CHAPTER 5

DESIGN OF PIPE NETWORKS

5.1 INTRODUCTION

When dealing with problems associated with pipelines (or pipe networks) for which all diameters, lengths, roughness coefficients, and demands are known, then the nodal HGL elevation, or *H*'s, and pipe discharges are the unknown quantities to be found. Problems of this nature are classified as analysis problems since a known piping system is being ana-lyzed for a given demand pattern. Chapter 4 dealt with the analysis of networks. In an analysis problem for a network, the demands at all nodes of the network are specified, and the elevation of the HGL is known at one or more positions (where reservoirs exist), and the solution seeks to find the discharges (and head losses) in all pipes, and the HGL elevation, head, and pressure at each node in the network.

The focus of this chapter is on the design of pipe networks, which most frequently means that the pipe diameters are unknown and are to be determined. A brief introduction to design problems was presented in Chapter 4, where the equations for mass and energy conservation were used in determining any desired variables associated with the problem. This chapter will greatly expand upon these principles, but we start with a single pipe.

5.1.1. SOLVING FOR PIPE DIAMETERS

A typical design problem consists of sizing, i.e., determining the size of, as many pipes as the equations allow to meet specified pressures and discharges throughout the network. For such design problems the pressures at all nodes, the heads at all nodes, or the HGL elevations are typically specified. (Knowing any one of these allows the others to be com-puted if the nodal elevations are known.) In addition to finding pipe diameters, one might want to determine the heads that pumps must produce to satisfy the specified pressures.

Consider a single pipe that conveys water from a reservoir with a known water surface elevation H_1 to another reservoir with a known water surface elevation H_2, as shown in Fig. 5.1, as the simplest possible design problem. For this case there is one unknown diameter D, a known length L, and a known roughness e. The problem is to determine the smallest pipe diameter that will convey the known discharge Q between the two reservoirs.

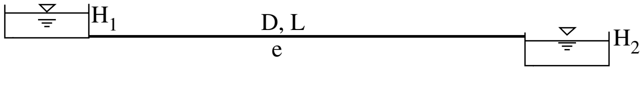


Figure 5.1 A simple two-reservoir design problem.

5.1.2. SOLUTION BASED ON THE DARCY-WEISBACH EQUATION

The Darcy-Weisbach equation will be used here to describe the head loss in a pipe as a function of the discharge in that pipe. The next section will base solution procedures on the Hazen-Williams equation. We recall the Darcy-Weisbach equation

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{Q^2}{2gA^2} \tag{5.1}$$

in which h_f is the head loss due to friction in units of energy per unit weight, i.e., a length, the friction factor f is in general a function of the Reynolds number and the relative roughness e/D of the pipe, and the cross-sectional area of the pipe is $A = \pi D^2/4$. Since nearly all water flows are in the transitional zone of the Moody diagram, the behavior of the friction factor can be defined by the implicit Colebrook-White equation in the form

$$\frac{1}{\sqrt{f}} = 1.14 - 2Log_{10}\left[\frac{e}{D} + \frac{9.35}{Re\sqrt{f}}\right] = 1.14 - 2Log_{10}\left[\frac{e}{D} + \frac{7.3434728\,\nu D}{Q\sqrt{f}}\right] \tag{5.2}$$

in which $Re = VD/\nu = 4Q/(\pi\nu D) = 1.27324\ Q/(\nu D)$ is the Reynolds number. Since Eq. 5.2 merges into the equation that describes the wholly rough zone on the Moody diagram well, and it also merges into the equation that describes hydraulically smooth flow, it will be used whenever the flow is turbulent. If the flow is laminar with Re below 2100, then Eq. 5.2 must be replaced by

$$f = 64/Re = 64\nu/(VD) = 81.487\nu D/Q \tag{5.3}$$

The basic problem that seeks to determine a diameter now requires that Eqs. 5.1 and 5.2 (or possibly Eq. 5.3) be solved simultaneously for the two unknowns D and f. Several methods will be applied to obtain a simultaneous solution of these equations. These methods will be implemented in the computer programs DIAPIP, DIAPIPA, DIAPIP2, and DIAPIP3. The reader will benefit most by printing a copy of these programs now and consulting the listings as the methods are described.

The first method uses the Newton method to solve simultaneously the Darcy-Weisbach and Colebrook-White equations for D and f. This approach is similar to that used in program DW_CW in Chapter 4, with the difference that D is chosen to be the second unknown in place of some other variable of the problem. In solving Eqs. 5.1 and 5.2 simultaneously by the Newton method, we first rewrite the original equations in the generic form $F(D, f) = 0$. One way of rewriting these equations is as follows:

$$F_1(D,f) = \frac{1}{\sqrt{f}} - 1.14 + 2Log_{10}\left[\frac{e}{D} + \frac{7.3434728\,\nu D}{Q\sqrt{f}}\right] = 0 \tag{5.2a}$$

$$F_2(D,f) = h_f - f\frac{L}{D}\frac{Q^2}{2gA^2} = 0 \tag{5.1a}$$

The Jacobian matrix for this system of equations is a 2x2 square matrix J:

$$J = \begin{bmatrix} \dfrac{\partial F_1}{\partial D} & \dfrac{\partial F_1}{\partial f} \\ \dfrac{\partial F_2}{\partial D} & \dfrac{\partial F_2}{\partial f} \end{bmatrix} \tag{5.4}$$

Program DIAPIP implements the Newton method to determine simultaneously the friction factor f and diameter D. Prompts in the program ask the user for the data that are required to define the problem. The acceleration of gravity is required so that problems in either of the ES or SI unit systems can be solved. If the solution is to be written to a disk file and also displayed on the monitor, then the Output unit number (the second input item) should not be 6. Microsoft's Fortran version 5 and higher versions prompt the user for the disk file if writing to a logic unit other than 6 and this unit is not already open. The next input statement requests values for the desired discharge Q, the roughness e, the pipe length L, and the frictional head loss h_f. For our problem the difference in the water surface elevations H_1 and H_2 is this frictional head loss. Since $1/\sqrt{f}$ occurs on both sides of Eq. 5.2a, let it be the unknown in place of f. In the program this variable is SF.

The Jacobian is defined by the expanded 2x3 array DJ. The first two columns in this array contain the Jacobian derivatives, and the third column contains the equation vector F. The derivatives are determined with respect to SF, rather than f, because this is slightly simpler. The two unknowns are SF and D, which are initialized to 8 and 0.5 ft, respectively, for the Newton method. The two equations are denoted by F1 and F2; after they are evaluated for the first time in each Newton iteration, they are stored in the third column of matrix DJ. Then the two statements that define the equations are evaluated twice more by the IF and GO TO statements. The last two times repeat the first computations with incremented values of SF and D. The program variable NCT counts the number of iterations. The number of Newton iterations should always be limited to avoid the possibility of an infinite loop in these computations. With two unknowns the solution by Gaussian elimination requires only one element D21 to be eliminated. Thereafter, the solution vector z is obtained by back substitution. Thus the approach is much like that in Chapter 4 to solve simultaneously for the discharge Q and the friction factor f (or SF). The major difference is the change in unknowns to D and f (or SF); when the unknowns are treated properly, the Newton method works in the same way.

If we want to find the diameter that will convey 2.0 ft^3/s when the difference is 40 ft in a 3000 ft long pipe of roughness 0.002 inches, the computer program DIAPIP will produce the solution $f = 0.01668$, D = 7.941 in, listed below as case 1. Although in practice these results would be rounded, we present them in this way to aid the checking of the computer output. To verify that the program works properly, the reader should use DIAPIP to solve the four problems in Table 5.1; these steps will also augment the reader's understanding of the program logic. We assume either $\nu = 1.41 \times 10^{-5}$ ft^2/s or $\nu = 1.31 \times 10^{-6}$ m^2/s.

Table 5.1 Test Problems

No.	**1**	**2**	**3**	**4**
L	3000 ft	1000 m	1000 m	10,000 ft
h_f	40 ft	8 m	80 m	15 ft
e	0.002 in	0.0001 m	0.0001 m	0.0004 ft
f	0.0168	0.01598	0.01679	0.01559
D	7.941 in (0.662 ft)	0.3664 m	0.3335 m	23.052 in (1.921 ft)

The attractive convergence behavior of the Colebrook-White equation, Eq. 5.2, using Gauss-Seidel iteration is the basis for an alternative to the simultaneous solution of Eqs. 5.1 and 5.2 by the Newton method. By starting with some reasonable value for f, Eq. 5.2 must only be solved a few times by always using the newly computed value of f to recompute f. When Gauss-Seidel iteration is used to solve Eq. 5.2, then Eq. 5.1 can be solved via the Newton method with f treated as if it were known in each Newton iteration. In this process the Newton method is therefore used to solve only one equation for the one unknown, D. Since D does affect the value of f, the Gauss-Seidel iteration must be repeated within each new Newton iteration, however. Therefore this alternative solution process consists of applying the Newton method to solve Eq. 5.1 for D, and within this iteration Gauss-Seidel iteration is used to resolve Eq. 5.2 for f. The Newton iteration is achieved via the equation

$$D^{(m+1)} = D^{(m)} - \frac{F(D^{(m)})}{dF(D^{(m)})/dD} \tag{5.5}$$

in which $F(D)$, under the assumption that f is known, is Eq. 5.1 written as follows:

$$F(D) = h_f - f(L/D)Q^2/(2gA^2) = 0 \tag{5.6}$$

This method is implemented by program DIAPIPA.

The approach in DIAPIPA can be used in a slightly modified manner in solving for D and f with an HP48 or equivalent handheld calculator. Retrieve both the Colebrook-White and Darcy-Weisbach equations from memory. Using an estimate for D, solve the Colebrook-White equation with the SOLVR function. Next solve the Darcy-Weisbach equation using SOLVR, and repeat this process until small changes in D occur between consecutive iterations.

A third alternative is to replace the Newton solution of the Darcy-Weisbach equation with a direct solution of this equation. Since the area $A = \pi D^2/4$, this equation can be written as

$$D = \left[\frac{fLQ^2}{2g(\pi/4)^2 h_f}\right]^{0.2} = \left[\frac{0.8105695 fLQ^2}{gh_f}\right]^{0.2} \tag{5.1b}$$

Because f depends upon D, Eq. 5.1b must be solved iteratively, with the Colebrook-White equation being solved either by the Gauss-Seidel method or the Newton method as soon as a new D is available. The program DIAPIP2 implements this solution method, applying the Gauss-Seidel method to the Colebrook-White equation. In previous programs a conversion factor CONV allowed D and e to be given in inches when using ES units, but program DIAPIP2 requires consistent units for all variables. One could use this same approach with an HP48 calculator. However, now one does not use SOLVR in obtaining the solution to the Darcy-Weisbach equation.

