, and parabolic interpolation is usually sufficient to cover the remaining situations.

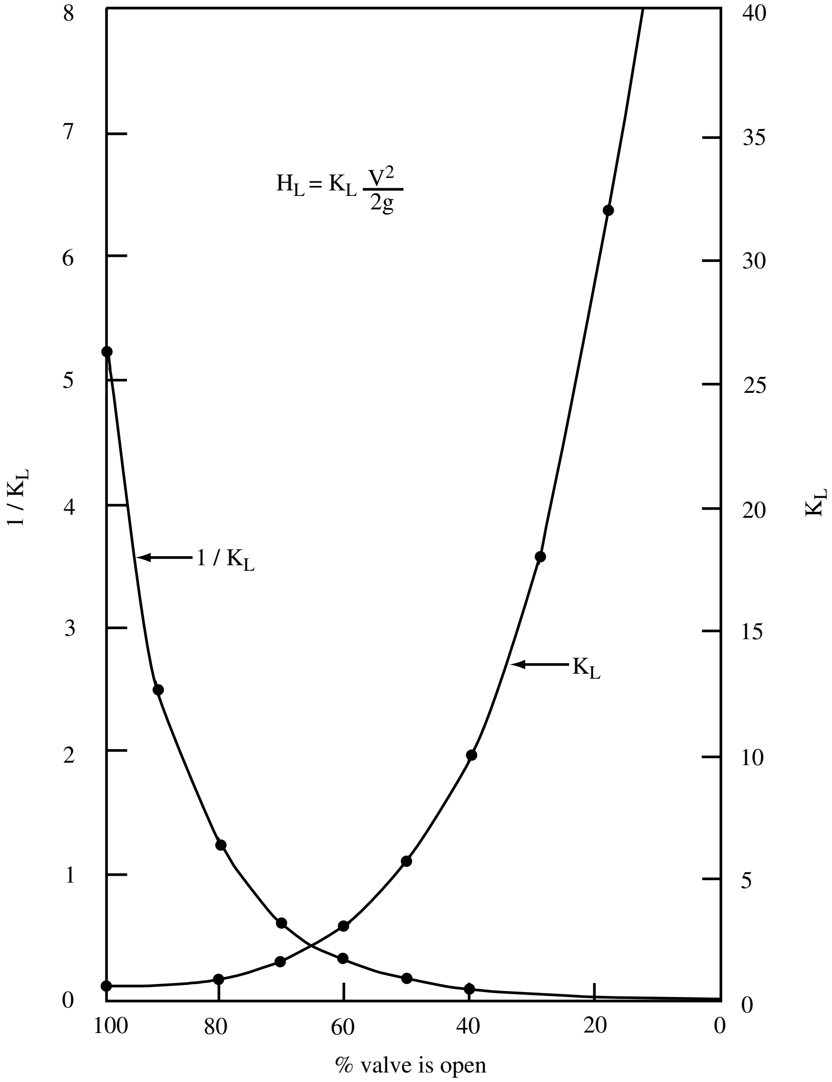


Figure 10.12 K_L and $1/K_L$ as functions of percent-open.

10.4.4. LINEAR INTERPOLATION

Because linear or straight-line interpolation is easiest to understand, we examine it first. For example, if the valve-closure schedule required, at a particular time, the determination of $1/K_L$ at 72.4% open, we may compute it from Table 10.2 with the following interpolation:

$$1/K_L = 0.625 + \frac{72.4 - 70}{80 - 70}(1.25 - 0.625) = 0.775 \qquad (10.55)$$

Now let us see how to program this step. The $1/K_L$ values for each percent-open are read into the program as data. The percent-open values are stored in an array called *PCT()* while the values of $1/K_L$ are stored in an array called *KI()*. The instantaneous value of

Table 10.2
Values of $1/K_L$ for Uniform Increments of Percent-open

Percent open	$1/K_L$
100	5.27
90	2.50
80	1.25
70	0.625
60	0.333
50	0.179
40	0.100
30	0.0556
20	0.0313
10	0.0167
0	0.0

percent-open is *OPEN* (72.4% in the example), and the desired value of $1/K_L$ is called *KLI* (0.775 above). Figure 10.13 presents the computer code which is inserted into the program to perform the interpolation.

```
      DO 32 I=1,10
      ITEST=(OPEN-PCT(I))*0.10
      IF(ITEST.EQ.0) GO TO 33
   32 CONTINUE
   33 FACT=(OPEN-PCT(I))*0.10
      KLI=KI(I)+FACT*(KI(I+1)-KI(I))
```

Figure 10.13. Linear interpolation computer code.

Owing to its simplicity, the linear interpolation procedure should be used whenever possible. However, for functions which are sharply curved or for table data which vary substantially, a higher-order interpolation should be considered.

10.4.5. PARABOLIC INTERPOLATION

The purpose of parabolic interpolation is to obtain a more accurate interpolation than is possible with linear interpolation. While this goal is generally achieved for smoothly varying functions, data sets which are highly curved or which possess points of inflection may not be represented well.

In this case we fit a parabola through subsets of three consecutive data points which cover the range over which interpolation is required (see Fig. 10.14). Once the parabolic equation has been found, the interpolated value is calculated by direct substitution. The value of x in Fig. 10.14 is the percent-open *OPEN* of the valve, and the value of $f(x)$ is *KLI*, the value of $1/K_L$ which is sought. A displaced local coordinate system is placed at x_n, and the parabolic equation is first written in this local coordinate system. The general form of the parabolic equation is

$$\eta = A\xi^2 + B\xi + C \tag{10.56}$$

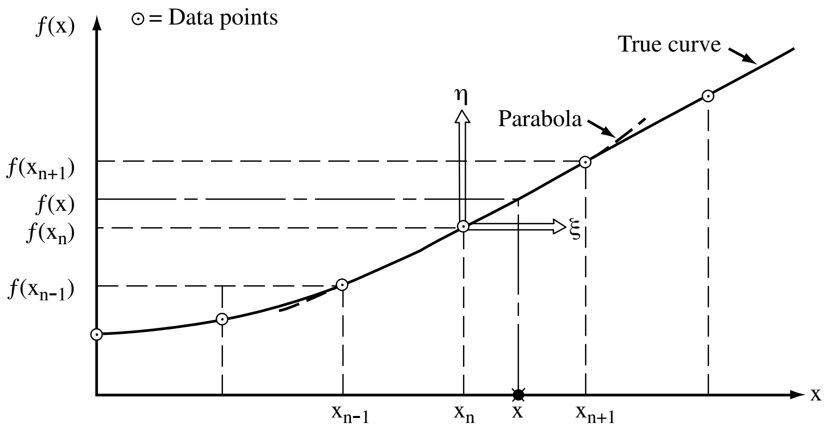


Figure 10.14 Definition sketch for parabolic interpolation.

