CHAPTER 2

REVIEW OF FUNDAMENTALS

This chapter will review the fundamental concepts and principles upon which the hydraulics of pipeline systems is based. The review is intended to be sufficiently complete that readers who have taken a good first course in elementary fluid mechanics, but not necessarily recently, will be reminded of, and updated in, the essential conceptual building blocks that are the foundation of the material in this book. We will begin with an introduction to the fundamental equations that are the foundation of most of the subsequent developments in the book. Because the concept of the energy grade line (EGL or simply EL) and the hydraulic grade line (HGL) is so useful, we shall look at this idea separately. Next we look at some length at various head loss formulas. How turbomachines with rotating impellers, particularly pumps, function is vitally important to the understanding of many parts of this book, so their theory of operation and basic characteristics will be examined. The chapter will conclude with several steady-flow examples and a range of problems that will allow readers to test their readiness for the coming chapters. If a thorough review is desired, one might consult Miller (1984).

2.1 THE FUNDAMENTAL PRINCIPLES

2.1.1. THE BASIC EQUATIONS

Conservation of mass is the most basic principle. In general, the fluid density ρ may vary in response to changes in the fluid temperature and/or pressure. For a fixed control volume $\forall$ enclosed by a surface S, a general statement of mass conservation is

$$\frac{\partial}{\partial t}\int_{\forall} \rho \, d\forall + \int_{S} \rho \vec{v} \cdot \vec{n} \, dS = 0 \tag{2.1}$$

in which $\vec{v}$ is a velocity at a point and $\vec{n}$ is an outer normal unit vector to the surface S, and t is time. The first term represents the accumulation of mass over time in the control volume; for steady flows it is zero. At a surface point the dot product $\vec{v} \cdot \vec{n}$ gives the component of the velocity which crosses the surface, so the second term computes the net outflow of fluid across the entire control surface. For steady incompressible flow of a liquid in a pipe, the conservation of mass is generally referred to as the continuity principle, or simply **continuity**, and it is written

$$Q = \int_{A} v \, dA = V_1 A_1 = V_2 A_2 \tag{2.2}$$

in which Q is the volumetric discharge through a pipe cross section, which can also be written as the product of the mean velocity V and cross-sectional area A of the pipe.

The second, equally important, principle is the **work-energy** principle, sometimes called simply the energy principle. Some also call it the Bernoulli equation, but in general it is distinctly more than that. For the steady one-dimensional flow of a liquid in a pipe, per unit weight of fluid, the principle can be written between two sections or stations as

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + \sum h_{L_{1-2}} - h_m \tag{2.3}$$

In this equation $V^2/2g$ is the velocity head or kinetic energy, p/γ is the pressure head or flow work, and z elevation head or potential energy, all per unit weight. If the last two terms on the right were absent, the equation would be the classical Bernoulli equation. The last two terms, however, are extremely important in the study of the hydraulics of pipe lines. The head loss term, or the accumulated energy loss per unit weight, Σh_L, is the sum, between sections 1 and 2, of the individual head losses in the reach caused by frictional effects. The last term, h_m, is the mechanical energy per unit weight added to the flow by hydraulic machinery. A pump adds energy to the flow so h_m is then positive and called h_p; a turbine extracts energy from the flow so h_m would then be negative and called h_t.

Fluid power, sometimes denoted by P, is the product of the energy gain or loss per unit weight h_m and the weight rate of flow $Q\gamma$, or $P = Q\gamma h_m$. A unit conversion factor can be applied to this result to express the power in, say, horsepower or kilowatts. Depending on the purpose of the computation, an efficiency factor η may be used as a multiplier or divisor of the power.

The last of the major principles considers **linear momentum**, which is governed by the impulse-momentum equation

$$\frac{\partial}{\partial t}\int_{\forall} \rho\vec{v}\, d\forall + \int_S \vec{v}(\rho\vec{v}\cdot\vec{n})\, dS = \vec{F}_{net} = \vec{F}_s + \vec{F}_b \tag{2.4}$$

in which the net force on the contents of the control volume, fluid and solid, which can be divided into surface forces and body forces, is equal to the rate of accumulation of momentum within the control volume plus the net flux of momentum through the surface of the control volume. In a steady flow the first term is again zero. For steady, incompressible, one-dimensional flow through a pipe, the component momentum equation along the direction of flow is

$$\vec{F}_{net} = \rho Q\left(\vec{V}_2 - \vec{V}_1\right) \tag{2.5}$$

in which we assume flow into the pipe at the left section, section 1, and flow from the pipe at the right section, section 2. If the pipe cross-sectional area is constant between the end sections and the pipe is straight, then the velocities are equal, and the equation simplifies further to $\vec{F}_{net} = 0$. Since Eq. 2.5 is a vector equation, it can always be written in component form; for two-dimensional flow in the x-y plane, the components of this equation are

$$\begin{aligned} \sum F_x &= \left(\rho Q V_x\right)_2 - \left(\rho Q V_x\right)_1 = \left(\rho A V_x^2\right)_2 - \left(\rho A V_x^2\right)_1 \\ \sum F_y &= \left(\rho Q V_y\right)_2 - \left(\rho Q V_y\right)_1 = \left(\rho A V_y^2\right)_2 - \left(\rho A V_y^2\right)_1 \end{aligned} \tag{2.5a,b}$$

2.1.2. ENERGY AND HYDRAULIC GRADE LINES

The Energy Grade Line, also called the Energy Line or simply EL, is a plot of the sum of the three terms in the work-energy equation, which is also the Bernoulli sum:

$$EL = \frac{V^2}{2g} + \frac{p}{\gamma} + z \tag{2.6}$$

Since each term has units of length, we can conveniently superimpose a diagram of the behavior of each energy term, and the sum, on a drawing of the physical flow problem. For example, a Pitot tube, inserted into a flow to cause locally at its tip a point of zero velocity so the velocity head is converted into additional pressure head there, will cause the liquid to rise to the elevation of the EL for that point in the flow.

The Hydraulic Grade Line, or HGL, is the sum of only the pressure and elevation heads. The sum of these two terms is also called the piezometric head, which can be conveniently measured by a piezometer tube inserted flush into the side of a pipe. It is also important to recognize that any HGL can quickly be located on a diagram if the EL has already been located; we simply measure downward by the amount of the local velocity head from the EL to locate the HGL.

Figure 2.1 portrays the relation of the individual head terms to the EL and HGL and the head that is lost between sections 1 and 2.

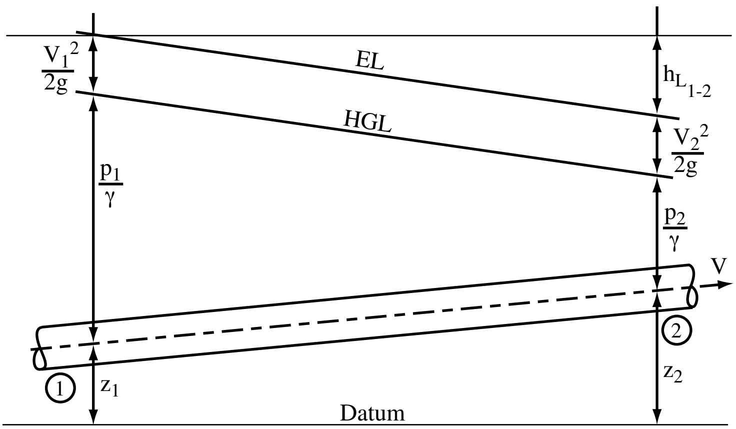


Figure 2.1 The EL and HGL in relation to individual heads and the head loss.

2.2 HEAD LOSS FORMULAS

The head loss term in Eq. 2.3 is responsible for representing accurately two kinds of real-fluid phenomena, head loss due to fluid shear at the pipe wall, called pipe friction, and additional head loss caused by local disruptions of the fluid stream. The head loss due to pipe friction is always present throughout the length of the pipe. The local disruptions, called local losses, are caused by valves, pipe bends, and other such fittings. Local losses may also be called minor losses if their effect, individually and/or collectively, will not contribute significantly in the determination of the flow; indeed, sometimes minor losses are expected to be inconsequential and are neglected. Or a preliminary survey of design alternatives may ignore the local or minor losses, considering them only in a later design stage. Each type of head loss will now be considered further.

2.2.1. PIPE FRICTION

If we were to select a small cylindrical control volume within a section of circular pipe, with coordinates s in the flow direction and r radially, in steady flow and subject this volume to analysis by the momentum equation, Eq. 2.4, we would find that the mean fluid shear stress τ, as a function of the radius r from the pipe centerline, is

$$\tau = -\frac{r}{2}\gamma\frac{\partial}{\partial s}\left(\frac{p}{\gamma}+z\right) \tag{2.7}$$

from which we learn two important facts:

1. The fluid shear stress τ varies linearly in a pipe cross-section, from zero at the centerline to a maximum, called τ_w, at the pipe wall where $r = D/2$.