Yet another possible approach is to eliminate the friction factor by solving for it in the Darcy-Weisbach equation and substituting the result into the Colebrook-White equation; then the resulting equation for D is solved by using the Newton method. The Darcy-Weisbach equation, with f on the left of the equal sign, is

$$f = h_f D(2g)A^2/(LQ^2) = h_f D(2g)/(LV^2) = 1.2337 h_f g D^5/(LQ^2) \tag{5.1c}$$

or

$$\frac{1}{\sqrt{f}} = \frac{Q\sqrt{L}}{A(2gh_f D)^{1/2}} = \left(\frac{L}{2gh_f}\right)^{1/2}\left(\frac{4Q}{\pi D^{2.5}}\right) = 0.90031632Q\left[L/(gh_f)\right]^{1/2}/D^{2.5} \tag{5.7}$$

The equation to be solved for D is obtained by replacing $1/\sqrt{f}$ in this last equation with the expression on the right wherever it appears in the Colebrook-White equation. In implementing the solution in a computer program it is better to use two lines of code, one for the above expression for $SF = 1/\sqrt{f}$ and the other for the Colebrook-White equation.

The program DIAPIP3 uses this method to determine the diameter, with the derivative of the equation with respect to diameter being obtained numerically. After the diameter has been found, Eq. 5.1c is used to determine f.

5.1.3. SOLUTION BASED ON THE HAZEN-WILLIAMS EQUATION

The empirical Hazen-Williams equation is widely used in practice to define the discharge-head loss relation for water flows in full pipes. The Hazen-Williams equation is

$$Q = KC_{HW}AR_h^{0.63}S^{0.54} \tag{5.8}$$

in which $K = 1.318$ for ES units and $K = 0.849$ for SI units, C_{HW} is the Hazen-Williams roughness coefficient which ranges from 150 for smooth-walled pipes to as low as 80 for old, corroded cast iron pipes (see Table 2, Chapter 2), R_h is the hydraulic radius, and S is the slope of the HGL or energy line so that $S = h_f/L$. Another convenient form of the Hazen-Williams equation is

$$h_f = \frac{K_1 L}{C_{HW}^{1.852} D^{4.87}} Q^{1.852} \tag{5.9}$$

in which $K_1 = 4.727$ with ES units, and $K_1 = 10.7$ with SI units. If the Hazen-Williams equation is solved directly for the pipe diameter D, it then appears as

$$D = \left[\frac{QK_1^{0.54}}{C_{HW}S^{0.54}}\right]^{0.380228} = K_2\left[\frac{Q}{C_{HW}S^{0.54}}\right]^{0.38} \tag{5.10}$$

in which $K_2 = 1.376$ for ES units and $K_2 = 1.626$ for SI units. As Eq. 5.10 indicates, use of the Hazen-Williams equation allows the pipe diameter to be found directly if the discharge Q, head loss h_f, length L, and roughness coefficient C_{HW} are known. This obvious computational advantage, simplicity, is the main reason for its popularity. Program DIAPIPH obtains a solution for D from the Hazen-Williams equation.

When computers (and programmable pocket calculators) are used, the ease of computation will be of minor importance in relation to the validity of the formula over a large range of flow conditions. The Hazen-Williams equation agrees closely with results produced by the Darcy-Weisbach equation for water flowing in relatively smooth-walled pipes with Reynolds Numbers in the range of 10^5 to 10^6 (the typical range for pipe design). However, it does not produce results that agree well with the Darcy-Weisbach equation over a range of flow conditions in rough-walled pipes. In fact, the Manning equation is a better empirical equation for the representation of flow in rough-walled pipes, especially if the pipe does not flow full.

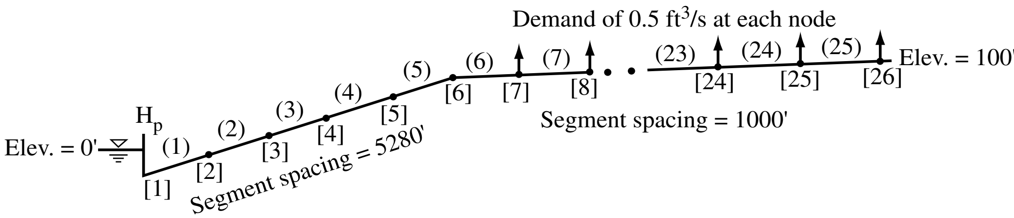
5.1.4. BRANCHED PIPE NETWORKS

In a branched pipe system it is easy to determine the discharge that must be carried by each pipe if all external demands are specified. If the pressures, heads, or HGL elevations are also known, then it is possible to use the methods described above to find the diameter of every pipe in the system, simply by repeating the computation for a single pipe. This can be done because the head loss and discharge for each pipe can be determined from simple preliminary computations. Thus no additional computational methods are needed to compute results for a branched system. Even though a more detailed look at the variables in pipe systems is presented later, it may be instructive to look at an example now.

Example Problem 5.1

As a consulting engineer you have been asked by an irrigation district to prepare a preliminary study of a pipeline using PVC pipe (Assume e = 0.000084 in.) that will bring irrigation water from a river that is 5 miles from the first farm. There are 20 farms. The turnout for each is to receive 0.5 ft^3/s, and these turnouts are spaced at 1000 ft intervals along the pipeline. The water level in the river is 100 ft below the elevation of the irrigated land, which is essentially flat. The water at the last turnout is to be delivered at a pressure of 40 lb/in^2. The pipeline will be laid on a constant grade between these two elevations, and a pump will be required at the river to provide sufficient head.

You decide to base computations on a 1-mile increment for the first 5 miles, and on a 1000 ft increment thereafter, with each turnout at a junction between pipe segments. A sketch of this pipe system is shown below. To determine the pipe size that will result in the least (or near least) cost, you decide to obtain a series of design solutions in which the slope of the HGL will vary. The sum of the pipe cost and the energy cost for pumping will be plotted as a function of the slope of the HGL, and the minimum cost on this graph will identify the best design for the piping system.



The following tables present the solution to this problem with the slope of the HGL specified to be $1.2424\text{x}10^{-3}$. The discharges in column 9 are obtained first. Thereafter the diameters are computed by using any of the methods described in this section. The last column lists the incremental head losses (because this is commonly given), but since the slope of the HGL has been specified here, they are directly related to the pipe lengths. You should verify some of these results. If the Hazen-Williams equation is used in place of the Darcy-Weisbach equation, then a solution such as that given below can easily be completed by using a spread sheet. If the spread sheet has the ability to solve an implicit equation, then the Darcy-Weisbach equation could also be used. The design solution is followed by an analysis, in which the nearest standard pipe sizes have replaced the computed values. The correctness of some of these head losses for the standard pipe sizes should be verified. The cost analysis assumes the life expectancy of 45 years and energy costs of \$0.09/kWh. A knowledge of engineering economic analysis will allow the pumping cost for this system to be verified. Pumping is assumed to occur 365 days per year and has a combined motor-pump efficiency of 70 percent.

DESIGN PIPE DIAMETERS

PIPE NO.	NODES FROM	TO	DIA. in	AREA ft^2	NOM. DIA. in	L ft	e x10^5 in	Q ft^3/s	VEL. ft/s	HEAD LOSS ft.
1	1	2	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
2	2	3	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
3	3	4	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
4	4	5	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
5	5	6	23.22	2.940	24.0	5280	8.4	10.0	3.40	6.56
6	6	7	23.22	2.940	24.0	1000	8.4	10.0	3.40	1.24
7	7	8	22.77	2.828	24.0	1000	8.4	9.5	3.36	1.24
8	8	9	22.31	2.715	24.0	1000	8.4	9.0	3.31	1.24
9	9	10	21.84	2.601	20.0	1000	8.4	8.5	3.27	1.24
10	10	11	21.34	2.484	20.0	1000	8.4	8.0	3.22	1.24
11	11	12	20.83	2.366	20.0	1000	8.4	7.5	3.17	1.24
12	12	13	20.29	2.246	20.0	1000	8.4	7.0	3.12	1.24
13	13	14	19.74	2.124	20.0	1000	8.4	6.5	3.06	1.24
14	14	15	19.15	2.000	20.0	1000	8.4	6.0	3.00	1.24
15	15	16	18.53	1.873	18.0	1000	8.4	5.5	2.94	1.24
16	16	17	17.88	1.743	18.0	1000	8.4	5.0	2.87	1.24
17	17	18	17.18	1.610	18.0	1000	8.4	4.5	2.79	1.24
18	18	19	16.44	1.474	15.0	1000	8.4	4.0	2.71	1.24
19	19	20	15.63	1.333	15.0	1000	8.4	3.5	2.63	1.24
20	20	21	14.75	1.187	15.0	1000	8.4	3.0	2.53	1.24
21	21	22	13.78	1.035	15.0	1000	8.4	2.5	2.42	1.24
22	22	23	12.67	0.875	12.0	1000	8.4	2.0	2.28	1.24
23	23	24	11.37	0.706	12.0	1000	8.4	1.5	2.13	1.24
24	24	25	9.77	0.521	10.0	1000	8.4	1.0	1.92	1.24
25	25	26	7.54	0.310	8.0	1000	8.4	0.5	1.61	1.24

NODE DATA

NODE	DEMAND ft^3/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	-10.0	100.	150.00	65.00	250.00
2	0.0	100.	143.44	62.16	243.44
3	0.0	100.	136.88	59.31	236.88
4	0.0	100.	130.32	56.47	230.32
5	0.0	100.	123.76	53.63	223.76
6	0.0	100.	117.20	50.79	217.20
7	0.5	100.	115.96	50.25	215.96
8	0.5	100.	114.72	49.71	214.72
9	0.5	100.	113.47	49.17	213.47
10	0.5	100.	112.23	48.63	212.23
11	0.5	100.	110.99	48.09	210.99
12	0.5	100.	109.75	47.56	209.75
13	0.5	100.	108.50	47.02	208.50
14	0.5	100.	107.26	46.48	207.26
15	0.5	100.	106.02	45.94	206.02
16	0.5	100.	104.78	45.40	204.78
17	0.5	100.	103.53	44.86	203.53
18	0.5	100.	102.29	44.33	202.29
19	0.5	100.	101.05	43.79	201.05
20	0.5	100.	99.81	43.25	199.81
21	0.5	100.	98.56	42.71	198.56
22	0.5	100.	97.32	42.17	197.32
23	0.5	100.	96.08	41.63	196.08
24	0.5	100.	94.84	41.10	194.84
25	0.5	100.	93.59	40.56	193.59
26	0.5	100.	92.35	40.02	192.35

An analysis based on the nearest standard pipe diameter yields the following results:

STANDARD PIPE DIAMETER SOLUTION

PIPE NO.	NODES FROM	NODES TO	L ft.	DIA. in	e x10^5 in	Q ft^3/s	VEL. ft/s	HEAD LOSS ft.	HLOSS /1000
1	1	2	5280.	24.0	8.4	10.0	3.18	5.59	1.06
2	2	3	5280.	24.0	8.4	10.0	3.18	5.59	1.06
3	3	4	5280.	24.0	8.4	10.0	3.18	5.59	1.06
4	4	5	5280.	24.0	8.4	10.0	3.18	5.59	1.06
5	5	6	5280.	24.0	8.4	10.0	3.18	5.59	1.06
6	6	7	1000.	24.0	8.4	10.0	3.18	1.06	1.06
7	7	8	1000.	24.0	8.4	9.5	3.02	0.96	0.96
8	8	9	1000.	24.0	8.4	9.0	2.86	0.87	0.87
9	9	10	1000.	20.0	8.4	8.5	3.90	1.90	1.90
10	10	11	1000.	20.0	8.4	8.0	3.67	1.70	1.70
11	11	12	1000.	20.0	8.4	7.5	3.44	1.51	1.51
12	12	13	1000.	20.0	8.4	7.0	3.21	1.33	1.33
13	13	14	1000.	20.0	8.4	6.5	2.98	1.17	1.17
14	14	15	1000.	20.0	8.4	6.0	2.75	1.01	1.01
15	15	16	1000.	18.0	8.4	5.5	3.11	1.43	1.43
16	16	17	1000.	18.0	8.4	5.0	2.83	1.20	1.20
17	17	18	1000.	18.0	8.4	4.5	2.55	0.99	0.99
18	18	19	1000.	15.0	8.4	4.0	3.26	1.93	1.93
19	19	20	1000.	15.0	8.4	3.5	2.85	1.52	1.52
20	20	21	1000.	15.0	8.4	3.0	2.44	1.15	1.15
21	21	22	1000.	15.0	8.4	2.5	2.04	0.83	0.83
22	22	23	1000.	12.0	8.4	2.0	2.55	0.61	1.61
23	23	24	1000.	12.0	8.4	1.5	1.91	0.96	0.96
24	24	25	1000.	10.0	8.4	1.0	1.83	1.11	1.11
25	25	26	1000.	8.0	8.4	0.5	1.43	0.94	0.94

AVE. VEL. = 2.87 ft/s, AVE. HL/1000 = 1.22, MAX. VEL. = 3.90 ft/s, MIN. VEL. = 1.43 ft/s

In one more table we can summarize the information that describes this solution fully by listing various data associated with each node.

NODE DATA

NODE	DEMAND ft³/s	gal/min	ELEV. ft.	HEAD ft.	PRESSURE lb/in²	HGL ELEV. ft.
1	- 10.00	- 4490.0	100.0	150.00	65.00	250.00
2	0.00	0.0	100.0	144.41	62.58	244.41
3	0.00	0.0	100.0	138.82	60.16	238.82
4	0.00	0.0	100.0	133.23	57.73	233.23
5	0.00	0.0	100.0	127.64	55.31	227.64
6	0.00	0.0	100.0	122.05	52.89	222.05
7	0.50	224.4	100.0	120.99	52.43	220.99
8	0.50	224.4	100.0	120.03	52.01	220.03
9	0.50	224.4	100.0	119.15	51.63	219.15
10	0.50	224.4	100.0	117.25	50.81	217.25
11	0.50	224.4	100.0	115.55	50.07	215.55
12	0.50	224.4	100.0	114.04	49.42	214.04
13	0.50	224.4	100.0	112.71	48.84	212.71
14	0.50	224.4	100.0	111.54	48.34	211.54
15	0.50	224.4	100.0	110.54	47.90	210.54
16	0.50	224.4	100.0	109.11	47.28	209.11
17	0.50	224.4	100.0	107.90	46.76	207.90
18	0.50	224.4	100.0	106.91	46.33	206.91
19	0.50	224.4	100.0	104.98	45.49	204.98
20	0.50	224.4	100.0	103.46	44.83	203.46
21	0.50	224.4	100.0	102.32	44.34	202.32
22	0.50	224.4	100.0	101.49	43.98	201.49
23	0.50	224.4	100.0	99.88	43.28	199.88
24	0.50	224.4	100.0	98.92	42.86	198.92
25	0.50	224.4	100.0	97.81	42.38	197.81
26	0.50	224.4	100.0	96.87	41.98	196.87

AVE. HEAD = 114.9 ft., AVE. HGL = 214.91 ft.,
MAX. HEAD = 150.0 ft., MIN. HEAD = 96.87 ft.

COSTS ASSOCIATED WITH THIS NETWORK

ITEM	TYPE	PRESENT WORTH	ANNUAL COST
1	ELEC. POWER	$ 101,898,590	$ 10,277,391
2	PIPE	2,969,690	299,520
	TOTAL	$ 104,868,280	$ 10,576,910

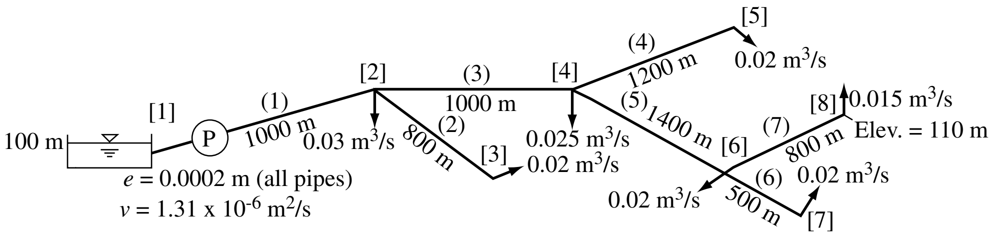
The solution was obtained by applying the NETWK program with this input data file:

```
EXAMPLE PROBLEM 5.1,  PIPE BRANCHED NETWORK
/*
$SPECIF IHGL=-2,NOMSOL=1,DESIGN=1,ICOST=1 $END
250. -10 100 0 .000084
1 6  .001242424 5280./
DEMAND
.5
6 26 .00124242 1000./
END
RUN
1 250.
END
```

* * *

Example Problem 5.2

This branched network is to be designed (i.e., pipe sizes determined) for the stated demands so the slope of the EL-HGL is 1/500 and a pressure of 50 kPa exists at node [8], the downstream node. What will be the cost per 30-day period for pumping if electricity costs $0.09/kWh and the combined efficiency of the motor and pump is 75 percent?



To determine the solution, first the discharge in each pipe is calculated by starting at the downstream nodes and working upstream, applying continuity at each node, and then the diameters are found by using any of the programs DIAPIP*. The results are given below in the table. The head that the pump must supply can be determined by starting at node [8] and computing successively the elevations of the HGL at the nodes that are farther upstream; finally the supply water surface elevation is subtracted to obtain the net rise that is needed in the HGL, or $h_p = 123.5 - 100 = 23.5$ m. The cost per month is the cost per kWh multiplied by the number of hours in 30 days and the power rate in kW; thus

$$\text{Cost} = 0.09(30\times24)(0.095\times9.81\times23.5)/0.75 = \$1892 \text{ per month.}$$

Pipe	**Q** m^3/s	**h_f** m	**D** m
1	0.095	2.0	0.370
2	0.020	1.6	0.206
3	0.075	2.0	0.339
4	0.020	2.4	0.206
5	0.055	2.8	0.301
6	0.020	1.0	0.206
7	0.015	1.6	0.184

Node	**HGL** m
1	123.5
2	121.5
3	119.9
4	119.9
5	117.5
6	116.7
7	115.7
8	115.1

* * *

5.2 LARGE BRANCHED SYSTEMS OF PIPES

Section 5.1 has shown how to determine the diameters of pipes in branched systems. First the discharges in all pipes are determined from the nodal external demands; second, once the discharge in each pipe is known, one of the methods described in Section 5.1 is applied repeatedly until all of the diameters have been computed. The discharges in all pipes of a branched system are obtained by satisfying the junction continuity equations. If we assume that the node which supplies the system is numbered (and that its demand is negative), then in general for a branched system there will be one more node or junction than there are pipes. Therefore the number of pipe flow equations will be NJ - 1, and a junction continuity equation will not be written for one of the nodes. The node that is omitted is seemingly arbitrary, but typically the omitted junction continuity equation is associated with either the last or first node. Let's examine how this approach can be implemented effectively in computer codes.

Three somewhat disparate methods can be used to obtain the discharges in a systematic manner that can be implemented in computer code. The three methods focus on either (1) the network layout, (2) the coefficient matrix produced by the junction continuity equations, or (3) the use of standard linear algebra. The reader can prepare best for the next three sections by obtaining now a listing of programs SOLBRAN, SOLBRAN2, and SOLBRAN3 from the CD.

5.2.1. NETWORK LAYOUT

The implementation of this method is based on the layout or topological connectivity of the network; it notes that pipes that have a dead end, i.e., that have at most one connection or nodal demand at one of their ends, must convey a discharge that is equal to the demand at that node. After the discharge in such a dead end pipe is determined, the demand at the other end of this pipe is modified to be the sum of the original nodal demand there and the discharge in the pipe, and then the dead end pipe is removed from the network of pipes. This reduced network will contain other dead end pipes, and the process is continued until the discharge is established for all pipes in the network. This process can be defined by the following steps:

1. Examine the network to find all nodes that have only one pipe connected to them, and assign the discharge in each such pipe to be the demand at this node.
2. Modify the demand at the node at the other end of each such pipe to reflect the original demand and the discharge in the pipe, and remove the pipe from the definition of the network.
3. Repeat steps 1 and 2 until the discharge is determined for all pipes in the branched network.