but here $C = 0$ owing to the choice of the local coordinate system origin. The values of A and B are found by using the known data values at points x_{n-1} and x_{n+1}. Assuming a constant x-spacing Δx for the data points, the equations for A and B become

$$A = \frac{f(x_{n+1}) + f(x_{n-1}) - 2f(x_n)}{2\Delta x^2} \tag{10.57}$$

$$B = \frac{f(x_{n+1}) - f(x_{n-1})}{2\Delta x} \tag{10.58}$$

Recognizing that the two coordinate systems are related by the equations

$$\eta + f(x_n) = f(x) \qquad \xi + x_n = x \tag{10.59}$$

the parabolic equations in the local coordinate system can be transformed back into the original coordinate system, giving

$$f(x) = f(x_n) + \frac{1}{2}\left[f(x_{n+1}) - f(x_{n-1})\right]\frac{x - x_n}{\Delta x} + \frac{1}{2}\left[f(x_{n+1}) + f(x_{n-1}) - 2f(x_n)\right]\left(\frac{x - x_n}{\Delta x}\right)^2 \tag{10.60}$$

While this equation looks bulky, the computer code is straightforward, and the computation is efficient.

If, however, x is in the first segment, then no x_{n-1} exists. In this instance we simply shift the local coordinate system so its origin is at x_{n-1}. For the first interval only we then use the following equation for $f(x)$:

$$f(x) = f(x_1) - \frac{1}{2}\left[f(x_3) + 3f(x_1) - 4f(x_2)\right]\frac{x - x_1}{\Delta x} + \frac{1}{2}\left[f(x_3) + f(x_1) - 2f(x_2)\right]\left(\frac{x - x_1}{\Delta x}\right)^2 \tag{10.61}$$

The computer code segment that carries out this interpolation is shown in Fig. 10.15; it uses the same symbols as the linear interpolation code. While this code could be placed in a subroutine, it is so brief that it seems unnecessary.

```
      IF(OPEN.LT.10.) GO TO 9000
      DO 9001 I=2,10
      ITEST=(OPEN—PCT(I))*0.10
      IF(ITEST.EQ.0) GO TO 9002
 9001 CONTINUE
 9002 FACT=(OPEN—PCT(I))*0.10
      KLI=KI(I)+0.5*FACT*(KI(I+1)-KI(I—1))+
     $0.5*FACT*FACT*(KI(I+1)+KI(I—1)—2.0*KI(I))
      GO TO 9004
 9000 FACT=OPEN*0.10
      KLI=KI(1)-0.5*FACT*(KI(3)+3.0*KI(1)—4.0*KI(2))+
     $0.5*FACT*FACT*(KI(3)+KI(1)—2.0*KI(2))
 9004 CONTINUE
```

Figure 10.15. Parabolic interpolation code.

A word of caution is appropriate when one considers the use of parabolic interpolation. One has no control over the shape of the local parabolic curve and could sometimes experience an odd result. Such is the case if, under certain circumstances, the parabolic code is used with the gate valve head loss data. At the point when the valve is just about closed, it is possible for the locally parabolic curve to dip below the axis and produce a negative value for $1/K_L$. This creates numerous problems in the analysis and is the major reason why programs in this text use linear interpolation for the computation of $1/K_L$.

10.4.6. TRANSIENT VALVE CLOSURE EFFECTS ON PRESSURES

The use of real valves in a transient situation has a more substantial impact on pressures than might be expected from our limited experience with valves that artificially vary the velocity linearly at the valve. This impact is even more pronounced with gate valves; in this case the valve must be nearly closed before it generates enough head loss to decrease the velocity by a significant amount. The result for simple pipe-reservoir systems is that the linear valve closure time must be substantially greater than $2L/a$ to reduce the transient pressure appreciably below that obtained for sudden valve closure. Example Problem 10.2 demonstrates this fact using three different closure schedules.