2. In the absence of a streamwise gradient of the piezometric head $(p/\gamma + z)$, the fluid shear stress will be zero, and consequently no flow will exist at that section.

If we now expand the control volume to fill the pipe cross-section and integrate Eq. 2.7 over a length L of pipe of constant diameter, we learn with a bit of further work that the frictional head loss h_L over that length is directly related to the wall shear stress τ_w via

$$\tau_w = \gamma\, h_L \frac{D}{4L} \tag{2.8}$$

But this equation does not relate head loss to the mean velocity V or the discharge Q.

2.2.2. DARCY-WEISBACH EQUATION

The completely general functional relation $\tau_w = F(V, D, \rho, \mu, e)$ between the wall shear stress τ_w and the mean velocity V, pipe diameter D, fluid density ρ, and viscosity μ, and the equivalent sand-grain roughness e can be reduced by dimensional analysis to

$$\frac{\tau_w}{\rho V^2} = F\left(\frac{VD\rho}{\mu}, \frac{e}{D}\right) = \frac{f}{8} \tag{2.9}$$

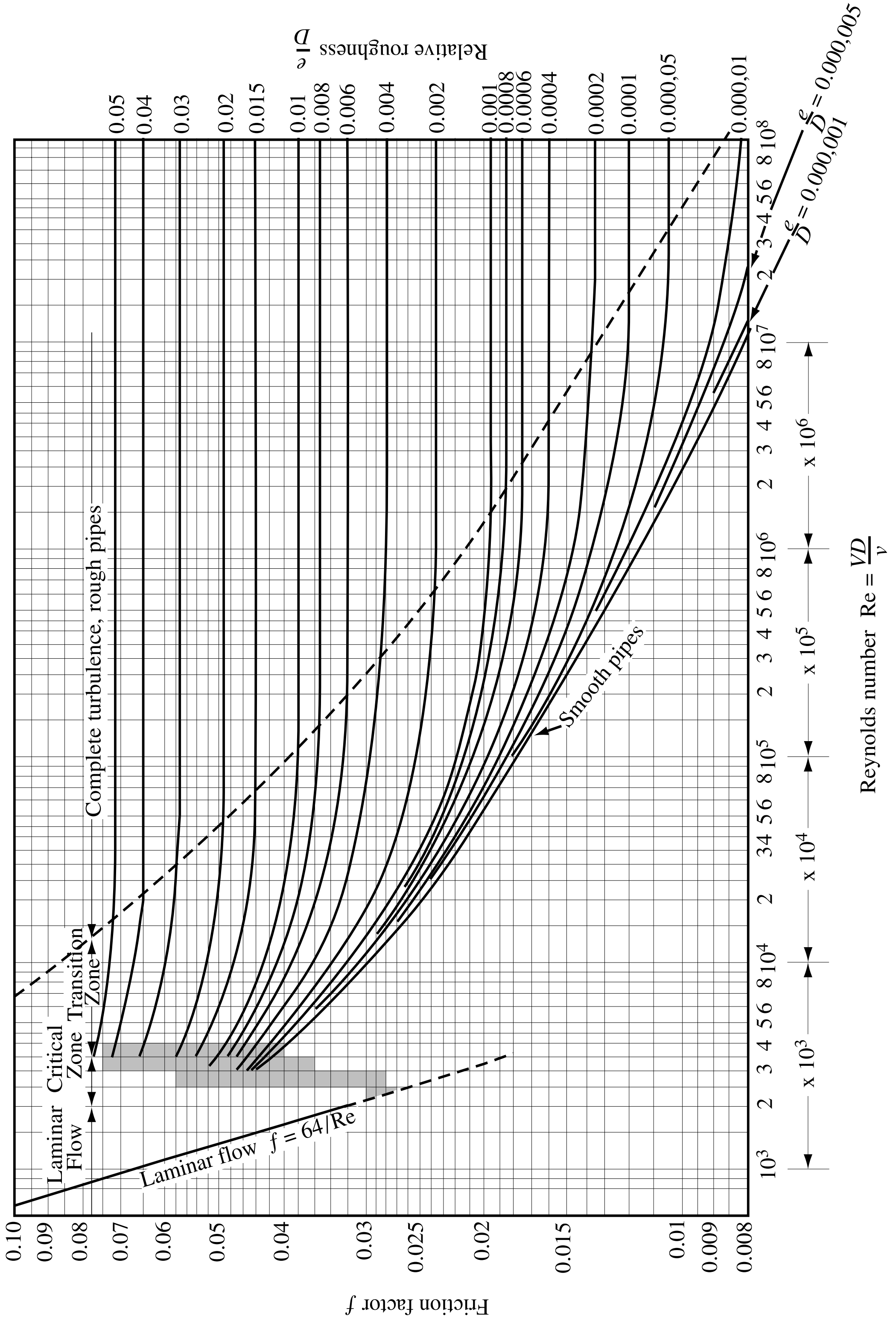
The combination of Eqs. 2.8 and 2.9 to eliminate the wall shear stress produces the fundamentally most sound and versatile equation for frictional head loss in a pipe, the Darcy-Weisbach equation:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{Q^2}{2gA^2} \tag{2.10}$$

In Eq. 2.9 the friction factor f (and the factor 8 to coincide with the historical development of the subject) is introduced as a shorthand notation for the function F. It is a function of the pipe Reynolds number $Re = VD\rho/\mu = VD/\nu$ and the equivalent sand-grain roughness factor e/D. For each pipe material either a single value or range of e/D values has been established; Table 2.1 presents common values for several materials.

Table 2.1 PIPE ROUGHNESSES

Material	e, mm	e, in
Riveted Steel	0.9 - 9.0	0.035 - 0.35
Concrete	0.30 - 3.0	0.012 - 0.12
Cast Iron	0.26	0.010
Galvanized Iron	0.15	0.006
Asphalted Cast Iron	0.12	0.0048
Commercial or Welded Steel	0.045	0.0018
PVC, Drawn Tubing, Glass	0.0015	0.000 06



From L. F. Moody, "Friction factors for Pipe Flow," *Trans.* A.S.M.E., Vol. 66, 1944, with permission.

Figure 2.2 The Moody diagram for the Darcy-Weisbach friction factor f.

Because commercially available pipes of any material display some heterogeneity or unevenness in roughness, any friction factor or its empirical equivalents can not be known with multiple-digit precision. The functional behavior of f is displayed fully in the Moody diagram in Fig. 2.2.

In the Moody diagram, which is Fig. 2.2, we see several zones that characterize different kinds of pipe flow. First we note that the plot is logarithmic along both axes. Below the Reynolds number $Re = 2100$ (some authors prefer 2300) there is only one line, which can be derived solely from the laminar, viscous flow equations without experimental input; the resulting friction factor for laminar flow is $f = 64/Re$. Because there is only one line in this region, we say all pipes are hydraulically smooth in laminar flow. Then for Reynolds numbers up to, say, 4000 is a so-called "critical" zone in which the flow changes from laminar flow to weakly turbulent flow. For still larger Reynolds numbers we find three flow zones that deserve comment:

1. A dashed line borders the upper right portion of the plot. In that zone, called wholly rough flow or the region of complete turbulence for rough pipes, the friction factor f is a function only of the roughness e/D and not of Re. For relatively rough pipes and/or large discharges this is a common flow type. Thus, if the pipe material is known so e/D is known, then the value of f follows immediately.

2. The lowest line is called the smooth-pipe line and is described by the empirical equation

$$1/\sqrt{f} = 2\log_{10}\left(Re\sqrt{f}\right) - 0.8 \tag{2.11}$$

This line continually slopes and never becomes horizontal, as in the wholly rough flow zone, so f always depends on Re. Since the flow in PVC pipe is described by this line, it has become increasingly important in some fields in recent years.

3. Between zones 1 and 2 is an important transitional band, called the turbulent transition zone, in which f depends on both Reynolds number and e/D. The Colebrook-White equation

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log_{10}\left(\frac{e}{D} + \frac{9.35}{Re\sqrt{f}}\right) \tag{2.12}$$

is used, especially in computer codes, to replicate numerically the data in this zone of the Moody diagram. In spite of our prior caution about limited precision in friction factors, we sometimes need to allow more significant figures in computations to assure that the computer algorithms do indeed converge. And additional significant figures in computed values are also an aid in checking the success of computational examples, so we will sometimes present results in this book with more digits for these reasons, even though practical considerations may not seem to warrant it.

Table 2.2 summarizes the relations that describe the Darcy-Weisbach friction factor f.

Early in Chapter 5 procedures will be described for the computer solution of the Colebrook-White and Darcy-Weisbach equations as an alternative to the use of the Moody diagram itself. Readers who own a pocket calculator with the ability to solve implicit equations should seriously consider writing the Colebrook-White equation, Eq. 2.12, into the calculator memory for use in routinely computing friction factor values.