The 10-pipe network shown in Fig. 5.2 will be used to illustrate this method. In step 1 we note that pipes 1, 5, 9, and 10 are dead end pipes, i.e., pipes connected to nodes that have only one pipe connected to them, and the discharges in these pipes equal the

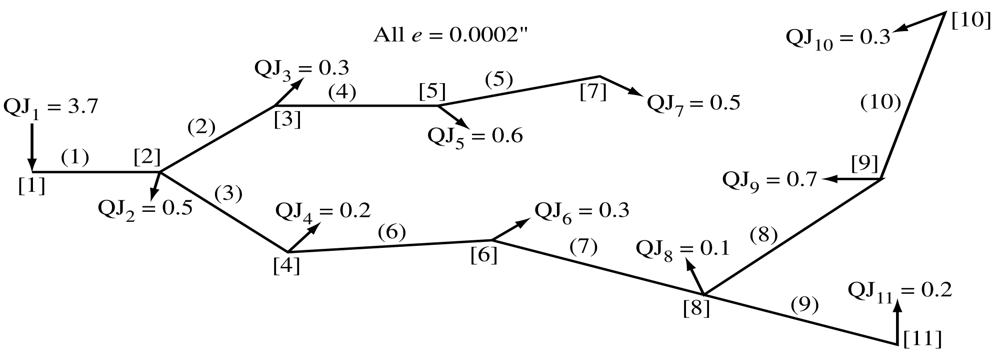


Figure 5.2 A 10-pipe network.

demands at these nodes: $Q_1 = QJ_1 = 3.7$ ft^3/s, $Q_5 = QJ_7 = 0.5$ ft^3/s, $Q_{10} = QJ_{10} = 0.3$ ft^3/s and $Q_9 = QJ_{11} = 0.2$ ft^3/s. Upon obtaining these discharges, step 2 is to reduce the branched system of pipes, by removing these pipes, to that shown in Fig. 5.3, in which the new demands account for the discharges in the pipes that have been removed:

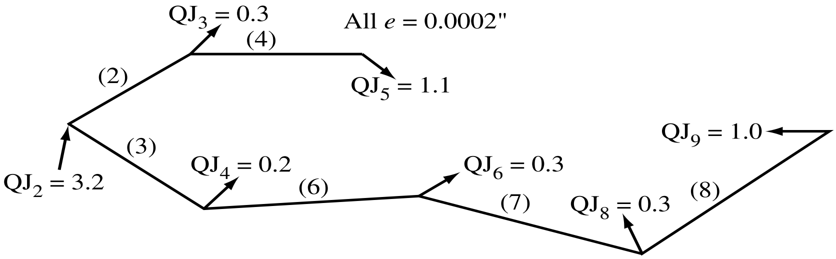


Figure 5.3 The reduced network.

For step 3 the process is repeated. After two additional applications (as shown below) there are only two pipes left, both of which are dead end pipes. The resulting discharges are Q_1 = 3.7 ft^3/s, Q_2 = 1.4 ft^3/s, Q_3 = 1.8 ft^3/s, Q_4 = 1.1 ft^3/s, Q_5 = 0.5 ft^3/s, Q_6 = 1.6 ft^3/s, Q_7 = 1.3 ft^3/s, Q_8 = 0.9 ft^3/s, Q_9 = 0.3 ft^3/s, and Q_{10} = 0.2

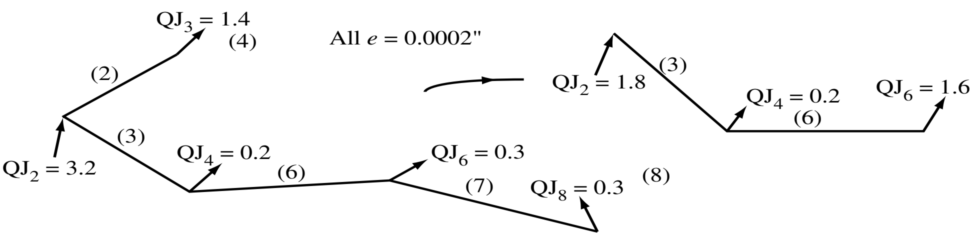


Figure 5.4 The final arrangement of the pipes.

Let's examine how this process can be implemented effectively in computer code. The details of the process will vary slightly, depending on the description of the network and one's sign convention. The description we will use for this purpose consists of a table with one line for each node. Each line contains the demand at the node, followed by a list of the pipes that join at this node. An extraction or outflow will be a positive demand, so if an external flow enters the network at a node it will be a negative demand. Pipes that receive flow from a node will be given positive numbers, whereas a pipe having flow into a node will be given a negative number. Using this nomenclature, the description of the branched network example is given by the two lists in Table 5.2. These lists are prepared in the order in which the nodes are numbered, and the entries under the second heading are the numbers of the pipes that join at this node. Thus dead end pipes are identified immediately by the single number on one row in this list.

Table 5.2

Demand QJ_i ft^3/s	Pipes at Node
- 3.7	1
0.5	- 1 2 3
0.3	- 2 4
0.2	- 3 6
0.6	- 4 5
0.3	- 6 7
0.5	- 5
0.1	- 7 8 9
0.7	- 8 10
0.3	- 10
0.2	- 9

The process for determining the discharge in each pipe can consist of these steps:

1. Scan the list "Pipes at Node." If only one pipe number appears in a row, assign the demand at this node to the discharge in this pipe. To account properly for the direction of flow, the discharge in this pipe k can be assigned as $Q_k = - QJ_i|k|/k$. The absolute value of the pipe number, divided by its number, will give the proper sign to the discharge.
2. Mark this node for deletion, as it is not needed during the next pass through the list.
3. Scan the list of nodes and note all other appearances of this same pipe number. Modify the demand at any node that has this pipe joining it by the discharge of this pipe, i.e. modify demand QJ_j by $(QJ_j)_{new} = (QJ_j)_{old} + Q_k|k|/k$, and remove this pipe from the list "Pipes at Node."
4. Delete all nodes that have been marked for deletion.
5. Repeat steps 1 through 4 until all nodes have been deleted from the list.

The program SOLBRAN executes the procedure that has just been described. After the discharges in the pipes are determined, then the diameters can be computed by the procedures described earlier. In this program these diameters are determined by solving the Darcy-Weisbach and Colebrook-White equations simultaneously; thus the previous program is now a subroutine that finds the diameter D (program variable DIA) given the discharge (program variable Q) and pipe roughness e. Then this subroutine finds D and f simultaneously.

The input to this program consists of the following:

1. The first line, which comes from the keyboard, gives the number of pipes NP (and in the C program the file names of the input and output units INPUT and IOUT);
2. The acceleration of gravity (32.2 for ES units or 9.81 for SI units) G, the kinematic viscosity of the fluid VISC, and the slope $S = h_f/L$ of the HGL line;
3. A list of pipe lengths;
4. A list of pipe roughnesses e in inches when using ES units and in meters when using SI units (by ending this list with / the missing e's will be equated to the last one supplied);
5. The list of demands and pipes at node as described above. Each line of item 5 must terminate with a / with the Fortran program. The program is dimensioned to allow up to four pipes to join at any node, but this can be changed by assigning PARAMETER N4 a different value.

The input file for this problem is presented in Fig. 5.5.

```
Input to FORTRAN program
32.2 1.41E-5 0.001
1000 1100 1200 1300 1400 1500 1600 1700 1800 1900
0.0002/
-3.7 1/
0.5 -1 2 3/
0.3 -2 4/
0.2 -3 6/
0.6 -4 5/
0.3 -6 7/
0.5 -5/
0.1 -7 8 9/
0.7 -8 10/
0.3 -10/
0.2 -9/
```

Figure 5.5 Input file for program SOLBRAN.

Table 5.3 Solution for a 10-pipe, 11-node Branched System

Pipe	Length ft.	e in x10^4	Dia. in.	Area ft^2	Discharge ft^3/s	Velocity ft/s	Head Loss ft.
1	1000.0	2.0	16.7139	1.52	3.7	2.43	1.00
2	1100.0	2.0	11.6044	0.73	1.4	1.91	1.10
3	1200.0	2.0	12.7509	0.89	1.8	2.03	1.20
4	1300.0	2.0	10.6023	0.61	1.1	1.79	1.30
5	1400.0	2.0	7.8967	0.34	0.5	1.47	1.40
6	1500.0	2.0	12.2000	0.81	1.6	1.97	1.50
7	1600.0	2.0	11.2867	0.69	1.3	1.87	1.60
8	1700.0	2.0	10.2308	0.57	1.0	1.75	1.70
9	1800.0	2.0	5.6149	0.17	0.2	1.16	1.80
10	1900.0	2.0	6.5282	0.23	0.3	1.29	1.90

5.2.2. COEFFICIENT MATRIX

This method writes the junction continuity equations in matrix form as [C]{Q} = {QJ}. The elements in the coefficient matrix [C] consist of three possible values, 0, 1, or - 1. The vector of unknowns {Q} contains the discharges in the pipes, and the known vector {QJ} lists the demands at the nodes. This method uses a very efficient method, rather than standard methods such as Gaussian or Gauss-Jordan elimination, to solve the linear algebra problem. The approach to the linear algebra problem can be very similar to the process employed in our first method, but the focus is on the coefficient matrix rather than the layout of the network. The steps can be identified as follows:

1. Examine the coefficient matrix for rows that contain only one element that is not zero, and solve this equation. (The solution of this equation will force the discharge in the pipe identified by the column in this coefficient matrix to be the demand at the downstream end of this pipe, i.e., equal to QJ in this row.) Then mark this equation as solved; i.e., remove this row from the existing linear equation system.
2. Find all other rows in the coefficient matrix that are not zero in this column; for each of these modify the known vector {QJ} in this row by multiplying the coefficient (1 or - 1) by the discharge determined in step 1, and subtract this amount from the existing value of QJ in this row. In effect this step removes this column from the coefficient matrix so that it has been reduced in size by one row and one column.
3. Repeat steps 1 and 2 until all rows and columns of the linear algebra problem have been removed.

The implementation of this method should not form the coefficient matrix as a N-row by N-column matrix, with N being the number of junctions NJ minus 1. Instead, identify which columns of the coefficient matrix contain the nonzero elements (the 1's or - 1's) for each of its rows, to save the storage needed for a two-dimensional array. Listing the pipe numbers that join at a node, as was done in implementing the first method, provides this identification, i.e. the node number identifies the row of the matrix, and the pipes joining at this node provide the column numbers that contain the non-zero elements. In program SOLBRAN this pipe information was read into the two-dimensional integer array JN(NJ,4) (the second subscript is the number of pipes that can join at any junction). Thus step 1 will identify those rows, i.e. the first subscript of JN, that have only one pipe and use only one position in the second subscript of JN. For these rows the Q's will be determined, and the row will be marked and eliminated. For step 2 all of the rows not yet marked as eliminated will be searched for the same pipe number, and whenever this number is found it will be removed, and the number of elements used in the second subscript will be reduced by one. Thus the actual solution process becomes very similar to the first method. The program SOLBRAN2 shows one way to implement the second method. The subroutine DIAPIP is unchanged from the listing in SOLBRAN.