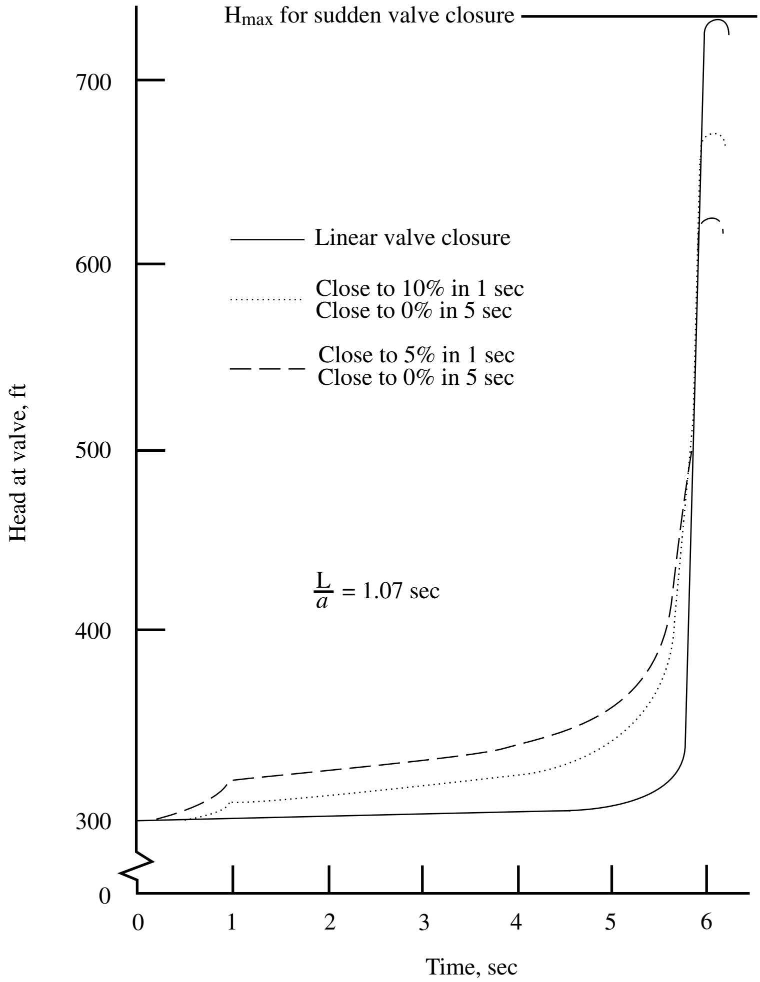
Example Problem 10.2

A pipe-reservoir system has a reservoir at the upstream end and a gate valve and reservoir at the downstream end. The steady-state pressure head at the valve is 300 ft. For sudden valve closure, Eq. 8.8 predicts an increase in pressure head of 431 ft.

Compute the pressure head at the valve for the following three closure schedules, and compare the results with the sudden-closure values. For this system, L/a = 1.07 seconds.

(1) Linear closure in 6 seconds.
(2) Close linearly to 10% open in 1 sec; close the remainder linearly over 5 sec so the valve is completely closed in 6 sec.
(3) Close linearly to 5% open in 1 sec; close the remainder linearly over 5 sec so the valve is completely closed in 6 sec.

The results of the analyses are shown in the following diagram. It is clear that the last 5% or less of the valve closure is critical in this case. Examining the results for case (1), it appears for all practical purposes that the valve in effect does not begin to close until the last 0.2 sec. Even though the valve is closed over approximately three times the critical closure time of $2L/a$ = 2.14 sec, we have achieved almost no reduction in pressure head increase. In fact, the fluid velocity doesn't change much in any of these cases until the valve is over 90% closed. For the gate valve it is the manner in which the last 5-10% of actuator movement is managed that will determine the pressure head increase.



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10.5 PRESSURE-REDUCING VALVES

Pressure reducing valves (PRV's) may routinely be placed in pipe systems, particularly water distribution networks. Because of their common occurrence and the fact that their computer models must be treated somewhat differently numerically than other internal boundary conditions, we will briefly examine their behavior.

10.5.1. QUICK-RESPONSE PRESSURE-REDUCING VALVES

In this case we assume the pressure reducing valve is spring actuated and undamped so that it responds instantaneously to changes in flow conditions. You will recall that the purpose of the PRV is to maintain a specified pressure on the downstream side of the valve within prescribed limits; the pressure there remains essentially unchanged so long as the upstream pressure is greater. However, if the upstream pressure drops so low that the prescribed downstream pressure cannot be maintained, then the PRV causes a major interior head loss with flow in the original direction. If under transient conditions the downstream pressure increases to the point where backflow could occur through the valve, the PRV then acts as a one-way check valve and prevents back flow.

The PRV could act in any one of three different modes. The first set of equations represents the normal mode of operation and assumes the PRV operates as intended. In solving this set of equations for the valve, if we discover that the PRV is not operating in the normal mode, we must then shift to another set of equations which describe one of the other two modes. The equations for each of these three modes follow.

For the normal mode of operation, the equations are similar to the major interior loss equations of Section 10.3. Figure 10.16 defines the variables applicable to this case. The

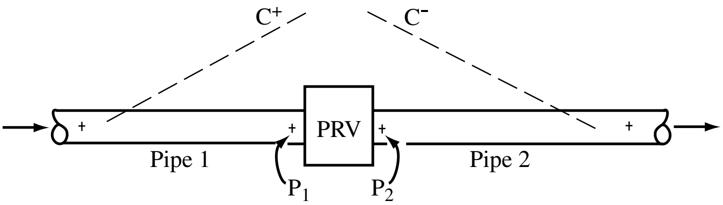


Figure 10.16 Definition sketch for the pressure reducing valve.

resulting equations are

Pipe 1, C^+
$$V_{P_1} = C_1 - C_2 H_{P_1} \tag{10.62}$$

Pipe 2, C^-
$$V_{P_2} = C_3 + C_4 H_{P_2} \tag{10.63}$$

Conservation of mass
$$V_{P_1} = V_{P_2} \tag{10.64}$$

Work-energy
$$H_{P_1} = H_{P_2} + K_{L_{PRV}} \frac{V_{P_2}^2}{2g} \tag{10.65}$$

The last equation must be altered for normal PRV operations when $H_{P_2} = H_{PRV} =$ constant. Consequently for normal operation

$$V_{P_2} = C_3 + C_4 H_{PRV} = V_{P_1} \tag{10.66}$$

and

$$H_{P_1} = \frac{C_1 - V_{P_1}}{C_2} \tag{10.67}$$

Before these calculations can be considered to be correct, we must verify that normal operation is indeed occurring. If, for example, V_{P_2} is negative, normal operation does not occur, and we must set both velocities to zero and solve Eqs. 10.62 and 10.63 for the proper heads. If, however, V_{P_2} is positive, we then must check whether the pressure head drop across the valve is larger than the minimum that is allowable with the valve wide open. This check can be made with Eq. 10.65 by using the wide-open value of $K_{L_{PRV}}$ to compute the drop in head ΔH_{min} for the given velocity and comparing it with the value obtained from Eq. 10.67, $H_{P_1} - H_{PRV} = \Delta H_{act}$. If $\Delta H_{act} \geq \Delta H_{min}$, then the valve is operating normally. If $\Delta H_{act} \leq \Delta H_{\min}$, then the PRV cannot sustain the downstream pressure requirement, even though the flow is still in the original direction. Now the velocities and heads must be computed from Eqs. 10.62 through 10.65 using $K_{L_{PRV}}$ for a fully-open PRV.