2.2.3. EMPIRICAL EQUATIONS

Empirical head loss equations have a long and honorable history of use in pipeline problems. Their initial use preceded by decades the development of the Moody diagram, and they are still commonly used today in professional practice. Some prefer to continue to use such an equation owing simply to force of habit, while others prefer it to avoid some of the difficulties of determining the friction factor in the Darcy-Weisbach equation.

As is common with empirical equations, each contains a constant that depends on the chosen unit system. Possibly the most widely used of these equations is the Hazen-Williams equation.

Table 2.2 DARCY-WEISBACH FRICTION EQUATIONS

Type of Flow	Equation for f	Range
Laminar	$f = 64/Re$	$Re < 2100$
Smooth pipe	$1/\sqrt{f} = 2\log_{10}\left(Re\sqrt{f}\right) - 0.8$	$Re > 4000$ and $e/D \to 0$
Transitional Colebrook-White Eq.	$\frac{1}{\sqrt{f}} = 1.14 - 2\log_{10}\left(\frac{e}{D} + \frac{9.35}{Re\sqrt{f}}\right)$	$Re > 4000$
Wholly Rough	$\frac{1}{\sqrt{f}} = 1.14 - 2\log_{10}\left(\frac{e}{D}\right)$	$Re > 4000$

To compute the discharge, the equation takes the forms

$$Q = 1.318\, C_{HW} A\, R^{0.63} S^{0.54} \qquad \text{ES units} \tag{2.13}$$

or

$$Q = 0.849\, C_{HW} A\, R^{0.63} S^{0.54} \qquad \text{SI units} \tag{2.14}$$

in which C_{HW} is the Hazen-Williams roughness coefficient, $S = h_f/L$ is the slope of the energy line, $R = A/P$ is the hydraulic radius, A is the cross-sectional area, and P is the wetted perimeter, so that pipes flowing full will always have $R = D/4$. Table 2.3 gives values for C_{HW} for some common pipe materials.

Another empirical equation, which was originally and primarily developed for flow in open channels, is the Manning equation

$$Q = \frac{1.49}{n} A\, R^{2/3} S^{1/2} \qquad \text{ES units} \tag{2.15}$$

or

$$Q = \frac{1}{n} A\, R^{2/3} S^{1/2} \qquad \text{SI units} \tag{2.16}$$

The pipe boundary roughness is described by the Manning n, for which some values are listed in Table 2.3.

Table 2.3 HAZEN-WILLIAMS AND MANNING ROUGHNESSES

Pipe Material	C_{HW}	n
PVC	150	0.009
Very Smooth	140	0.010
Cement-lined Ductile Iron	140	0.012
New Cast Iron, Welded Steel	130	0.014
Wood, Concrete	120	0.016
Clay, New Riveted Steel	110	0.017
Old Cast Iron, Brick	100	0.020
Badly corroded Cast Iron	80	0.035

A comparison of the Hazen-Williams and Manning equations with the Darcy-Weisbach equation would show conclusively that the empirical equations are much more limited in their ranges of applicability. Each is applicable only to the turbulent flow of water. The Manning equation is only valid for flows which correspond to the wholly rough flow regime in pipes. If the Hazen-Williams equation were plotted on the doubly-logarithmic Moody chart, it would appear as a family of sloping (the slope is - 0.15) straight lines across the turbulent transitional flow portion of the Moody diagram; hence each choice of a Hazen-Williams coefficient can at most replicate only a part of an individual *e/D* line on the Moody diagram.

2.2.4. EXPONENTIAL FORMULA

It will later be advantageous to express the head loss in each pipe in a network by an exponential formula so one presentation of the theory covers all cases, regardless of whether the Darcy-Weisbach equation, the Hazen-Williams equation or the Manning equation is used to express the head loss as a function of discharge:

$$h_f = KQ^n \tag{2.17}$$

The values for K and n change, depending on whether the Darcy-Weisbach, Hazen-Williams, or Manning equation is used.

The Hazen-Williams and Manning equations can be solved for h_f and put in the form of the exponential formula. For the Hazen-Williams equation the exponent is $n = 1.852$ and the coefficient K is

$$K = \frac{C_K L}{C_{HW}^{1.852} D^{4.87}} \tag{2.18}$$

For the Manning equation the exponent is $n = 2$ and K is

$$K = \frac{C_K n^2 L}{D^{5.33}} \tag{2.19}$$

in which the dimensional constant C_K is given for various choices of units in Table 2.4.

Table 2.4 The Coefficient C_K

Units of D	Units of L	Hazen-Williams C_K in Eq. 2.18	Manning C_K in Eq. 2.19
ft	ft	4.73	4.66
in	ft	8.53×10^5	2.65×10^6
m	m	10.67	10.29

To find K and n for the Darcy-Weisbach equation, we note that f can be approximated over a limited range on the Moody diagram by an equation of the form

$$f = a / Q^b \tag{2.20}$$

This equation plots as a straight line on the Moody diagram (a log-log plot) if a and b are constant. Substituting Eq. 2.20 into the Darcy-Weisbach equation and grouping terms gives

$$n = 2 - b \tag{2.21}$$

and

$$K = \frac{aL}{2gDA^2} \tag{2.22}$$

Hence a determination of K and n for use in Eq. 2.17 is equivalent to a selection of values for a and b in Eq. 2.20 which cause that equation to approximate f over the expected discharge range. If the chosen range is too large, then K and n will cause Eq. 2.17 to produce frictional head losses that differ slightly from predictions that are obtained directly from the Darcy-Weisbach and Colebrook-White equations. If the chosen range is too small, then the actual discharge may fall outside this range, and K and n should be redetermined. To obtain a and b, select an appropriate Reynolds number (discharge, or velocity) range that brackets the expected discharge Q. Solve the Colebrook-White equation with these two Reynolds numbers Re_1 and Re_2, obtaining both f_1 and f_2 and the corresponding discharges Q_1 and Q_2. Taking the logarithm (either natural or base-10 logarithms can be used) of both sides of Eq. 2.20 now gives two equations for a and b:

$$\begin{aligned} \ln f_1 &= \ln a - b \ln Q_1 \\ \ln f_2 &= \ln a - b \ln Q_2 \end{aligned} \tag{2.23}$$

Subtracting the second equation from the first and solving for b produces

$$b = \frac{\ln(f_1 / f_2)}{\ln(Q_2 / Q_1)} \tag{2.24}$$

Then a can be obtained as

$$a = f_1 Q_1^b \tag{2.25}$$

Calculations to determine K and n can readily be done with a pocket calculator, but if many are needed, the computations should be implemented in a spreadsheet or a computer program. FORTRAN program 2.1, PIPK_N, is included on the CD for this purpose.

Example Problem 2.1

Determine the values of K and n in the exponential formula for the three pipes in the table which follows ($\nu = 1.217 \times 10^{-5}$ ft^2/s or $\nu = 1.13 \times 10^{-6}$ m^2/s):

Pipe	**Type**	**Length**	**Diameter**	$e \times 10^4$	C_{HW}	n	Approx. Q
1	PVC	1000 ft	8 in	0.08 in	150	0.009	2.5 ft^3/s
2	Riveted steel	1000 ft	8 in	2.5 ft	110	0.015	0.8 ft^3/s
3	Ductile iron	3000 m	300 mm	500 mm	140	0.011	0.4 m^3/s

Only the solution details for pipe 1 are given here, but for practice the other answers should be verified. For the Hazen-Williams and Manning equations K and n are computed from Eqs. 2.18 and 2.19, respectively. For the Hazen-Williams equation n_1 =

1.852 and K_1 = 4.73(1000)/($150^{1.852}0.667^{4.87}$) = 3.17; for the Manning equation n_1 = 2.000, K_1 = 4.66$(0.009)^2$1000/$0.667^{5.33}$ = 3.27. For the Darcy-Weisbach equation first select two discharges that span the expected range, say Q_1 = 2 ft^3/s and Q_2 = 3 ft^3/s. Next, from the Colebrook-White equation find the friction factors f corresponding to these two discharges, or f_1 = 0.01435815, f_2 = 0.01332301. For the accuracy that we require, these values must be obtained from a pocket calculator or other computational equipment and not just read from a Moody Diagram. Next compute $b = \{\log f_1/f_2\}/\{\log Q_2/Q_1\}$ = 0.18454, leading to $n = 2 - b$ = 1.81546, and $a = f_1 Q_1^b$ = 0.016317, from which $K = aL/(2gDA^2)$ = 3.2649. The remainder of the computations for each determination of K and n for these three pipes is summarized in the following pair of tables:

Pipe	Q_1	Q_2	Re_1	Re_2	f_1	f_2	b	a
1	2.0	3.0	314000	471000	0.0145	0.0133	0.1845	0.0163
2	0.4	1.2	62800	188000	0.0200	0.0160	0.1993	0.0166
3	0.2	0.6	751000	2250000	0.0146	0.0134	0.0543	0.0133

	Darcy-Weisbach		**Hazen-Williams**		**Manning**	
Pipe	K	n	K	n	K	n
1	3.2649	1.8155	3.1773	1.852	3.2649	2.000
2	9.0692	1.8007	5.6431	1.852	9.0691	2.000
3	2296.1	1.9957	1194.7	1.852	2296.1	2.000

* * *

In summary, the best equation for computing the frictional head loss in a given pipe for a given discharge, or the best equation for the discharge if the head loss is known, regardless of the fluid, is the Darcy-Weisbach equation. The range of applicability for the empirical equations is much more restricted. Consequently, all engineers should consider using the Darcy-Weisbach equation in professional practice even if it is sometimes more difficult to use than the empirical equations.