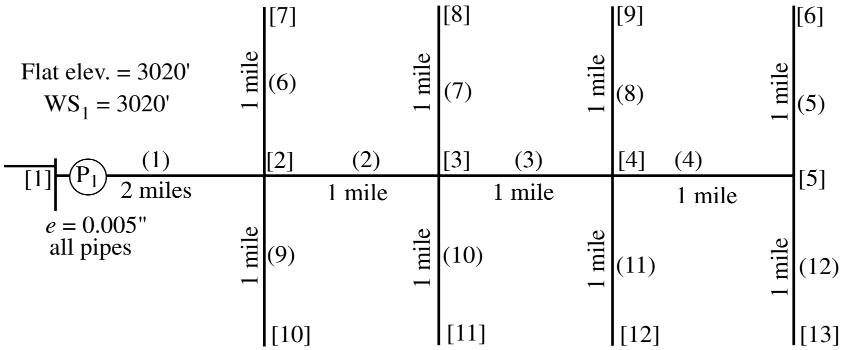
5.2.3. STANDARD LINEAR ALGEBRA

In this method the junction continuity equations are written as a coefficient matrix that multiplies the vector of unknown discharges (of length NP pipes) in the system. This product equals the known vector which consists of the demands at NP = NJ - 1 nodes of the network. This method requires the coefficient matrix to be a square matrix with NP rows and columns. The coefficient matrix elements will have the values 0, 1, or - 1. The row numbers correspond to the junction numbers for which the NP junction continuity equations are written, and the column numbers correspond to the pipe numbers. Upon properly defining the coefficient matrix and the known vector, a standard linear algebra subroutine (function) is called to solve the linear system of equations. One implementation of such a solution is given below in program SOLBRAN3. In this program the junction continuity equation is not written at the last junction of the network. Since the linear algebra solver SOLVEQ (see Appendix A) returns the solution in the same array that originally contained the known vector, the demands are now placed in the array Q at the outset, and the array QJ has been removed. In studying this listing you should strive to understand how the coefficient matrix is stored as 0's, 1's, or - 1's in the two-dimensional array C.

This method can be implemented easily by using spread sheets and general-purpose mathematics application software such as MathCAD, MATLAB, or TK-Solver. While the use of such software will result in computationally inefficient solutions, as is the case with SOLBRAN3, especially for large branched networks, the near-zero cost associated with such computations and the large PC RAMS makes it a viable approach. The CD contains a TK-Solver model and a brief description of it as files SOLBRAN3.TK2 and SOLBRAN3.DOC. A variation of the C program SOLBRAN.C is also on the CD under the name SOLBRAN4.C. This C program calls special pointer functions to allocate arrays beginning with 1, rather than 0, as is standard in C. (See Appendix A and the file SOLVEQC.DOC on the CD for more information.)

Example Problem 5.3

Water from a reservoir with a water surface elevation of 3020 ft passes through a pump to a pipeline that supplies twelve center-pivot irrigation sprinklers, each receiving a discharge of 1.5 ft^3/s at elevation 3020 ft and having a 1-mile spacing, as shown in the diagram. A pressure of 60 lb/in^2 or more is needed at each pivot location. Design the system to minimize costs. The capital cost of the pump is $100,000. Electrical energy costs $0.0935/kWh (actually $0.11/kWh, accounting for the 85% pump efficiency).



The cost per unit length for different pipe sizes is as follows (The NETWK program uses these default values.):

Diameter, in	10	12	15	18	20	24	30	36
Cost, cents/ft	10.67	16.67	24.00	43.33	56.67	80.00	100.00	120.00

The life expectancy of all components is 50 years, and the interest rate for acquiring capital for the project is 11 percent.

The cost of a system with pipes that are too small will be excessive, owing to the large energy cost of pumping the water. On the other hand the capital recovery cost for the pipes will be excessive if they are too large. The minimum total cost will be somewhere between these two extremes and will be determined by solving this branched system for several slopes of the HGL along the main line from node 1 though node 6 so that the pressure at node 6 is 60 lb/in^2. Likewise the pressures at nodes 7 through 13 will be specified as 60 lb/in^2. Thus a number of tentative designs will be required, and for each of these the costs will be determined. Since standard pipe sizes will be used, the nearest standard pipe size will be used in computing these costs.

The solution procedure will consist of the following steps:

1. Select a slope for the HGL along the main branch.
2. With a pressure of 60 lb/in^2 at node 6, or HGL_6 = 3020 + 60(144)/62.4 = 3158.46 ft, and the slope chosen in step 1, find the HGL slopes of pipes 6 through 12.
3. Compute all of the pipe diameters based on these HGL slopes.
4. Select standard pipe sizes that are nearest to the computed diameters.
5. Analyze the system that is composed of these standard pipe sizes, and compute the head and power that the pump must supply; then compute the electrical energy cost.
6. Determine the cost of the pipes, and convert this cost to an equivalent uniform annual cost by applying the capital recovery factor.
7. Repeat steps 1 through 6 until the least total cost is found.

SOLBRAN can not be used to seek this solution in a single run because the slope of the HGL is not the same for all pipes. The code would require modification to allow different slopes for different pipes. In its present form it could use the following input data to size pipes 1 through 6, but separate runs would be needed for the pipe pairs 6 and 9, 7 and 10, and 8 and 11 owing to the different HGL slopes. It is an instructive exercise to use the following input with SOLBRAN to compute the diameters of the pipes; those results can then be compared with those from NETWK.

Input to SOLBRAN

```
32.2  1.41E-5  0.001
10560  5280  5280  5280  5280/
0.005/
-18.  1/
4.5  -1 2/
4.5  -2 3/
4.5  -3 4/
3.0  -4 5/
1.5  -5/
```

The program NETWK will accomplish steps 1 through 6 with the input file below. In this input file the option IHGL = - 2 allows the main branch to be described by 2 lines of input, and the regular input is added to describe the lateral pipes. This input file has a HGL slope of 0.001 (and this slope results in the least cost). To obtain a solution for a different slope, this value (0.001) is changed; additional required changes are the HGL elevation at the beginning node (3190.14) and, on the line after the RUN command, a beginning HGL elevation for the analysis that is requested with the option NOMSOL=1. To pursue this solution process further, you should now obtain a solution from NETWK. The input file is on the CD under the name EXP5_3.IN. In obtaining the solution you should note that NETWK first computes a design solution in which the pipe diameters are

```
Example Problem 5.3
/*
$SPECIF IHGL=-2,NOMSOL=1,DESIGN=1,ICOST=1 $END
3190.14 -18. 3020. 1.5 .005
1 6 0.001  10560. 5280./
END
PIPES
6 2 7 5280. 0. .005
7 3 8/
8 4 9/
9 2 10/
10 3 11/
11 4 12/
12 5 13
NODES
7 1.5 3020. 3158.46
8 1.5
9 1.5
10 1.5
11 1.5
12 1.5
13 1.5
RUN
1 3190.14
PUMPS
UNIT=0.11
CAPI=100000
END
```

determined. Then the nearest standard pipe sizes are used to "analyze" the network. The final cost is based on this analysis and should agree with the data in this table:

COSTS ASSOCIATED WITH THIS NETWORK

ITEM	TYPE	PRESENT WORTH	ANNUAL COST
1	PIPE	$ 2,749,243	$ 277,286
2	ELEC. ENERGY	2,575,937	259,857
	TOTAL	$ 5,325,180	$ 537,143

The least cost is $537,143 per year with the energy costing $259,857 per year and the amortized cost of the pipes being $277,256 per year. Pipes 1 and 2 should be 30 inches in diameter, pipe 3 should be 24 inches in diameter, pipe 12 should be 12 inches in diameter, and the other pipes should be 10 inches in diameter.

* * *

5.3 LOOPED NETWORK DESIGN SOLUTION CRITERIA

This section will discuss the means for establishing equations to determine diameters and other desired quantities associated with looped pipe network problems. As background it is appropriate to review some of the fundamental relations that apply to the analysis of pipe networks, whether looped or branched. If the number of pipes that exist in a network is denoted by NP, the number of nodes (or junctions) is denoted by NJ, and the number of independent loops is denoted by NL, then this basic relation must be satisfied:

$$NP = NJ + NL \quad \text{if the network has two or more supply sources}$$

or

$$NP = (NJ - 1) + NL \quad \text{if the network has fewer than two supply sources.}$$

Actually a network can never be devoid of supply sources, but often problems are shown without a supply source. Instead the supply source is simply a node that has a negative demand or a flow into the system. If a network has only one supply source, it can always be shown as a network with no reservoir, or source pump, by obtaining the sum of the other demands and then indicating that this discharge amount enters at a particular point. For this relation to apply we tacitly assume that supply sources are not numbered as nodes.

The two kinds of *basic* equations are (1) junction continuity equations (NJ or NJ - 1 in number) that simply give mathematical expression to the fact that the mass rate of flow (or volumetric discharge for an incompressible fluid) from a junction must equal the mass rate of flow (or discharge) to a junction, and (2) equations that describe the relation between head loss and discharge in a pipe, e.g., the Darcy-Weisbach or Hazen-Williams equations. Of course other equations could be written and may be needed, but these are not considered to be basic equations. For example, in using the Darcy-Weisbach equation a friction factor f is introduced for each pipe, but alternative equations such as the Colebrook-White equation could express this relation. In a similar way pipe cross-sectional areas could be introduced as variables, and for each such area an additional equation becomes available. These secondary equations will not be included in the subsequent discussion.

One might wonder whether the equations around the loops constitute additional independent equations? The answer is no; they are not independent if all of the pipe head loss equations are written. The connectivity of the network, in conjunction with the pipe head loss equations, can be used to obtain the loop equations around both pseudo and real loops. To demonstrate this situation, consider the 16-pipe, 9-node network in Fig. 5.6.

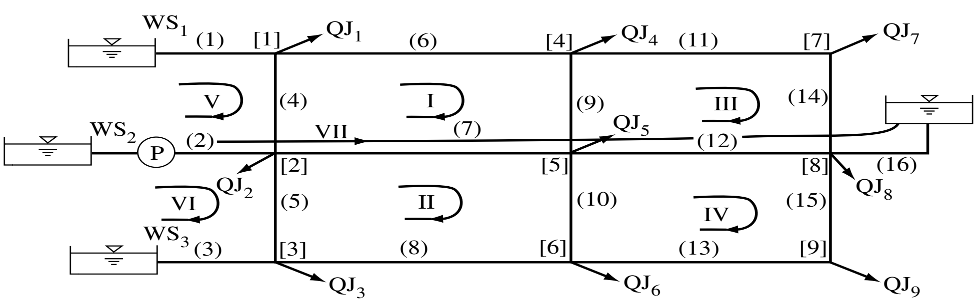


Figure 5.6 A 16-pipe, 9-node network.

The number of independent loop equations that can be written is NP - NJ = 16 - 9 = 7. These 7 loops are clear; four of them are real loops and three are pseudo loops connecting the four supply sources in some manner. However, the total number of basic equations consists of NJ = 9 junction continuity equations, and NP = 16 pipe head loss equations, for a total of 25. Thus 25 variables might be regarded as unknown, and if the other variables of the problem were all known, a solution for them could be sought.