10.5.2. SLOWER ACTING PRESSURE-REDUCING OR PRESSURE-SUSTAINING VALVES

Most valves of this type are operated by a pilot system which senses the pressure on the downstream (or upstream) side of the valve and actuates a valve system in the pilot piping which moves a diaphragm to change the valve setting and maintain the required pressure. Because the pilot operation requires fluid to move through the pilot system, there is a discrete or finite response time to sudden and large pressure changes in the pipeline. These valves are designed to respond to much more slowly fluctuating pressures than occur when transients are present. In addition, the time-varying response of these valves is unknown and varies from valve to valve as well as with the magnitude of the transient pressure. In most instances these uncertainties prevent any attempt at a sophisticated analysis; instead the system is simply conservatively assumed to respond instantaneously, as was described in the previous section.

10.6 WAVE TRANSMISSION AND REFLECTION AT PIPE JUNCTIONS*

In many instances it is desirable to be able to estimate what portions of pressure waves are reflected and transmitted at pipe junctions. We already know at reservoirs that none of the pressure wave is transmitted into the reservoir. We will now briefly look into the reflection and transmission properties of series pipe junctions and tee junctions. In all cases we assume that the head lost at the junction is negligible.

10.6.1. SERIES PIPE JUNCTIONS

The equations of mass and linear momentum conservation can be applied to flow at a junction as a pressure head increase ΔH reaches a junction. At that instant ΔH_1 passes through the junction (is transmitted) and ΔH - ΔH_1 is reflected. Figure 10.17 depicts the

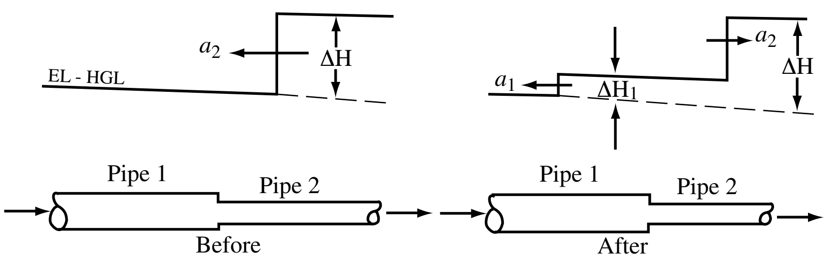


Figure 10.17 Wave transmission and reflection at a series pipe junction.

configuration of the EL-HGL before and after the pressure wave reaches the junction. The analysis produces the following equation for transmission and reflection:

* This section is adapted from Elementary Fluid Mechanics, by R. L. Street, G. Z. Watters, and J. K. Vennard, Ed. 7, Copyright 1996 by John Wiley & Sons, Inc. Reprinted by permission.

$$\Delta H_1 = \frac{2a_1 A_2}{a_2 A_1 + a_1 A_2} \Delta H \tag{10.68}$$

Here A is the cross-sectional area of the pipes. When the wave speeds a are approximately the same, we obtain

$$\Delta H_1 = \frac{2A_1}{A_1 + A_2} \Delta H \tag{10.69}$$

Example Problem 10.3

A 24-in-diameter pipeline with a wave speed of 3300 ft/s reduces to a 6-in-diameter pipe with a wave speed of 3700 ft/s. The velocity in the 24-in pipe is 1.0 ft/s which corresponds to a velocity in the 6-in pipe of 16 ft/s. The head difference ΔH for sudden flow stoppage in the 6-in pipe is 1838 ft. Find the portion of this wave which is transmitted through the junction.

From Eq. 10.69,

$$\Delta H_1 = \frac{2a_1 A_2}{a_2 A_1 + a_1 A_2} \Delta H = \frac{2(3300)(\pi/4)(6/12)^2(1838)}{3700(\pi/4)(24/12)^2 + 3300(\pi/4)(6/12)^2} = 194 \text{ ft}$$

With only some 10% of the head difference being transmitted upstream, it appears that the upstream pipe acts much like a reservoir.

* * *

10.6.2. TEE JUNCTIONS

A tee junction is shown in Fig. 10.18. Using the same analysis techniques as before

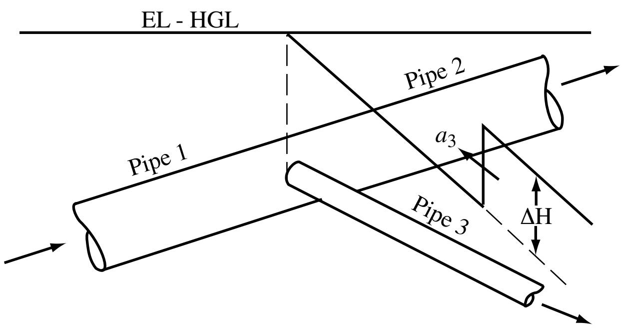


Figure 10.18 Wave transmission and reflection at a tee junction.

leads to the following equations

$$\Delta H_1 = \Delta H_2 = \frac{2a_1 a_2 A_3}{a_2 a_3 A_1 + a_1 a_3 A_2 + a_1 a_2 A_3} \Delta H \tag{10.70}$$

or, for pipes with similar wave speeds,

$$\Delta H_1 = \Delta H_2 = \frac{2A_3}{A_1 + A_2 + A_3} \Delta H \tag{10.71}$$

Example Problem 10.4

A 24-inch-diameter main line has a 6-in-diameter takeoff (similar to Fig. 10.18) which has a velocity of 10 ft/s. The velocities in the main line are 4.0 ft/s before the takeoff and 3.38 ft/s after the takeoff. A sudden flow stoppage in the 6-in-diameter takeoff causes a head difference $\Delta H = 1150$ ft to occur.

Assuming that the wave speeds in all pipes are similar, compute the portion of ΔH that passes into the 24-in-diameter pipe.