2.2.5. LOCAL AND MINOR LOSSES

A local loss is any energy loss, in addition to that of pipe friction alone, caused by some localized disruption of the flow by some flow appurtenances, such as valves, bends, and other fittings. The actual dissipation of this energy occurs over a finite but not necessarily short longitudinal section of the pipe line, but it is an accepted convention in hydraulics to lump or concentrate the entire amount of this loss at the location of the device that causes the flow disruption and loss. If a loss is sufficiently small in comparison with other energy losses and with pipe friction, it may be regarded as a minor loss. Often minor losses are neglected in preliminary studies or when they are known to be quite small, as will often happen when the pipes are very long. However, some local losses can be so large or significant that they will never be termed a minor loss, and they must be retained; one example is a valve that is only partly open.

Normally, theory alone is unable to quantify the magnitudes of the energy losses caused by these devices, so the representation of these losses depends heavily upon experimental data. Local losses are usually computed from the equation

$$h_L = K_L \frac{V^2}{2g} \tag{2.26}$$

in which $V = Q/A$ is normally the downstream mean velocity. For enlargements the following alternative formula applies:

$$h_L = K_L \frac{(V_1 - V_2)^2}{2g} \tag{2.27}$$

in which V_1 and V_2 are, respectively, the upstream and downstream velocities. In Eq. 2.27 the loss coefficient K_L is unity for a sudden enlargement and takes on values between 0.2 and 1.2 for assorted gradual conical enlargements. The head loss for flow from a pipe into a reservoir is a special but important case of Eq. 2.27, called the exit loss; in this case $K_L = 1$ and $V_2 = 0$, independent of the geometric details of the pipe exit shape.

Local loss coefficients K_L for some common valve and pipe fittings are listed in Table 2.5. The energy losses for these fittings are mostly a consequence of fluid turbulence caused by the device rather than by secondary motions which persist downstream. Normally a locally accelerating flow will cause much less energy loss than does a decelerating flow. If deceleration is too rapid, it causes separation, which results in additional turbulence and a high velocity in the non-separated region. Some additional loss coefficients from specific valve manufacturers and coefficient values as a function of the amount of the valve opening can be found in Appendix C.

Table 2.5 Loss Coefficients for Fittings

Fitting	K_L
Globe valve, fully open	10.0
Angle valve, fully open	5.0
Butterfly valve, fully open	0.4
Gate valve, fully open	0.2
3/4 open	1.0
1/2 open	5.6
1/4 open	17.0
Check valve, swing type, fully open	2.3
Check valve, lift type, fully open	12.0
Check valve, ball type, fully open	70.0
Foot valve, fully open	15.0
Elbow, 45°	0.4
Long radius elbow, 90°	0.6
Medium radius elbow, 90°	0.8
Short radius (standard) elbow , 90°	0.9
Close return bend, 180°	2.2
Pipe entrance, rounded, $r/D < 0.16$	0.1
Pipe entrance, square-edged	0.5
Pipe entrance, re-entrant	0.8

An abrupt contraction has first a region of accelerating flow, followed by a region of decelerating flow caused by flow separation. Though the region of accelerating flow may be larger, the head loss is attributable principally to the deceleration and separation which occurs immediately downstream from the contraction. The local loss coefficient for a pipe contraction is given in Fig 2.3.

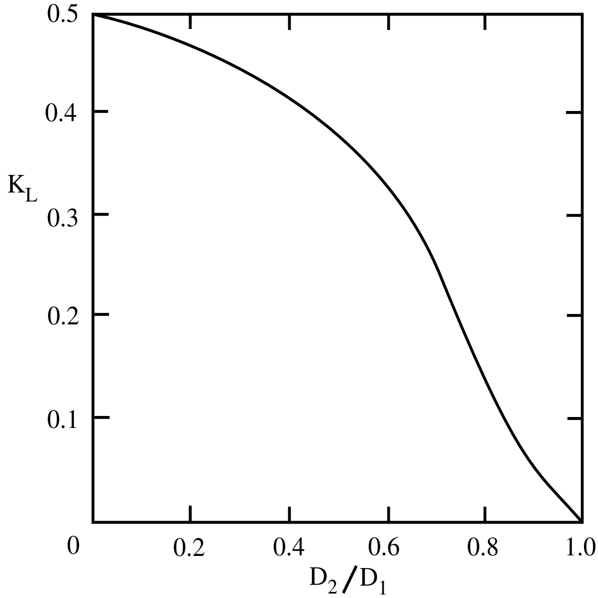


Figure 2.3 Local loss coefficient for a sudden contraction as a function of diameter ratio.

2.3 PUMP THEORY AND CHARACTERISTICS

The addition of mechanical energy $h_m = h_p$ per unit weight to a fluid stream is accomplished by pumps, as was mentioned with Eq. 2.3. Although positive displacement pumps sometimes play a role, by far the more important class of pumps contains a rotating impeller to inject energy, in the form of an increased pressure head, into the flowing fluid in the pipe. The characteristic shape of the impeller varies with the operating regime of the pump. The energy addition is called the net head h_p of the pump. The water power P_w that is delivered to the fluid stream is the product of the net head, the discharge, and the unit weight of the fluid, or $P_w = Q\gamma h_p$. The mechanical power to operate the pump must be larger; it is called the brake horsepower or $bhp = T\omega$, in which T and ω are the torque and angular velocity of the pump drive shaft. The ratio $\eta = P_w/bhp$ is the pump efficiency, which may be larger than 0.8 for large and/or efficient pumps that are operating near their best efficiency point (bep), also called the design point, but which may be much lower for small, old or worn pumps.

Pumps are sufficiently complex that they cannot be designed on the basis of theory alone. To refine a new or revised design, model experiments are first conducted, and after success is achieved with the model, then the full-scale or prototype pump is built. The results of dimensional analysis are used to relate the model and prototype to each other. First we assume that the model and prototype are similar in shape, called geometric similarity, and second that the velocity fields also have a similar shape, called kinematic similarity. Devices satisfying these requirements are called homologous. The nondimensional parameters that are used to complete the scaling process are called affinity or scaling laws. They are three in number and are called the head, discharge, and power coefficients C_H, C_Q, and C_P, respectively:

$$C_H = \frac{gh_p}{N^2D^2}; \quad C_Q = \frac{Q}{ND^3}; \quad C_P = \frac{P}{\rho N^3 D^5} \tag{2.28}$$

The diameter of the rotating impeller is D. These coefficients may be computed in any consistent set of units. If plots of one nondimensional coefficient vs. another are constructed, homologous units having different sizes and/or rotative speeds can be related to each other. Or one can say for homologous units that

$$\left(\frac{h_p}{N^2D^2}\right)_1 = \left(\frac{h_p}{N^2D^2}\right)_2; \quad \left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2; \quad \left(\frac{P}{\rho N^3D^5}\right)_1 = \left(\frac{P}{\rho N^3D^5}\right)_2 \qquad (2.29)$$

In a way these relations are more versatile than Eqs. (2.28) because the units no longer must lead to a truly nondimensional group so long as each variable is measured in the same units. Thus rotative speed can be in rad/s, rev/s or rev/min. If pumps 1 and 2 have the same diameter, Eqs. 2.29 show how h_p, Q, and P respond to changes in N, or for fixed N we see how the variables scale with the diameter D.

The specific speed N_S is a parameter for homologous pumps that contains the important pump variables, the discharge Q and head h_p, without containing the unit size D; different ranges of this parameter therefore capture the essential differences in shape, not mere size, that separates the performance of one kind of pump from another type of pump. The nondimensional form of pump specific speed, with N in rad/s, is

$$N_S = \frac{NQ^{1/2}}{\left(gh_p\right)^{3/4}} \qquad (2.30)$$

In the United States, however, for many years it has been customary instead to use

$$N_S' = \frac{(rev/min)(gal/min)^{1/2}}{\left[h_p(ft)\right]^{3/4}} \qquad (2.31)$$

which is clearly far from dimensionless. Based on specific speed, pumps can be classified into three categories, based on impeller shape, as given in Table 2.6.