To verify that the loop equations do not constitute additional independent equations, consider the four pipes (6, 9, 7, and 4) in loop I, using the exponential formula to express the head loss in each pipe:

$$H_1 - H_4 = K_6 Q_6^{n_6} \tag{5.11}$$

$$H_4 - H_5 = K_9 Q_9^{n_9} \tag{5.12}$$

$$H_2 - H_5 = K_7 Q_7^{n_7} \tag{5.13}$$

$$H_2 - H_1 = K_4 Q_4^{n_4} \tag{5.14}$$

Adding Eqs. 5.11 and 5.14 gives

$$H_2 - H_4 = K_4 Q_4^{n_4} + K_6 Q_6^{n_6} \tag{5.15}$$

Subtracting Eq. 5.12 from 5.13 gives

$$H_2 - H_4 = K_7 Q_7^{n_7} - K_9 Q_9^{n_9} \tag{5.16}$$

Now the subtraction of Eq. 5.16 from Eq. 5.15 produces

$$K_4 Q_4^{n_4} + K_6 Q_6^{n_6} + K_9 Q_9^{n_9} - K_7 Q_7^{n_7} = 0 \tag{5.17}$$

which is the loop equation for loop I. In a similar way writing the pipe head loss equations for pipes 1, 4, and 2 leads to

$$WS_1 - H_1 = K_1 Q_1^{n_1} \tag{5.18}$$

$$H_2 - H_1 = K_4 Q_4^{n_4} \tag{5.19}$$

$$WS_2 + h_p - H_2 = K_2 Q_2^{n_2} \tag{5.20}$$

Subtracting Eq. 5.19 from 5.18 results in

$$WS_1 - H_2 = K_1 Q_1^{n_1} - K_4 Q_4^{n_4} \tag{5.21}$$

Finally subtract Eq. 5.20 from 5.21 to eliminate H_2 and obtain

$$WS_1 - WS_2 - h_p + K_2 Q_2^{n_2} + K_4 Q_4^{n_4} - K_1 Q_1^{n_1} = 0 \tag{5.22}$$

which is the loop equation for pseudo loop V.

If a pipe head loss equation were written for every pipe in the network and the H's were then eliminated from these equations, an independent set of loop equations would be obtain-ed. Thus we see that loop equations are not independent of the pipe head loss equations and cannot also be used if the head loss equations are used. It is the way in which pipes are connected in a network that allows the loop equations to replace the pipe head loss equa-tions. This realization was the basis for the development of the Q-

equations in Chapter 4 to analyze a network. If one desires, it is always possible to omit pipe head loss equations and use loop equations in their place. Doing this, however, generally results in more arithmetic in obtaining the solution.

For the present we regard a design problem as one in which pipe diameters are to be determined. The definition of a design problem could be given a broader meaning, but at this time we are not concerned with the sizing of other components of a pipe system. Design problems can be further divided into two categories: (1) those in which we seek to determine as many diameters as there are nodes in the network (branched networks are a special case here); and (2) those in which we seek only certain individual pipe diameters to meet specified pressures. The latter category of problems will be treated in a later section. In the first category it is not possible to solve for more pipe diameters than there are nodes because the number of unknowns would then exceed the number of available equations. If the maximum possible number of pipe diameters is to be found (category 1), then it is assumed that the HGL elevations, or the heads H (pressure heads, or pressures), are specified at all nodes of the network. The number of basic equations is then NP + NJ (or NP + NJ - 1 if no supply sources are specified), but some of these must be used to determine other variables, usually the individual pipe discharges. Thus a basic difference between the first type of design problem and an analysis problem is that the H's at the nodes are known (specified) rather than unknown, and pipe diameters are to be found in place of the H's. The discharges are unknown variables in both the first type of design problem and the analysis problem. Thus diameters replace H's in the list of unknowns. The number of diameters in the list of unknowns must equal the number of H's which are specified. Looking again at the most recent network as an example, if the H's at all 9 nodes are given, one can in principle determine 9 pipe diameters. In this case the 25 independent equations would be used to determine 16 discharges plus 9 diameters.

To gain further insight into how this interchange of unknowns for knowns occurs, and what works and what won't work, consider the three-pipe looped system in Fig. 5.7, for

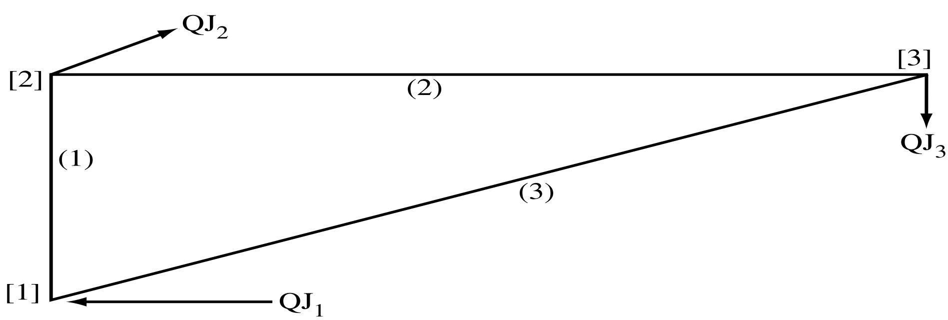
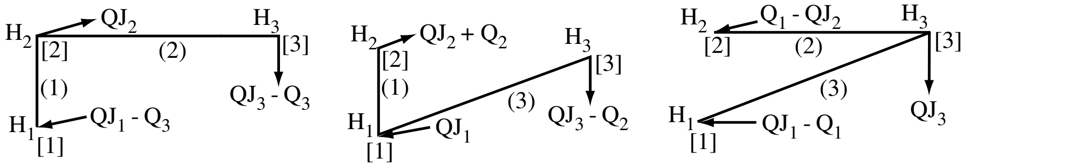


Figure 5.7 The three-pipe looped system.

which there exist two independent junction continuity equations and three head loss equations. If this were an analysis problem, all pipe diameters (and their lengths and roughnesses) would be given, and the five unknowns to be found would be Q_1, Q_2, Q_3, H_2, and H_3 (assuming H_1 is known). For the design problem H_2 and H_3 are given, along with H_1, and two diameters can then be found. The unknowns in the design problem would be Q_1, Q_2, Q_3, and two diameters. There are three possible combinations of two diameters: D_1 and D_2, D_1 and D_3, and D_2 and D_3. In the first combination D_3 must be given, in the second D_2, and in the third D_1. Specifying a diameter plus the head at both ends of a pipe establishes from the head loss equation the discharge in that pipe. These three combinations of diameters create the three problem cases shown in Fig. 5.8.



Case 1. D_3 known (Q_3 fixed) Case 2. D_2 known (Q_2 fixed) Case 3. D_1 known (Q_1 fixed)

Figure 5.8 The three cases.

One approach to the solution of these three cases is to write the 5 basic equations (plus the secondary equations), specify the knowns and solve for the unknowns. In other words the independent equations are simultaneously solved for as many unknowns as there are equations. This approach is illustrated by the "Rule Sheet" from TK-Solver, shown below with the three variable sheets for these three cases. The diameter that is regarded as known is listed in the "Input" column, and the diameters that are to be found are listed in the "Output" column.

However, from these cases one may be able to see a computationally more efficient means of solving the problem. First, the discharges in the pipes with given diameters can be computed by solving a head loss equation. Next, by removing these pipes and

RULE SHEET

```
S Rule-----------------------------------------------------------------------
 Q2+Q3=QJ3
 Q1-Q2=QJ2
   H1-H2=f1*(L1/D1)*Q1^2/(G2*(pi()/4.*D1^2)^2)
   H2-H3=f2*(L2/D2)*Q2^2/(G2*(pi()/4.*D2^2)^2)
   H1-H3=f3*(L3/D3)*Q3^2/(G2*(pi()/4.*D3^2)^2)
     1/sqrt(f1)=1.14-2*log(e/D1+7.34347283*v*D1/(Q1*sqrt(f1)))
     1/sqrt(f2)=1.14-2*log(e/D2+7.34347283*v*D2/(Q2*sqrt(f2)))
     1/sqrt(f3)=1.14-2*log(e/D3+7.34347283*v*D3/(Q3*sqrt(f3)))
```

Case 1

VARIABLE SHEET

St Input----	Name---	Output---
	D1	.1209918
	D2	.1244984
.125	D3	
	Q1	.0459944
	Q2	.0159944
	Q3	.0290056
150	L1	
400	L2	
550	L3	
.045	QJ3	
.03	QJ2	
100	H1	
85	H2	
80	H3	
.0000	e	
1		
1.3E-6	ν	
19.62	G2	
	f2	.0176878
	f1	.0148337
	f3	.0159636

Case 2

VARIABLE SHEET

St Input----	Name---	Output---
	D1	.1211631
.125	D2	
	D3	.1247196
	Q1	.0461667
	Q2	.0161667
	Q3	.0288333
150	L1	
400	L2	
550	L3	
.045	QJ3	
.03	QJ2	
100	H1	
85	H2	
80	H3	
.0000	e	
1		
1.3E-6	ν	
19.62	G2	
	f2	.0176642
	f1	.0148277
	f3	.0159747

Case 3

VARIABLE SHEET

St Input----	Name---	Output---
.12	D1	
	D2	.1215503
	D3	.1265911
	Q1	.0450043
	Q2	.0150043
	Q3	.0299957
150	L1	
400	L2	
550	L3	
.045	QJ3	
.03	QJ2	
100	H1	
85	H2	
80	H3	
.0000	e	
1		
1.3E-6	ν	
19.62	G2	
	f2	.0178295
	f1	.0148689
	f3	.0159017

Figure 5.9 The TK-Solver variable and rule sheets for the three cases.

modifying the demands on the reduced network, the discharges in the remaining pipes can be determined so they satisfy the junction continuity equations. Finally, for the remaining two pipes whose discharges are now known, the head loss equations can be solved for the diameters. Thus for all three cases the problem can be reduced to the solution of three separate equations, in proper order, each with only one unknown. (If the Darcy-Weisbach equation is selected for use, then actually pairs of equations must be solved, because the Colebrook-White equation for f must also be employed.)

For Case 1 this procedure would consist of the following steps if the heads are specified as $H_1 = 100$ m, $H_2 = 85$ m and $H_3 = 80$ m, each $e = 0.00001$ m, $L_1 = 150$ m, $L_2 = 400$ m, and $L_3 = 550$ m:

(a) Find Q_3 from $H_1 - H_3 = f_3(L_3/D_3)Q_3^2/(2gA_3^2)$ and the Colebrook-White equation using $D_3 = 0.15$ m, $L_3 = 550$ m, $e_3 = 0.00001$ m, $QJ_2 = 0.03$ m^3/s, and $QJ_3 = 0.045$ m^3/s; the solution is $Q_3 = 0.029$ m^3/s, $f_3 = 0.016$.

(b) From continuity (i.e. inspection) $Q_1 = 0.0460$ m^3/s, $Q_2 = 0.0160$ m^3/s.

(c) Seek D_1 from $H_1 - H_2 = f_1(L_1/D_1)Q_1^2/(2gA_1^2)$ and the Colebrook-White equation; the result is $D_1 = 0.1210$ m.