We calculate the transmitted portion of the wave from Eq. 10.71:

$$\Delta H_1 = \Delta H_2 = \frac{2A_3}{A_1 + A_2 + A_3}\Delta H = \frac{2(\pi/4)(6/12)^2(1150)}{(\pi/4)(24/12)^2 + (\pi/4)(24/12)^2 + (\pi/4)(6/12)^2} = 70 \text{ ft}$$

With a tee connection only 6% of the pressure wave passes into the 24-in-diameter pipe. It is easy to see why transients in pipe networks are absorbed so rapidly.

* * *

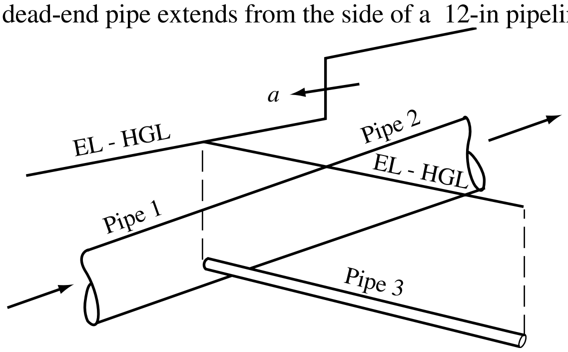
10.6.3. DEAD-END PIPES

If a pipe system contains a member which carries no discharge and terminates in a dead end, e.g., a closed valve, then a unique situation exists which could cause unexpectedly high pressures. This is actually a special case of the tee junction in the previous section. As a high-pressure wave passes the junction from which the dead-end pipe extends, a portion of the wave is transmitted into the dead-end pipe, increasing the pressure there by an increment ΔH_1 and inducing a flow velocity ΔV_1 toward the closed end. When the pressure wave reaches the dead end, the induced velocity is abruptly stopped, thereby increasing the pressure head at the dead end by $2\Delta H_1$.

While pipe system geometry, pipe size, and friction losses all affect the overall pressure increase in varying amounts, the maximum effect at a dead end occurs when the dead-end pipe is very small in comparison with the main pipe. For this condition with small frictional effects, the pressure head increase is at most twice the value of the ΔH that initially passes the junction. The following example demonstrates the dead-end pipe effect for two extreme cases.

Example Problem 10.5

A 3000-ft-long dead-end pipe extends from the side of a 12-in pipeline.



(a) If the dead-end pipe has a diameter of 1.0 in, find the maximum pressure head increase in this pipe if the mainline velocity of 5 ft/s in pipe 2 is suddenly halted. Assume a wave speed of 3000 ft/s for all pipes and neglect friction.

(b) If part (a) were solved with a friction factor of 0.020 in the 1-in pipe, what would be the result?

(c) What is the result if all three pipes are 12 in in diameter and friction is neglected?

(a) From Eq. 8.4 the incremental head increase in the main line is

$$\Delta H = -\frac{a}{g}\Delta V = -\frac{3000}{32.2}(-5) = 466 \text{ ft}$$

From Eq. 10.71 the head increment in the dead-end pipe is

$$\Delta H_3 = \frac{2A_2}{A_1 + A_2 + A_3}\Delta H = \frac{2 \times 12^2}{12^2 + 12^2 + 1^2}(466) = 464 \text{ ft}$$

when the common constants in both numerator and denominator are canceled. The maximum possible head increase would be 2(464) = 928 ft, according to our earlier reasoning. A computer analysis gives an identical 928 ft, thus verifying our earlier conclusion.

(b) The pressure head increment moving up the 12-in pipe would again be 466 ft, and the head increment entering the 1-in pipe would be 464 ft. Although the maximum possible pressure head increment would remain 928 ft, a computer analysis shows that friction effects have reduced the maximum head increment to 770 ft.

(c) The head increment moving into pipe 3 is again computed from Eq. 10.71 as

$$\Delta H_3 = \frac{2A_2}{A_1 + A_2 + A_3}\Delta H = \frac{2 \times 12^2}{12^2 + 12^2 + 12^2}(466) = 311 \text{ ft}$$

The maximum possible head increase is 2(311) = 622 ft. A computer analysis also gives 622 ft as the actual head increase.

* * *

We conclude in the absence of friction that the rule that the head increment doubles is valid. The presence of friction reduces the head increase by an undetermined amount. We note that the neglect of friction when estimating a dead-end pressure increment gives conservative results.

10.7 COLUMN SEPARATION AND RELEASED AIR

It is common knowledge that excessively-high pressures resulting from transients in pipes can cause damage. It is also generally recognized that low pressures could cause the collapse of pipes with thin walls or high external loads. What is not so commonly known or understood is the phenomenon of *column separation* and the consequences of its occurrence.

10.7.1. COLUMN SEPARATION AND RELEASED AIR

When transients in a pipe system cause the pressure to approach the vapor pressure of the liquid, gases in solution begin to come out of solution and dramatically affect the flow behavior. If the drop in pressure is severe enough to cause the local pressure to reach the vapor pressure of the liquid, then the liquid boils (cavitates, vaporizes), forming large pockets of undissolved gases and vapor. This phenomenon is called *column separation.*

One consequence of this occurrence is a substantial change in the wave speed caused by the presence of entrained gases and vapor bubbles which affect the compressibility of the liquid (see Section 8.4). A second consequence is the fact that the liquid "column" is no longer homogeneous and in fact may have large cavities. This means that the analyses we have developed no longer apply directly.

Whenever the pressure at any point in the pipeline drops below the pressure at the pipeline source, the saturation pressure of the dissolved gases may be reached, and these gases will begin to come out of solution. This is one of the reasons for placing air release valves at pipeline summits. The amount of gas that comes out of solution depends on the degree of initial saturation and the severity and extent of the low pressure. If the pressure drops to the fluid vapor pressure for an extended time period, large cavities of vapor and gases may form.