Table 2.6
Pump Type vs. Specific Speed

	Radial Flow	Mixed Flow	Axial Flow
N_S	$N_S < 1.46$	$1.46 < N_S < 3.7$	$3.7 < N_S$
N'_S	$500 < N'_S < 4000$	$4000 < N'_S < 10000$	$10000 < N'_S$

For relatively low specific speed the most efficient pump uses a radial-flow impeller, that is, the primary flow direction through the impeller is radially outward from the axis of rotation of the impeller; this pump type has several names but is usually called a centrifugal pump. For the highest specific speed range, the flow through the impeller is nearly parallel to the axis of rotation and is called axial flow in pumps that are termed propeller pumps. The transition from radial to axial flow occurs over the intermediate range called mixed flow; the pumps are called turbine pumps. Certainly there is some overlap between regions, and different authorities cite somewhat differing values for the ends of the ranges.

The performance of an individual pump, or a family of pumps having the same pump casing and several impellers that differ only in size, is usually described by a set of pump characteristic curves, or simply pump curves, that are developed by manufacturers. Appendix B contains eight sets of pump characteristic curves. Across the upper portion of each figure is a plot of head (per stage) vs. discharge; although these curves are usually approximated as straight lines or parabolic curves for subsequent analysis, the reader will quickly notice that the actual head curves are more complex. A change in the shape of a curve normally means that the flow pattern within the pump has also changed. Crossing the set of head curves are contour lines of constant efficiency. By each contour is the numerical percentage value of the efficiency; usually the values are between 70 and 85%. Across the bottom of each plot is a set of curves that relate brake horsepower to the discharge; we see that straight lines would fit most of these lines rather well but not perfectly. Finally, in the upper right corner of each plot is a plot of NPSH vs. discharge.

The Net Positive Suction Head (NPSH) for a pump is used to determine the head z_i that is needed at the pump inlet so that cavitation is avoided in the pump. Cavitation is the conversion of liquid into vapor by locally low absolute pressure. The onset of cavitation can also be inferred from tests to note impaired operational efficiency, excessive noise and possibly damage to the pump. A useful form of the NPSH relation is

$$NPSH = \frac{p_{atm}}{\gamma} - \frac{p_v}{\gamma} - h_L - z_i \qquad (2.32)$$

in which p_{atm} and p_v are the atmospheric and vapor pressure of the liquid, h_L is the head loss in the inlet piping (often included in NPSH itself), and z_i is the highest allowable or safe elevation for the pump impeller inlet. For the operating discharge, read NPSH from the pump curve, and z_i can then be computed.

2.4 STEADY FLOW ANALYSES

This section will touch on several kinds of steady flow problems. Although the exponential formula or the empirical head loss equations could be used for this purpose, we choose to employ the versatile Darcy-Weisbach formula here, sometimes simplifying by assuming the value of the friction factor. We will look at series pipe flow first, with and without consideration of local losses and a pump in line. Flow through parallel pipes will follow, and the section concludes with a look at multiple-reservoir problems.

2.4.1. SERIES PIPE FLOW

The basic tools for analysis here are Eqs. 2.2, 2.3 and 2.10, which are the continuity, work-energy and Darcy-Weisbach equations. All series pipe flow problems fit one of three computational categories, depending on which factors are known or given and which is sought, as listed in Table 2.7:

Table 2.7 Problem Types

Category	Known Quantities:	To Find:
1	Q, pipeline properties	h_L
2	h_L, pipeline properties	Q
3	Q, h_L	Smallest size D

The problems in categories 1 and 2 are analysis problems; analysis of type 1 problems is direct, without iteration, but iteration may be required for the second group. Category 3 is a design problem, which normally requires more assumptions and more iterative computations to solve. Pipeline properties include the length, diameter and material type so that the relative roughness is known.

Example Problem 2.2

A cast iron pipe connects two reservoirs. The line is 1200 ft long and has a diameter of 12 in. If it were to convey 8 ft^3/s, what would be the frictional head loss for this pipe? [In this and following examples in this chapter, we assume the fluid is 60°F water with a kinematic viscosity $\nu = 1.2\text{x}10^{-5}$ ft²/s.]

This problem is a type 1 problem. The mean velocity in the pipe is

$$V = \frac{Q}{A} = \frac{8}{\pi/4} = 10.18 \text{ ft/s.}$$

Thus the pipe Reynolds number is

$$Re = \frac{VD}{\nu} = \frac{(10.18)(1)}{1.2x10^{-5}} = 8x10^5$$

Upon consulting Table 2.1 for cast iron pipe, we determine $e/D = 0.010/12 = 0.00083$. From the Moody diagram, Fig. 2.2, we find $f = f(Re,\ e/D) = 0.0185$. The Darcy-Weisbach equation, Eq. 2.10, then produces

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = (0.0185)\frac{1200}{12/12}\frac{(10.18)^2}{2(32.2)} = 35.7\ ft$$

* * *

Example Problem 2.3

The pipe in Example Problem 2.2 actually connects two reservoirs having a difference in water surface level of only 20 ft, so that pipe is clearly incapable of conveying 8 ft³/s.

Now a new pipe has been installed between the reservoirs. It is made of welded steel and has a diameter of 18 in.

(a) If only pipe friction is considered, what is the new discharge?

(b) If local losses for a sharp-edged entrance, a fully open gate valve near the pipe exit, and the pipe exit itself are also considered, how much does the computed discharge change?

(c) If the gate valve in part (b) were only 1/4 open, what would then be the discharge?

All parts of this problem belong to category 2, since now Q and not h_L is sought.

(a) We are told to assume in this case that

$$z_1 - z_2 = 20ft = h_f = f\frac{L}{D}\frac{V^2}{2g}$$

From Table 2.1 for welded steel, we find $e/D = 0.0018/18 = 0.0001$. If the flow is assumed to be in the wholly rough flow zone of the Moody diagram, Fig. 2.2, $f = 0.012$. Hence

$$h_f = 20 = (0.012)\frac{1200}{18/12}\frac{V^2}{2(32.2)}$$

and $V = 11.6$ ft/s. Now we must check $Re = VD/\nu = 11.6(18/12)//1.2\text{x}10^{-5} = 1.4\text{x}10^6$, which is not in the wholly rough zone; this Re and the value of e/D imply $f = 0.013$. Using 0.013 in place of 0.012 leads to $V = 11.1$ ft/s. The small change in Re will cause no further change in f, so the discharge can now be computed as

$$Q = VA = (11.1)\frac{\pi}{4}\left(\frac{18}{12}\right)^2 = (11.1)(1.77) = 19.6\ ft^3/s$$

(b) In this case

$$20 = \sum h_L = \left(K_{ent.} + f\frac{L}{D} + K_{valve} + K_{exit} \right) \frac{V^2}{2g}$$

The velocity head factors out only because each loss term is associated with the same pipe size, area and velocity. Table 2.5 supplies 0.5 and 0.2 for the entrance and valve loss coefficients; always $K_{exit} = 1.0$. From part (a) we take our first estimate of the friction factor as 0.013, leading to

$$20 = \left(0.5 + 0.013\frac{1200}{18/12} + 0.2 + 1.0 \right) \frac{V^2}{2g}$$

and yielding V = 10.3 ft/s. Again check $Re = VD/\nu = 10.3(18/12)//1.2\text{x}10^{-5} = 1.3\text{x}10^6$, so the initial estimate of f is adequate. Now Q = (10.3)(1.77) = 18.2 ft^3/s so the discharge has decreased by 1.4 ft^3/s, a bit under 8%, as a consequence of considering the local losses.

(c) When the gate valve is only 1/4 open, we find from Table 2.5 that the valve loss coefficient has increased from 0.2 to 17.0. The valve loss remains a local loss, but it is no longer in any way a minor loss, since it will cause more head loss than the pipe friction term. Replacing 0.2 in part (b) by 17.0, we recompute and find V = 6.68 ft/s. The new, lower Reynolds number is $Re = 8.4\text{x}10^5$, so the new friction factor is $f = 0.0135$. A recomputation of the velocity gives V = 6.63 ft/s, and so Q = 11.7 ft^3/s, a decrease of about one third from the discharge in part (b).

* * *

2.4.2. SERIES PIPE FLOW WITH PUMP(S)

The solution of pipeflow problems involving pumps normally requires us to read data from pump characteristic curves. However, if we prefer to use a computer to solve these problems, such readings can no longer be done in this way. But the resolution of this problem is not difficult. As part of the computer solution of this kind of problem, we supply sufficient data to the program so that the head h_p can be expressed as a polynomial in discharge that fits the pump-curve data.