(d) Finally, find D_2 from $H_2 - H_3 = f_2(L_2/D_2)Q_2^2/(2gA_2^2)$ and the Colebrook-White equation; the result is $D_2 = 0.1245$ m.

When our discussion indicated that the number of pipe diameters that can be sought is equal to the number of junction continuity equations, one might infer that a simultaneous solution of continuity equations would provide all of the unknown diameters. This is not the case. In fact a simultaneous solution of the junction continuity equations provides the discharges in the pipes with unknown diameters (since this case uses only one continuity equation at a time), and the head loss equations (e.g., the Darcy-Weisbach equation) are used to find the unknown diameters and also to determine the discharges in pipes whose diameters are specified. In other words, all of the equations were used.

Before moving on to additional and more complex networks, we must note that it is quite possible to create combinations of specifications that lead to impossible situations. In this three-pipe network, for example, if the diameter of pipe 3 in case 1 were specified to be too large so that the discharge it conveys in response to the head loss $H_1 - H_3$ exceeds the demand QJ_3, an impossible problem is defined in which the flow in pipe 2 must be from node 3 to node 2, but this is not possible because H_2 is greater than H_3. A specified diameter which is too small can also create impossible conditions: if in case 3 $D_1 = 0.1$ m, then the discharge in pipe 1 must be (with $H_1 = 100$ m and $H_2 = 85$ m) $Q_1 = 0.028$ m^3/s, which is less than the demand QJ_2, and so the flow in pipe 2 must be from node 3 to node 2, but this is not possible because $H_2 = 85$ m is larger than $H_3 = 80$ m. The prescribed diameters and the heads at the pipe ends must be within certain limits so the flow pattern is consistent with what is required by continuity at both ends of these pipes and with the head distribution in nearby pipes.

Consider next the design of a simple network consisting of only two pipes with reservoirs at both ends, as shown in Fig. 5.10. If this network is viewed as a design

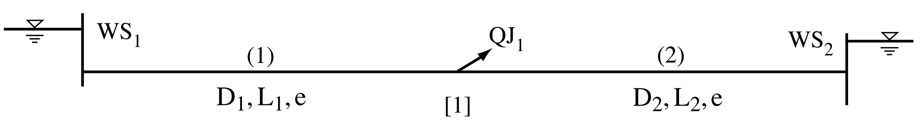


Figure 5.10 A simple two-pipe, two-reservoir network.

problem in which the head is specified at the one node, then only one pipe diameter can be found. There are three basic equations available, one junction continuity equation and two head loss equations, since NJ + NP = 1 + 2 = 3. Below are results from TK-Solver for a set of known values for two cases; in the first case D_1 is unknown, and in the second case D_2 is unknown. This simple network with a pseudo loop (because there are two supply sources) shows that the same principles govern how many diameters can be found for a looped network with two or more supply sources and for a looped network with one supply source. Clearly the number of diameters that can be regarded as unknown equals the number of junction continuity equations. This same principle applies to branched networks.

RULE SHEET

```
S Rule------------------------------------------------------------------------
  Q1-Q2=QJ1
    WS1-H1=f1*(L1/D1)*Q1^2/(G2*(pi()/4.*D1^2)^2)
    H1-WS2=f2*(L2/D2)*Q2^2/(G2*(pi()/4.*D2^2)^2)
      1/sqrt(f1)=1.14-2*log(e/D1+7.34347283*v*D1/(Q1*sqrt(f1)))
      1/sqrt(f2)=1.14-2*log(e/D2+7.34347283*v*D2/(Q2*sqrt(f2)))
```

Case 1

VARIABLE SHEET

St Input-----	Name----	Output------
	D1	.67179122
.5	D2	
	Q1	2.0806609
	Q2	.58066088
1200	L1	
1000	L2	
1.5	QJ1	
85	H1	
.0001	e	
1.217E-5	v	
64.4	G2	
	f1	.01569440
	f2	.01840944
100	WS1	
80	WS2	

Case 2

VARIABLE SHEET

St Input-----	Name----	Output------
.8	D1	
	D2	.76556319
	Q1	3.299349
	Q2	1.799349
1200	L1	
1000	L2	
1.5	QJ1	
85	H1	
.0001	e	
1.217E-5	v	
64.4	G2	
	f1	.01494752
	f2	.01613281
100	WS1	
80	WS2	

Figure 5.11 The rule and variable sheets for the network of Fig. 5.10.

Another view of this two-pipe network problem might be as in Fig. 5.12; now a desired pressure at the downstream end is sought. This specified pressure could equally well be interpreted as a reservoir with a specified water surface elevation, as in the previous example. If so, then the demand at node 2, QJ_2, is unknown since it is the discharge into the downstream reservoir. If the diameters of both pipes are specified, then this is an analysis problem.

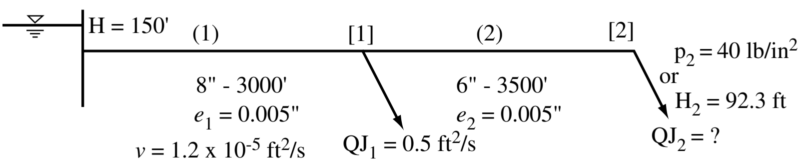


Figure 5.12 An alternative view of the network in Fig. 5.10.

The unknowns are Q_1, Q_2, and QJ_2, and the three equations that are to be solved to determine these unknown values are

Node 1 continuity	$Q_1 - Q_2 - QJ_1 = 0$	(5.23)
Node 2 continuity	$Q_2 - QJ_2 = 0$	(5.24)
Pseudo loop equation	$h_{f1} + h_{f2} = 150 - 92.3 = 57.7$	(5.25)

For this problem the pressure specification in place of the demand at node 2 allows this unknown demand to be computed. The solution requires the first continuity equation and the loop equation to be solved simultaneously (if the Darcy-Weisbach equation is used, then one Colebrook-White equation must be added for each f; so we actually require the simultaneous solution of four equations for the four unknowns Q_1, Q_2, f_1, and f_2), followed by noting from the second continuity equation that $Q_2 = QJ_2$. The results are $Q_2 = QJ_2 = 0.815$ ft^3/s, $Q_1 = 1.315$ ft^3/s, $f_1 = 0.01935$, and $f_2 = 0.02056$. We encourage you to verify this solution.

An alternative would be to pose the question: What pipe diameter D_2 would be needed if the demand QJ_2 were to be 0.6 ft^3/s and the pressure at node 2 were to be $p_2 = 40$ lb/in^2 (HGL = 92.3 ft)? This is now a design problem; in our three equations QJ_2 is known, and the unknowns are Q_1, Q_2, and D_2. A logical sequence in solving this problem would first note that $Q_2 = 0.6$ ft^3/s (the specified demand); next find $Q_1 = 1.1$ ft^3/s from the first continuity equation, and with Q_1 known compute $h_{f1} = 13.77$ ft, leading to $h_{f2} = f_2(L_2/D_2)(Q_2/A_2)^2/(2g) = 31.7 f_2/D_2^5 = 43.9$ ft from the loop equation, which when solved with the Colebrook-White equation would produce $f_2 = 0.02157$ and $D_2 = 5.194$ in. If the pressure is also specified at node 2, then both pipe diameters can be found. Then the problem is converted into a branched system with demands known at all three nodes, and the heads are also known at these nodes.

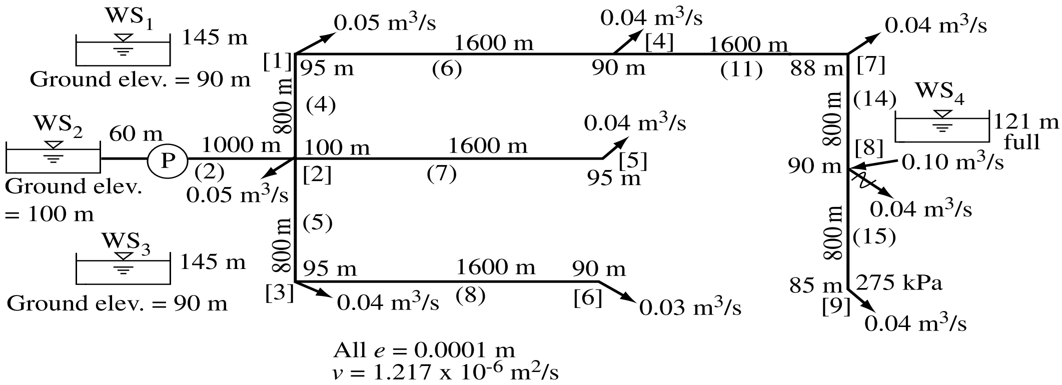
This example illustrates a principle that can be applied to our second looped-network category: each alternate specification allows us to regard another variable as a member of the set of unknowns. In this case if pressures are specified, then diameters can be left unspecified, and the resulting equations can be used to determine these diameters. However, if the pipe roughness coefficients are unknown, then we must specify the diameters. In brief, any variable in a pipe network may be left unspecified while another is specified in its place, so long as the number of independent equations equals the number of unspecified variables, or unknowns, for which a solution is sought.

Flow through a single pipe illustrates this principle. For the Darcy-Weisbach approach six variables appear in the problem: L, D, e, Q, f, and h_f. Two independent equations are available, the Darcy-Weisbach equation $h_f = f(L/D)Q^2/(2gA^2)$ and the Colebrook-White equation $1/\sqrt{f} = 1.14 - 2\,\mathrm{Log}\{e/D + 9.35/(Re\sqrt{f})\}$, that allow two unknowns to be found. Any pair of variables may be selected as unknown, so long as the other four variables are given values. Other equations may appear in this process, such as $A = \pi D^2/4$, $Re = VD/\nu$, and $V = Q/A$; these equations define secondary quantities. More fundamentally, however, these additional variables may be added to the list of variables, and the equations may be added to the list of equations. Then V may be counted as an unknown, for example. For each additional pipe in a network one can add variables to the list of unknowns, and at the same time equations are added to that list. Thus for two pipes the list of six variables becomes 12, and the number of unknowns that can be found increases to four, etc. Almost any combination of variables may be chosen as unknowns.

In summary: (1) We use two basic fluid mechanics principles in the design of pipe systems, the continuity principle (conservation of mass) and the energy principle. (2) The continuity principle assures that the discharge into each junction (or node) in the network equals the discharge from that junction. Mathematically, ΣQ_i - $QJ_j = 0$, in which the subscript on Q denotes the pipe numbers that join at that junction, and QJ is the demand at this junction. (QJ is positive from, and negative to, the junction. The reverse convention applies for pipe discharges; Q_i is positive if to the junction, and negative if from the junction.) (3) The energy principle accounts for the head loss that occurs in a pipe, $H_i - H_j = h_{fk}$, in which subscript k denotes the pipe number and subscripts i and j denote the upstream and downstream node numbers. If every pipe head loss equation is used, then the network connectivity guarantees that the head losses around loops sum to zero and through pseudo loops equals the difference in water surface supply elevations. (4) These two principles provide all of the basic equations that are available. (5) The number of unknowns and independent equations must match for a unique solution to exist. (6) Any variable may be selected as an unknown. Once the unknowns have been chosen, then the remaining variables must be specified. (7) It is possible, however, to assign values to known variables in such a way that physically impossible situations are created.