If we look carefully at the consequences of closing a valve, we find a simple example of how column separation can form. Upon sudden valve closure, the pressure head just downstream of the valve attempts to drop by an amount (given by Eq. 8.8) which should be just enough to bring the liquid column to rest. However, if this pressure drop is greater than that required to reach the fluid vapor pressure, a vapor cavity will form because a liquid cannot remain a liquid at a pressure which is lower than its vapor pressure (see Fig. 10.19). Because the pressure drop is limited, there is not a sufficient pressure gradient to stop the flow, so the flow separates at the valve and forms a vapor cavity. Analysis of the ensuing transient can become exceedingly complex, requiring at the least a means of representing the cavity formation, growth and decay over time. Owing to the large difference in density

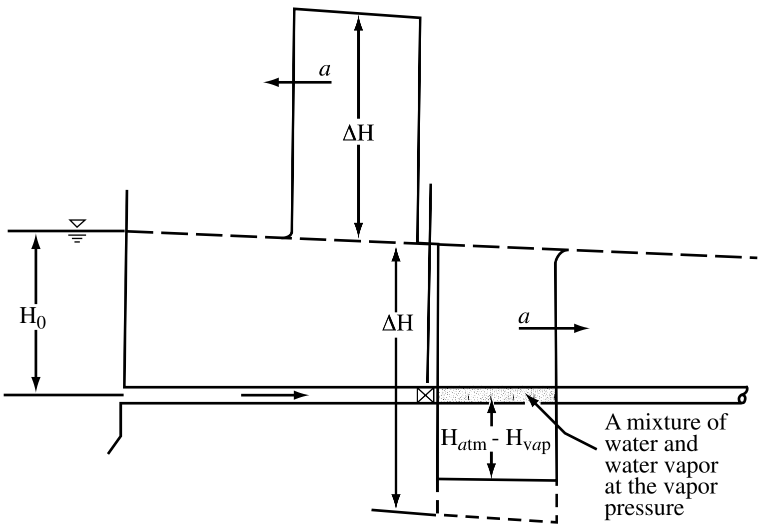


Figure 10.19 Column separation caused by sudden valve closure.

between the liquid and the gases, buoyancy effects encourage a gaseous cavity to lie over the liquid rather than fill the pipe cross section, which calls into question the assumption of the existence of a one-dimensional flow.

10.7.2. ANALYSIS WITH COLUMN SEPARATION AND RELEASED AIR

Tullis et al. (1976) thoroughly discuss the effects of air release at low pressures as well as column separation at vapor pressure. They suggest that volumes of released air may be either uniformly distributed throughout the flow or concentrated in pockets. In the first case Eq. 8.40 can be used to find the reduced wave speed. Because the change in wave speed will cause large interpolations if a rectangular grid is used, they suggest the use of a method whereby the characteristic lines are followed as closely as possible to minimize interpolation errors and maintain numerical stability.

If it is undesirable to use this approach, then the regular wave speed is used, and the air or vapor is assumed to be concentrated in discrete sections along the pipeline with internal boundary conditions imposed at the ends of each cavity. The growth and decline of the cavities is monitored; if they disappear, the regular analysis technique can then resume.

Other investigators who cite experience with the modeling of column separation are Martin et al. (1976), Ewing (1980), and Marsden and Fox (1976). Most studies address the modeling of the vapor cavity, the mechanism of release and re-absorption of air and water vapor, and numerical techniques. Wylie and Streeter (1993) offer a compact summary of the state of the art.

To address the problem of column separation, we must first create a model of the phenomenon. The simplest model of column separation ignores the existence of dissolved gases that might come out of solution at low pressures. Instead it is assumed that the liquid remains intact until the vapor pressure is reached. When that point is reached, it is postulated that the vapor cavity will grow at a constant cavity pressure equal to the vapor pressure. Eventually, when the cavity closes, it is presumed that the vapor re-absorbs so that it disappears at the instant of cavity closure. In effect, the vapor cavity is treated much like a vacuum.

This simple model also requires some assumptions regarding the form of the cavity. In reality, this form is quite complicated and nearly impossible to simulate accurately. Therefore we might as well use the simplest possible model. We assume the walls of the cavity remain normal to the pipe cross section, and the growth or decay of the cavity depends entirely on the relative velocity of the cavity endwalls. This in turn requires an internal boundary condition to be imposed at each node within the cavity where the pressure is fixed at the vapor pressure. Thus, at each node where column separation occurs, there are two velocities, one associated with the upstream face of the cavity and one associated with the downstream face. The relative magnitudes and directions of these velocities determine the growth or decay of the cavity. All of the cavity behavior is concentrated at the computational nodes in the pipeline with the liquid between the nodes intact and retaining the original wave speed. This model is illustrated in Fig. 10.20.

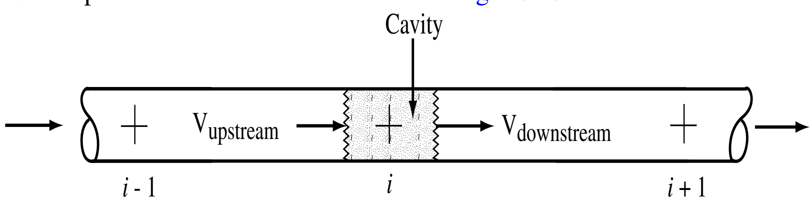


Figure 10.20 Basic model for column separation analysis.

The determination of the consequences of cavity closure is an important prediction of the model. To analyze this feature we apply conservation of momentum to the collision of the two collapsing cavity walls that are moving at different velocities. The result is an equation for the head increase which results from the collision:

$$\Delta H = \frac{a}{2g}\left(V_{upstream} - V_{downstream}\right) \tag{10.72}$$

This head increase ΔH is added to the vapor pressure head at the node to determine the new pressure immediately after cavity closure. PROG8 employs this model of column separation occurring in pumped pipelines.