Let the pump characteristic curve for the head h_p be expressed by a second-order polynomial $h_p = AQ^2 + BQ + C$, in which the coefficients A, B, and C are determined by the use of three (h_p, Q) data pairs that bracket the expected range of operation of the pump. To obtain the coefficients, we write three equations by substituting each data pair into the polynomial to obtain

$$\begin{aligned} AQ_1^2 + BQ_1 + C &= h_{p1} \\ AQ_2^2 + BQ_2 + C &= h_{p2} \\ AQ_3^2 + BQ_3 + C &= h_{p3} \end{aligned} \tag{2.33}$$

In matrix notation Eq. 2.33 becomes

$$\begin{bmatrix} Q_1^2 & Q_1 & 1 \\ Q_2^2 & Q_2 & 1 \\ Q_3^2 & Q_3 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} h_{p1} \\ h_{p2} \\ h_{p3} \end{Bmatrix} \tag{2.34}$$

which can be solved in various ways to determine the coefficients.

An alternative approach is to use the Lagrangian interpolation. Lagrange's formula is

$$h_p = \frac{(Q-Q_2)(Q-Q_3)}{(Q_1-Q_2)(Q_1-Q_3)} h_{p1} + \frac{(Q-Q_1)(Q-Q_3)}{(Q_2-Q_1)(Q_2-Q_3)} h_{p2} + \frac{(Q-Q_1)(Q-Q_2)}{(Q_3-Q_1)(Q_3-Q_2)} h_{p3} \quad (2.35)$$

The head h_p is again expressed as a quadratic equation in Q, but the terms are rearranged from the earlier approach. The coefficients A, B, and C can be found by expanding the numerators. Letting

$$\begin{aligned} c_1 &= h_{p1}/(Q_1-Q_2)(Q_1-Q_3) \\ c_2 &= h_{p2}/(Q_2-Q_1)(Q_2-Q_3) \\ c_3 &= h_{p3}/(Q_3-Q_1)(Q_3-Q_2) \end{aligned} \quad (2.36)$$

we find

$$\begin{aligned} A &= c_1 + c_2 + c_3 \\ B &= -2\left[(Q_2+Q_3)c_1 + (Q_3+Q_1)c_2 + (Q_1+Q_2)c_3\right] \\ C &= Q_2Q_3c_1 + Q_3Q_1c_2 + Q_1Q_2c_3 \end{aligned} \quad (2.37)$$

Irrespective of which approach is used, the results can be inserted in a computer program so that the need to read data from a pump characteristic curve during the solution process is avoided. Additional uses of such polynomial representations and interpolations will be found in later chapters, including Chapters 4, 5, and 10.

Example Problem 2.4

A single-stage Ingersoll-Dresser 15H277 pump, outfitted with the largest impeller (Refer to Appendix B for the pump characteristic curves), is used to pump water from a reservoir at elevation 1350 ft to another reservoir at elevation 1425 ft. The line is 6000 ft long and 18 in. in diameter with an equivalent sand grain roughness $e = 0.015$ in. ($\nu = 1.14\text{x}10^{-5}$ ft^2/s) Neglecting local losses, compute the discharge in the pipeline.

We begin by applying the work-energy equation, Eq. 2.3, between the two reservoir water surfaces, points 1 and 2:

$$1350 = 1425 + h_f - h_p$$

or

$$h_p = 75 + f\frac{L}{D}\frac{Q^2/A^2}{2g} = 75 + f\frac{6000}{1.5}\frac{Q^2}{2g(1.767)^2} = 75 + 19.9fQ^2$$

There are three unknowns in this equation: h_p, Q, and f. They must be determined by using this equation, the pump curve and the Colebrook-White equation. We shall obtain the solution in two ways, first by hand and then with the aid of a computer.

The hand solution begins by (a) selecting a trial discharge, (b) solving the Colebrook-White equation, Eq. 2.12, for f, (c) calculating h_p from the above work-energy equation, (d) comparing this h_p with the value that is read from the pump characteristic curve, and (e) repeating the process until the h_p's match, as summarized in the table:

(a) Q gal/min	Q ft^3/s	(b) f	(c) h_p ft	(d) hp, curve ft
3000	6.68	0.01961	92.4	103
3500	7.80	0.01950	98.6	88
3300	7.35	0.01951	96.0	95
3280	7.31	0.01954	95.8	95.8

The discharge is 3280 gal/min by this method.

The pump curve must be defined by an algebraic equation if the computer is to be used in solving for h_p, Q, and f. A second-order polynomial can be fit to the Ingersoll-Dresser 15H277 pump curve by applying Eqs. 2.33 and 2.34 and using the three data pairs (103.0, 6.68), (95.0, 7.35), and (88.0, 7.80). Equation 2.34 gives the matrix form of this problem as

$$\begin{bmatrix} Q_1^2 & Q_1 & 1 \\ Q_2^2 & Q_2 & 1 \\ Q_3^2 & Q_3 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{bmatrix} 44.62 & 6.68 & 1 \\ 54.02 & 7.35 & 1 \\ 60.84 & 7.80 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 103 \\ 95 \\ 88 \end{Bmatrix}$$

yielding a solution $A = -3.224$, $B = 33.293$, and $C = 24.472$ so that the pump curve is approximately

$$h_p = -3.224Q^2 + 33.293Q + 24.472$$

Using MathCAD, TK-Solver or some other mathematics application software to solve our three simultaneous equations leads to $h_p = 95.7$ ft, $Q = 7.30$ ft^3/s $= 3280$ gal/min, and $f = 0.019546$.

* * *

Example Problem 2.5

Repeat the problem in Example Problem 2.4 with two three-stage Ingersoll-Dresser 15H277 pumps in parallel; assume the smallest of the three impellers is used in each pump stage.

The pipeline analysis itself in unchanged; hence

$$h_p = 75 + 19.9 f Q^2$$

In this case h_p is the total head developed in the three stages of either of the two pumps. In addition, only half of the pipeline discharge passes through each pump. The table of trial solutions can be developed as

Pump Q gal/min	**Pipe Q** gal/min	f	**Right Side h_p** ft	**h_p/stage** ft	**Total h_p** ft
3000	6000	0.01921	143.3	67	201
3500	7000	0.01915	167.7	45	135
3300	6600	0.01917	157.5	54	162
3320	6640	0.01917	158.5	53	159

The total discharge is 6640 gal/min.

To set up the computer solution for this problem, we first obtain the polynomial approximation to the pump curve by setting up the matrix

$$\begin{bmatrix} Q_1^2 & Q_1 & 1 \\ Q_2^2 & Q_2 & 1 \\ Q_3^2 & Q_3 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{bmatrix} 44.69 & 6.685 & 1 \\ 54.02 & 7.35 & 1 \\ 60.84 & 7.80 & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 67 \\ 55 \\ 45 \end{Bmatrix}$$

resulting in $h_{p1} = -3.74643Q_1^2 + 34.536Q_1 + 3.552$ for one stage. To account for the number of stages, we multiply the coefficients by 3 so that $h_p = 3h_{p1}$. Since only half of the pipe flow passes through each of the parallel pumps $Q_1 = Q/2$. The final composite pump curve is therefore

$$\begin{aligned} h_p &= 3(-3.7464)(Q/2)^2 + 3(34.536)Q/2 + 3(3.552) \\ &= -2.8098Q^2 + 51.804Q + 10.656 \end{aligned}$$

Solving this equation, the Colebrook-White equation and the work-energy equation simultaneously gives $h_p = 159.4$ ft, $Q = 14.878$ ft^3/s = 6680 gal/min, and $f = 0.01917$.

* * *

2.4.3. PARALLEL PIPE FLOW, EQUIVALENT PIPES

In the flow of fluid in parallel pipes the roles of energy loss and discharge are reversed from their roles in series pipe flow: for a series of pipes, as we have seen earlier, the discharge is identical in each pipe of the series while the head losses are additive; for a set of parallel pipes between two common junctions the head loss between the two junctions is identical for each pipe while the total discharge is the sum of the individual discharges.

Since the analysis of flow in a series of pipes is more straightforward than the analysis of flow through a combination of pipes that includes parallel pipes as a part of the combination, it is advantageous to replace the set of parallel pipes by a single "equivalent pipe." This equivalent pipe, which is devised so it has the same head loss as the original set of parallel pipes and conveys the same total discharge, will in some cases allow the analyst to avoid the use of iteration in seeking a solution. In other cases the amount of iteration will be reduced.

The equivalent pipe formula can be constructed so it can be used with any pipe combination having head loss characteristics that can be described by the exponential formula, Eq. 2.17. Assume that pipes 1 and 2 are two parallel pipes with frictional losses described by KQ^n; then the equivalent pipe (unsubscripted) must satisfy

$$h_f = KQ^n = K_1Q_1^n = K_2Q_2^n \tag{2.38}$$

and

$$Q = Q_1 + Q_2 \tag{2.39}$$

By solving Eqs. 2.38 for Q_1 and Q_2 and inserting the results into Eq. 2.39, we find

$$\left(\frac{1}{K}\right)^{1/n} = \left(\frac{1}{K_1}\right)^{1/n} + \left(\frac{1}{K_2}\right)^{1/n} \tag{2.40}$$

For the remainder of the problem the equivalent variables K and Q are then used in place of the original parallel pipes. Once Q has been found, then a back-substitution into Eq. 2.38 determines Q_1 and Q_2. To treat several parallel pipes rather than two, simply add one additional term to the right side of Eq. 2.40 for each pipe that is in parallel.

Example Problem 2.6

Two reservoirs have a difference in water surface elevation of 40 ft. Water flows from the higher reservoir through 4000 ft of 12-in-diameter pipe, which then joins a pair of parallel 2000-ft-long pipes which end at the lower reservoir. One parallel pipe has a 10-in diameter; the diameter of the other pipe is 8 in. For simplicity, assume $f = 0.02$ for all pipes. Find the discharge in each pipe between the two reservoirs.

In this problem we use the exponential formula for head loss. For each pipe $n = 2$ and

$$K = f\frac{L}{D}\frac{1}{2g}\frac{1}{A^2}$$

With the given data $K_{12} = 2.01$, $K_{10} = 2.51$ and $K_8 = 7.65$. The equivalent pipe coefficient K_e is found from

$$\left(\frac{1}{K_e}\right)^{1/2} = \left(\frac{1}{K_{10}}\right)^{1/2} + \left(\frac{1}{K_8}\right)^{1/2} = \left(\frac{1}{2.51}\right)^{1/2} + \left(\frac{1}{7.65}\right)^{1/2}$$

or $K_e = 1.014$. Omitting local losses, the work-energy equation for the change in water surface elevation ΔWS between the reservoirs is

$$\Delta WS = 40 = \left(K_{12} + K_e\right)Q^2$$

and $Q = 3.64$ ft^3/s. From Eq. 2.38 we then find $Q_{10} = 2.31$ ft^3/s and $Q_8 = 1.33$ ft^3/s.

* * *

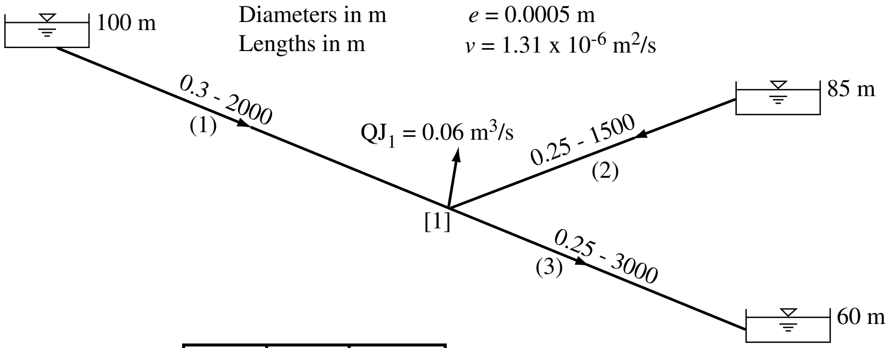
If the friction factor is known, no iteration is needed in such a problem. This will be the case for problems involving large discharges and rough pipes, for the friction factor will then come from the wholly rough flow region of the Moody diagram. For problems in which the friction factors are found in the transitional turbulent region of the Moody diagram, some iteration to determine the friction factors will be required, but it is iteration only to determine the correct friction factors.

2.4.4. THREE RESERVOIR PROBLEM

Problems involving pipe flow between more than two reservoirs will always require some form of iterative solution. Here we examine briefly an economical solution strategy for these problems. In Chapter 4 a computer-oriented solution to such problems will be detailed.

Example Problem 2.7

The figure below is a diagram of the three reservoir problem; the reservoirs are connected by three pipes with an external demand at the common junction of the pipes. The highest reservoir has a water surface elevation of 100 m; the middle reservoir water surface elevation is 85 m; and the lowest reservoir has a water surface elevation of 60 m. Determine the discharge in each pipe.



Pipe	K	n
1	1469	1.974
2	2432	1.927
3	5646	1.971

It is clear that flow is out of the upper reservoir and into the lowest reservoir. What is unclear is the direction of flow in the pipe that connects the middle reservoir to the system. The key step is to determine that direction in only one trial.

Let H_J be the head at the junction. The discharges in pipes 1 and 2 can then be found from these two head loss equations:

$$100 - K_1 Q_1^{n_1} = H_J \qquad H_J - K_3 Q_3^{n_3} = 60$$

Now select $H_J = 85$ m, the water surface elevation of the middle reservoir, so that there is no flow in pipe 2 for the first trial solution. Inserting values of K and n from the table, we find $Q_1 = 0.0980$ m^3/s and $Q_3 = 0.0639$ m^3/s. These values, combined with the external demand QJ_1, do not satisfy continuity at the junction J. To satisfy junction continuity we need more inflow to the junction, so H_J must be less that 85 m; thus we find that the flow in pipe 2 will be toward the junction and will be governed by

$$85 - K_2 Q_2^{n_2} = H_J$$

The junction continuity error for each trial will be $Q_e = Q_1 + Q_2 - Q_3 - QJ_1$. Now we select trial values for H_J, use the three head loss equations to compute the discharges and finally compute the error Q_e. Each trial outcome can be compactly recorded in a table:

H_J m	Q_1 m^3/s	Q_2 m^3/s	Q_3 m^3/s	Q_e m^3/s
85.0	0.0980	0.0	0.0639	- 0.0259
80.0	0.1134	0.0403	0.0571	0.0366
83.0	0.1045	0.0251	0.0613	0.0083
83.5	0.1029	0.0216	0.0620	0.0025
83.7	0.1023	0.0200	0.0622	0.0001

The systematic assignment of values to the head at the junction, which is itself usually not of great interest, is the step which allows us to search methodically for the solution. This approach can also be applied productively to similar problems which may even contain more than three reservoirs. The repeated manual intervention in selecting the trial values of H_J may make other procedures more attractive for solutions by computer, however.

2.5 PROBLEMS

2.1 For the following pipe flows determine whether the flow is laminar, turbulent smooth, turbulent rough, or turbulent transition, using the Moody diagram, Fig. 2.2.

(a) A velocity of 3.05 m/s (10 ft/s) occurs in a cast iron pipe having $e = 2.6 \times 10^{-4}$ m (8.5×10^{-4} ft) which is 2.54 cm (1 in) in diameter. The fluid kinematic viscosity is $\nu = 9.29 \times 10^{-5}$ m^2/s (10^{-3} ft^2/s).

(b) A velocity of 2.44 m/s (8 ft/s) occurs in a cast iron pipe having $e = 2.6 \times 10^{-4}$ m (8.5×10^{-4} ft) which is 0.15 m (6 in) in diameter. Use $\nu = 9.29 \times 10^{-8}$ m^2/s (10^{-6} ft^2/s).

(c) The velocity is 2.44 m/s (8 ft/s) in a 0.91 m (3 ft) diameter welded steel pipe having $e = 4.6 \times 10^{-5}$ m (1.5×10^{-4} ft). Use $\nu = 9.29 \times 10^{-5}$ m^2/s (10^{-3} ft^2/s).

(d) The velocity is 2.44 m/s (8 ft/s) in a 0.91 m (3 ft) diameter welded steel pipe having $e = 4.6 \times 10^{-5}$ m (1.5×10^{-4} ft). Use $\nu = 9.29 \times 10^{-7}$ m^2/s (10^{-5} ft^2/s).

2.2 A 250 mm diameter pipe is 1500 m long. When the discharge is 0.095 m^3/s in this pipe, the pressure drop between the ends of the pipe is measured as 98.06 kPa. The elevation at the end of the pipe is 10 m below its beginning. What type of flow is this? What is the equivalent sand-grain roughness of the pipe wall? What is the Hazen-Williams roughness coefficient? How much energy is dissipated by fluid friction during each hour that this flow continues? Use $\nu = 1.31 \times 10^{-6}$ m^2/s.

2.3 Find the pressure drop in 1000 m (3280 ft) of 0.10 m (0.33 ft) diameter pipe carrying 0.015 m^3/s (0.53 ft^3/s) of olive oil at 10 ^{o}C (50 ^{o}F). The downstream end of the pipe is 10 m (32.8 ft) below the upstream end.

2.4 Determine the discharge that will occur in a 450 mm diameter pipe that is 1000 m long connecting two reservoirs with a difference in water surface elevations of 25 m. The wall roughness of the pipe is $e = 0.12$ mm, and $\nu = 1.31 \times 10^{-6}$ m^2/s. How much head must a pump supply to reverse the flow, i.e. cause the same discharge to flow from the lower to the upper reservoir? What power must be supplied by electricity to this pump if the combined efficiency of the pump and motor is 75%?

2.5 A 0.305 m (1 ft) diameter concrete pipe that is 366 m (1200 ft) long carries water from a reservoir with surface elevation 1086 m (3560 ft) to a ditch at elevation 1041 m (3415 ft). If the Hazen-Williams roughness coefficient is 120, find the discharge through the pipe.