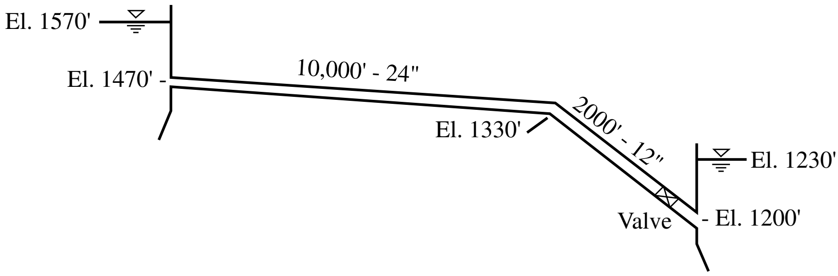
While this model of column separation seems very primitive, it is widely used in practice. Although much research over recent years sought to improve on this basic model, no one has developed a model and analysis which is sufficiently more general and accurate to attract the user community. Commenting on this state of affairs, Wang and Locher (1991) note that this vapor cavity model is a "very simplified formulation of what is really a highly complex problem ... that works surprisingly well in spite of well-grounded theoretical objections to the approach." And in applying this method they caution that it is "essential to understand the formulation of the method, its limitations, and to interpret the results in the light of this knowledge and past experience". Clearly, experience and an understanding of the physical phenomena are crucial ingredients in successfully applying this model.

10.8 PROBLEMS

Note: Use PROG2 for computer analyses in this chapter unless instructed otherwise.

10.1 The gate valve in the pipeline below closes linearly from wide open to completely closed in 30 sec. The diameters shown are inside diameters. The pipe is welded steel with a wall thickness of 0.135 in and Case (b) restraint. Assume the gate valve has the same loss coefficients as shown in Table 10.2.

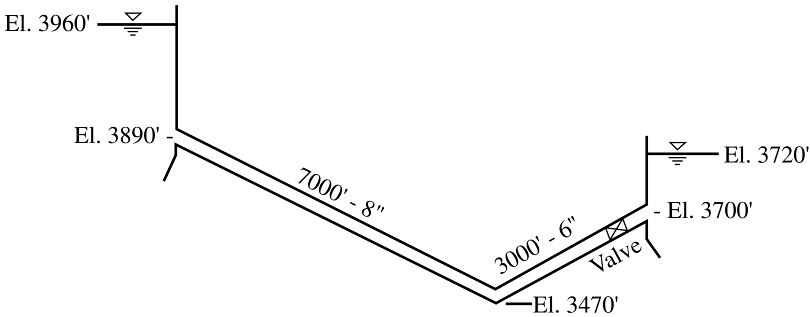
Find the maximum and minimum pressures in the system and where and when they occur. If column separation occurs, identify when and where it first appears.



10.2 The engineer in charge of project design wants you to answer the following questions regarding the proposed pipeline shown below.

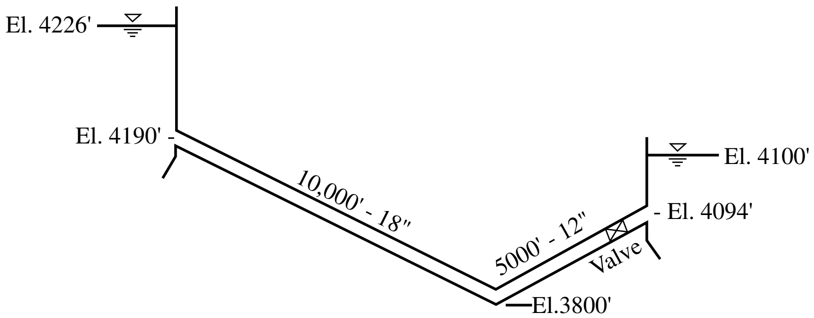
(a) What will be the maximum pressure in the pipeline?
(b) Where and when will it occur?
(c) Will column separation occur?
(d) If so, where and when will it occur?

The pipe diameters shown are inside diameters. The pipe is 14-ga. (0.0747-in) welded steel with Case (b) restraint. Use a Hazen-Williams coefficient of 140 in your calculations. The Pratt butterfly valve, which has the loss characteristics given in Appendix C, closes at a uniform angular rate in 20 sec.



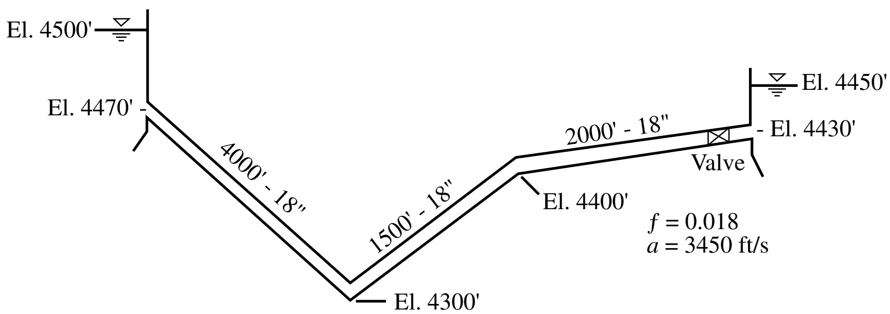
10.3 The reservoir of surface elevation 4226 ft supplies a city water supply through a 15,000-ft pipeline. The pipe is 7-ga (0.1793-in) welded steel with Case (b) restraint; the diameters are inside diameters. The working pressure in the 18-in pipe is 270 lb/in^2 and in the 12-in pipe 420 lb/in^2. The design engineer wants to close the Cla-Val globe valve linearly without exceeding the working pressure in either pipe. Assume the valve loss characteristics in Appendix C for Cla-Val valves follow the GA Industries curve for variation with percent open.

As a consultant, your task is to analyze the transient behavior of this system for valve closure times of 20, 40, and 60 sec and determine whether the working pressure in the pipeline is exceeded in any of these cases. As part of your report, you should find the maximum and minimum pressures in the pipeline and where and when they occur. Also note whether column separation occurs.



10.4 Water flows from the upper reservoir by gravity through a 7500-ft-long steel pipeline. Discharge is controlled by an angle valve at the downstream reservoir. The system is to be shut down as quickly as possible without exceeding the allowable working pressure of 200 lb/in^2 or causing column separation.

The GA Industries angle valve, with loss characteristics given in Appendix C, is programmed to close at two different rates. Recalling from Example Problem 10.2 how gate valves behave, use this knowledge to adjust the closure stages of the angle valve to minimize the closure time.



10.5 The 24-in (23.65 in inside diameter) pipeline is 7-ga (0.1793-in) welded steel pipe with a working pressure of 200 lb/in^2. Water flows between the two reservoirs shown atop the following page, controlled by a valve at the downstream reservoir. The valve has the following loss coefficients K_L for the different openings:

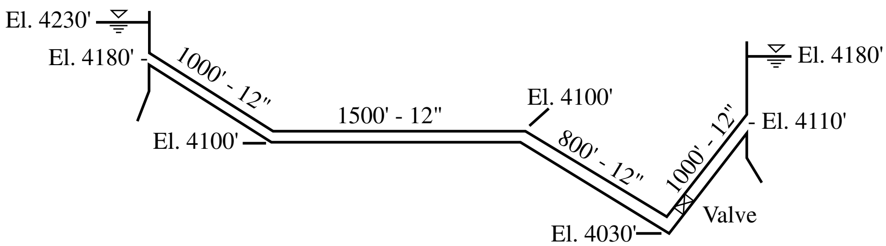
% Open	100	75	50	37.5	25	12.5
K_L	0.07	0.42	2.20	5.10	12	56

As a consultant to the project engineer, your task is to determine a closure schedule which will close the valve as quickly as possible without exceeding the working pressure or causing column separation. The valve is designed to close at two different rates.

10.6 The pipeline connecting the two reservoirs is 12-inch CL 200 Transite pipe with an inside diameter of 11.56 in and a wall thickness of 1.26 in. The pipe sections are joined with couplings and ring gaskets. Assume a friction factor of 0.014 in your calculations.

The discharge rate is controlled by a GA Industries globe valve at elevation 4030 ft. To determine the valve loss coefficients, use the plot of C_v vs. %-open for the GA Industries valves found in Appendix C.

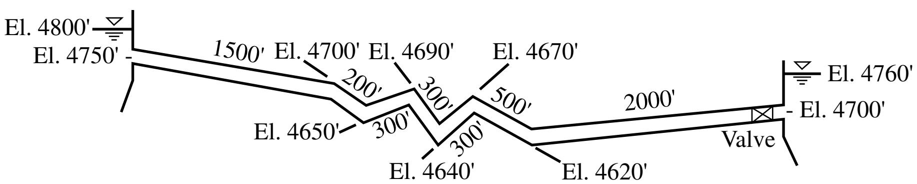
As a consultant, your task is to recommend a fast closure schedule which will not cause pressures in excess of the pipe class rating (200 lb/in^2) or column separation. Use PROG2A for this analysis.



10.7 The pipeline supplying water from the upper reservoir is 12-in Class 200 PVC pipe (wall thickness = 0.61 in, outside diameter = 12.750 in). The pipe is considered hydraulically smooth so use a Hazen-Williams coefficient of 150. The pipe is joined by bell-and-spigot connections, and anchor blocks are installed at all bends and fittings.

Two valves are being considered for use by the design engineer. Valve *A* is an expensive servo-controlled valve which can cause the velocity at the valve to vary linearly. Valve *B* is a lower-cost globe valve with a wide-open C_v of 1750. The variation of C_v with %-open is given in Appendix C for GA Industries valves. The globe valve can close at only one rate.

Each valve closes in 30 sec. You are to conduct analyses of the two alternatives and determine the maximum and minimum pressures to be expected in each case. Make a recommendation as to which valve to use.

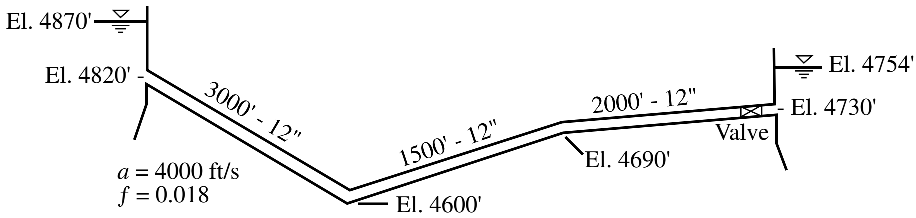


10.8 Water flows from the reservoir at surface elevation 4870 ft shown below at a rate of 6.28 ft^3/s. The system is to be designed to shut down in the least possible time without developing excessively high pressures or column separation. Current plans are to use a gate valve that can be programmed to close at two different rates. The loss characteristics of the valve are given in Table 10.2.

The project design engineer has decided to try the following schedules:

	Stage 1	Stage 2
(a)	90% closed at 5 sec	100% closed at 15 sec
(b)	95% closed at 5 sec	100% closed at 15 sec
(c)	90% closed at 10 sec	100% closed at 20 sec
(d)	95% closed at 5 sec	100% closed at 20 sec
(e)	95% closed at 10 sec	100% closed at 20 sec

Your task is to identify the high and low pressures, their location, and the time they occur.



10.9 The pipeline below is constructed of 7-ga. (0.1793-in) welded steel with a working stress of 13,500 lb/in^2, which corresponds to a working pressure of 200 lb/in^2. Water flows from the reservoir at 4800 ft through a 6000-ft pipeline 24 in in diameter (23.65 in inside diameter). The discharge is controlled by a Pratt ball valve at the lower reservoir whose loss characteristics are given in Appendix C.

The system is to be designed to shut down as quickly as possible without developing pressures greater than the working pressure or causing column separation. It is possible to purchase a valve which can be programmed to close at two different uniform angular rates. The project design engineer has decided to try the following five schedules:

	Stage 1	Stage 2
(a)	90% closed at 4 sec	100% closed at 15 sec
(b)	95% closed at 4 sec	100% closed at 15 sec
(c)	98% closed at 4 sec	100% closed at 15 sec
(d)	90% closed at 2 sec	100% closed at 20 sec
(e)	98% closed at 2 sec	100% closed at 20 sec

Your task is to identify the high and low pressures, their location, and the time they occur.