2.6 Determine the minimum pipe size to convey 0.028 m^3/s (1 ft^3/s) of water at 15^{o}C (60^{o}F) for new cast iron pipe that is 914 m (3000 ft) long with a change in HGL between the ends of 15.2 m (50 ft).

2.7 Determine the values of K and n in the exponential formula $h_f = KQ^n$, based on the Darcy-Weisbach and Hazen-Williams formulas for these pipes:

(a) $L = 1000$ ft, $D = 6$ inches, $e = 0.002$ inches, $C_{HW} = 110$, $V \approx 8$ ft/s.

(b) $L = 1000$ m, $D = 0.2$ m, $e = 0.005$ m, $C_{HW} = 140$, $V \approx 2$ m/s.

(c) $h_f = 50$ ft, $L = 3000$ ft, $D = 8$ inches, $e = 0.0102$ inches, $C_{HW} = 120$.

2.8 Plot the K and n values that were found in Example Problem 2.1 from the Darcy-Weisbach equation on a Moody diagram. How close are the slopes of these lines on the Moody diagram to the slopes of the Hazen-Williams and Manning equations? From this comparison develop some guidelines for when the Hazen-Williams equation is most appropriate, and when the Manning equation may be more appropriate.

2.9 Use the K and n values that were found in Example Problem 2.1 from the Darcy-Weisbach, Hazen-Williams and Manning equations to compute the head losses associated with discharges that are 50 and 200% of the given approximate Q, and compare the results.

2.10 If the friction factor is held constant, show that the Darcy-Weisbach equation indicates that the head loss is proportional to the velocity squared, or the discharge squared, just as the Manning equation does. For what flow type(s) is such a relation appropriate?

2.11 Determine the coefficient K and the exponent n in $h_f = KQ^n$ for the pipes in the table which follows by using both the Darcy-Weisbach and Hazen-Williams equations. The water flows in the pipe at about 6 m/s and has a temperature of 10°C.

Pipe No.	Type	Dia. m	Length m	Darcy-Weisbach K	Darcy-Weisbach n	Hazen-Williams K	Hazen-Williams n
1	Smooth concrete	2.50	1000				
2	PVC	0.25	1500				
3	Old cast iron	0.08	800				
4	e = 0.005 mm	0.35	2000				

2.12 For pipes 1 and 3 in Problem 2.11, determine the equivalent length of pipe that could be used to replace the minor loss caused by a globe value (K = 10). If needed, assume a velocity of 6 m/s in the pipe.

2.13 Determine the discharge of water at 20°C (68°F) through a 10 cm (4 in) diameter concrete pipe that is 457 m (1500 ft) long. Assume the wall roughness is e = 0.61 mm (0.002 ft). The pipe connects two reservoirs with a 6.1 m (20 ft) difference in water surface elevation.

2.14 One-tenth m^3/s (3.53 ft^3/s) of water at 20°C (68°F) flows through a 0.25 m (0.82 ft) diameter cast iron pipe. Find the head loss in 200 m (656 ft) of this pipe.

2.15 Compare the head loss for a discharge of 0.1 m^3/s (3.53 ft^3/s) of water at 20°C (68°F) through (a) a 0.20 m (8 in) diameter concrete pipe with (b) a 0.20 m (8 in) diame-ter PVC pipe.

2.16 Water at 10°C (50°F) flows between reservoirs through a 0.30 m (1 ft) diameter cast iron pipe that is 1 km (3280 ft) long. Find the difference in elevation between the reservoirs if the discharge is 0.2 m^3/s (7.1 ft^3/s).

2.17 Water is to be pumped from a lake to a canal which is 200 m (656 ft) distant and 20 m (65.6 ft) higher in elevation. If 0.5 m^3/s (17.66 ft^3/s) of water at 20°C (68°F) is to be delivered through a 0.5 m (1.64 ft) concrete pipe, what power must the pump deliver to the water?

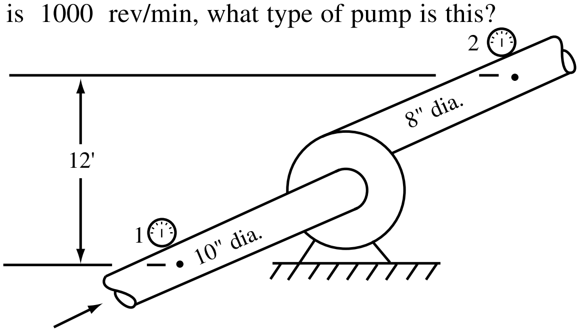
2.18 Find the power which pumps must supply to 3 m^3/s (106 ft^3/s) of water at 20°C (68°F) which is to be delivered from the Snake River to the plateau 180 m (591 ft) above the river through 1100 m (3610 ft) of 1 m (3.28 ft) asphalt-dipped cast iron pipe.

2.19 Use the Hazen-Williams formula to find h_f when 0.013 m^3/s (0.46 ft^3/s) of water at 20°C (68°F) flows through 300 m (984 ft) of 75 mm (0.25 ft) diameter smooth pipe.

2.20 A power plant is 16 km (52,500 ft) from a reservoir. A discharge of 25 m^3/s (883 ft^3/s) is to be delivered to the plant at an elevation that is 1120 m (3,670 ft) below the reservoir surface. What size of riveted steel pipe is required? Assume a temperature of 4°C (40°F).

2.21 What diameter of commercial steel pipe will convey 0.003 m^3/s (0.106 ft^3/s) of crude oil at 40°C (104°F) with a pressure drop of 15 kPa (2.18 lb/in^2) per 30 m (98 ft)?

2.22 The pump shown below delivers 8 ft^3/s of water. The recorded pressures at sections 1 and 2 on the gauges are - 5.0 lb/in^2 and + 35.0 lb/in^2. (a) Draw a diagram of the system and locate the EL and HGL at sections 1 and 2 in the diagram. (b) Determine the required h_p and power that must be supplied by the pump to the water to deliver this discharge. Neglect pipe friction and local losses. (c) If the rotative speed of the pump impeller is 1000 rev/min, what type of pump is this?



2.23 You are asked to design a pipe line for a farmer which will carry 0.2 m^3/s of water from a lake on a mountainside at elevation 1905 m to a farm sprinkler system 6 km away at elevation 1795 m. The sprinklers require a pressure of 400 kPa to operate properly. PVC pipe is to be used. Assume a temperature of 10°C.

2.24 A farmer wants you to design his irrigation pipe line so it can be used in the winter to generate electricity for his home. He wants to run a 20 kW turbine-generator (70% efficient) from the 0.05 m^3/s stream. The PVC pipe line is 1050 m long, and the up-stream end is 75 m above the turbine. What pipe diameter should be selected? Assume a temperature of 10°C.

2.25 Use a computer program to generate several tables of f versus Re for different values of relative roughness e/D, and use these to plot several curves on a Moody diagram with a spreadsheet or other graphing software.

2.26 How much energy per unit weight would be saved by using a long radius elbow instead of a short radius elbow in a 0.30 m (1 ft) diameter pipe with a discharge of 0.23 m^3/s (8 ft^3/s) of water at 20°C (68°F)?

2.27 What loss is caused by a close return bend in a 0.15 m (0.49 ft) diameter pipe carrying a discharge of 0.1 m^3/s (3.53 ft^3/s) of gasoline at 20°C (68°F)? How does this loss compare with the use of two short radius bends? Two long radius elbows?

2.28 A discharge of 0.283 m^3/s (10 ft^3/s) flows in a 0.30 m (1 ft) diameter pipe. Compare the head losses for a completely open (a) angle valve, (b) gate valve, and (c) globe valve. Under what conditions would you select the gate valve? One of the other valves?

2.29 An irrigation siphon tube is 76 mm (3 in) in diameter and 3 m (9.84 ft) long. Estimate the discharge for a head difference of 0.5 m (1.64 ft), assuming a re-entrant entrance, an equivalent sand-grain roughness $e = 0.06$ mm ($2.36\text{x}10^{-3}$ in), and two bends with loss coefficients of 0.2. Draw the system, including the EL and HGL.

2.30 To obtain more electrical energy during the day when there is a shortage and use it during the late night when there is a surplus, a power company plans to pump water from a lake to a reservoir through a 0.5 m diameter pipe that is 2500 m long ($e = 0.001$ m); when the power is needed, the company will run that water through a turbine. The elevation difference between the reservoir and lake water surfaces is 90 m. Surplus electrical energy costs \$0.02/kWh, prime time energy is worth \$0.10/kWh, and the efficiencies of the pump and turbine are 80 percent. Analyze the hydraulics and economics of the proposed plan. Suggest the discharges that should be used.

2.31 Write a program for a computer or calculator for determining the unknown discharge Q in a pipe line (Category 2), including local losses.

2.32 Write a program for a computer or calculator for determining the unknown diameter of a pipe (Category 3), including local losses.