No one set procedure exists for the design of looped networks. Professional judgment is required to balance the concern for redundancy (i.e., the ability to satisfy large emergency demands, or to allow components to be pulled out of service) with the desire to minimize costs. Since the equations will allow only NJ pipe diameters to be determined, one workable procedure would first select NL = NP - NJ pipes, for which we specify the diameters. The selection of these pipes should be such that, if they were to be removed, the remaining network would be a branched network. Normally there are several pipe combinations that could be selected to reduce a network to a branched system, and this branched system should be considered to be the main transmission lines. The specification of diameters for the pipes (NL in number) that have been selected is also based on judgment; if these pipes are secondary, they might be given diameters that are the minimum size that is allowed for this network. Second, with the heads known at the ends of these pipes, compute their discharges, and then modify the demands at the two pipe ends to include these discharges in defining the branched system. Third, solve the branched network. The diameters that are found for this branched system are normally then replaced by the nearest standard pipe sizes, but they may be rounded up to the next larger standard pipe size. Fourth, conduct analyses that cover a variety of conditions that the proposed network is expected to encounter, and study these results. If deficiencies are noted, adjust the pipe diameters (or other network components) so these deficiencies no longer exist.

To illustrate this procedure, assume that the 16-pipe network in Fig. 5.6 is the subject of a design study. The supply sources denoted by WS_1 and WS_3 are imported from another water supplier with a head of 50 m, but this water is costly and will be used only when demands are large. The other source, WS_2, is from a groundwater well with an aquifer water surface elevation that is 40 m below the ground surface. Lastly, WS_4 is a storage tank with a 45-m diameter, a bottom elevation at 118 m and a maximum depth of 3 m. The average demands are given in the accompanying table, and the demands (in m^3/s) for the hour of greatest demand, on which the design is to be based, are twice these values.



Node	1	2	3	4	5	6	7	8	9
Demand m^3/s	0.025	0.025	0.020	0.020	0.020	0.015	0.020	0.020	0.020

Figure 5.13 Another view of the network of Fig. 5.6.

First NP - NJ = 16 - 9 = 7 pipes must be given diameters, say 150 mm, the smallest size allowed in this system. Pipes 1, 3, 9, 10, 12, 13, and 16 are selected, based on judgment. Pipes 1 and 3 are chosen because they supply the expensive water and will be shut off during this design process. Using the maximum capacity of the pumping station, which is 0.22 m^3/s with all pumps on, the demands are summed, and it is determined that the storage tank must supply 0.14 m^3/s. Therefore, the demand at node 8 is changed from 0.04 m^3/s to an inflow of 0.10 m^3/s, and pipe 16 is removed. Elimination of these pipes results in the branched system in the figure. Of course, depending upon the choice for the main transmission system, there are several alternatives that could be explored. For this reduced system, node 9 is the farthest downstream, and its pressure should be set to the minimum allowable pressure, say 275 kPa, which corresponds to $H_9 = 113$ m for the elevation of the HGL. The pipe discharges in this branched network can now be determined directly, as given below in the first table. Based on energy-line slopes, which are also in this table, and on economic considerations, the heads at the nodes can be computed; they are listed in the second table. By solving the Darcy-Weisbach and Colebrook-White equations simultaneously for the 9 single pipes, the diameters can then be computed. These computed diameters, also listed in the first of Tables 5.4, should be replaced by sizes chosen from a set of standard sizes (such as the following: 150, 205, 255, 305, 355, and 405 mm). As a final step, analyses of the full network should be completed for several different demand levels, storage tank levels, fire flows, etc.

Table 5.4

Pipe	Q m^3/s	S_f	h_f m	Dia. mm
2	0.23	0.005	5.0	428.6
4	0.07	0.006	4.8	263.1
5	0.07	0.005	4.0	272.7
6	0.02	0.010	16.0	148.1
7	0.04	0.002	4.0	253.0
8	0.03	0.005	8.0	197.9
11	0.02	0.0025	4.0	194.8
14	0.06	0.005	4.0	257.5
15	0.04	0.005	4.0	220.6

Node	H m
1	125.0
2	129.8
3	125.8
4	109.0
5	125.8
6	117.8
7	113.0
8	117.0
9	113.0

The newly-found diameters in Table 5.4 ignore any influence of the pipes that were removed. This approach assumes that the discharges in the other (ignored) pipes assist the network in performing adequately under the variety of conditions that will occur, but the branched system can by design supply the required demands without the other pipes. In a sense it also assumes that the discharges carried by the pipes that are ignored is small in comparison to the discharges in the pipes that are retained throughout the computations.

Next let us explore the design process further by attempting another design without ignoring the flows in pipes 1, 3, 9, 10, 12, and 13 that were removed to form the branched network. Assume these all have a diameter of 150 mm and that pipes 1 and 3 (that bring water from the more costly water supply) are open. With the additional flow from these two pipes let us also assume that no flow enters or leaves the storage tank through pipe 16, and that the head losses in the pipes are as specified in the third part of Tables 5.5. Under these assumptions the flow in pipes 11 and 14 now reverse direction from what they were previously. Consequently the heads at the nodes are as given in the second part of Tables 5.5, and with these heads the discharges in the other pipes can be computed, as listed in the first of Tables 5.5:

Tables 5.5

Pipe	h_f m	L m	Dia. mm	Q m^3/s
1	16.0	800	150	0.030
3	15.8	800	150	0.029
9	4.8	800	150	0.016
10	8.0	800	150	0.021
12	12.8	1600	150	0.018
13	8.8	1600	150	0.015

Node	H m
1	129.0
2	133.8
3	129.8
4	125.0
5	129.8
6	121.8
7	121.0
8	117.0
9	113.0

Pipe	h_f m	Q m^3/s	Dia. Mm
2	5.0	0.311	480.8
4	4.8	0.131	334.1
5	4.0	0.034	207.5
6	4.0	0.111	372.4
7	4.0	0.095	351.1
8	8.0	0.025	161.2
11	4.0	0.087	339.5
14	4.0	0.047	234.5
15	4.0	0.025	184.7

If the assumption was, as before, that the storage tank was supplying $0.14\ m^3/s$ along with the supply through pipes 1 and 3, and the HGL-elevations at the nodes was as before, then the results in Tables 5.6 would be obtained:

Tables 5.6

Pipe	h_f m	L m	Dia. mm	Q m^3/s
1	20.0	800	150	0.033
3	19.2	800	150	0.033
9	8.8	800	150	0.022
10	8.0	800	150	0.021
12	8.8	1600	150	0.015
13	4.0	1600	150	0.011
16	4.0	1200	384	0.140

Node	H m
1	125.0
2	129.8
3	125.8
4	109.0
5	125.8
6	117.8
7	123.0
8	117.0
9	113.0

Pipe	h_f m	Q m^3/s	Dia. Mm
2	5.0	0.114	480.8
4	4.8	- 0.011	*
5	4.0	0.027	190.2
6	8.0	- 0.028	*
7	4.0	0.098	355.2
8	8.0	0.020	169.8
11	4.0	0.046	266.8
14	4.0	0.086	294.9
15	4.0	0.029	195.4

* The heads do not allow a negative Q.

The newer set of assumptions has led to an impossible situation in which the junction con-tinuity equations require flows in pipes 4 and 6 in the opposite direction from what the heads at their ends require. The specified flow from the storage tank was too large to be compatible with the heads and pipe diameters that were specified. In the earlier case the absence of flow from the storage tank avoided the impossible situation that was created in the last set of specifications. However, we see clearly that various combinations of specified variables can lead to situations in which the direction of flow is inconsistent with the heads at some nodes.

One even simpler example of an inappropriately specified diameter consists of two pipes which meet at junction [2]; the HGL at this junction is smaller than the HGL at the other ends of these pipes, as shown in Fig. 5.14. The discharge for each pipe must be toward the common junction. If the diameter of either pipe is specified so that the resulting discharge in that pipe exceeds the demand QJ_2, then an impossible situation has been created, since the direction of the discharge in the other pipe must oppose the direction of flow implied by the HGL for that line.

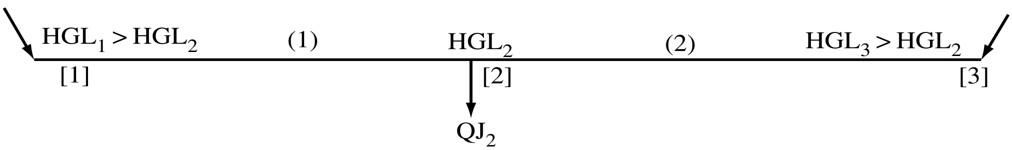
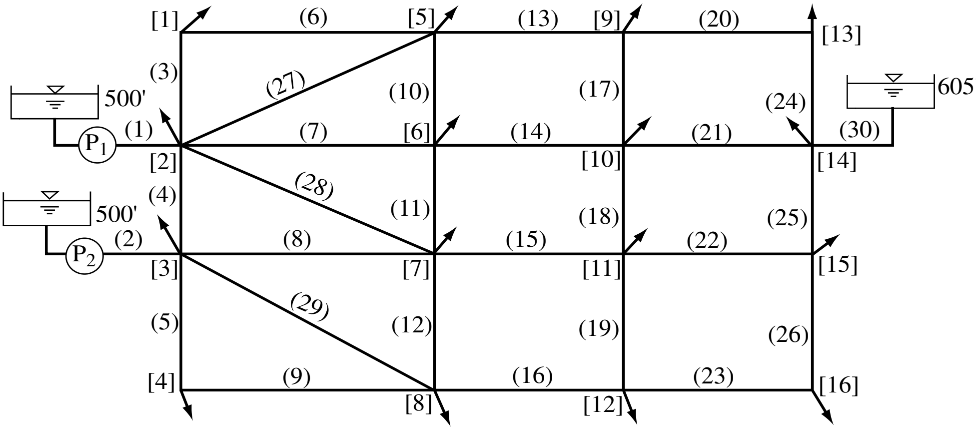


Figure 5.14 A problem with an inappropriate diameter.

For example, let $HGL_1 = 100$ ft., $HGL_2 = 88$ ft., $HGL_3 = 90$ ft., $L_1 = 2000$ ft., $L_2 = 2500$ ft. and $QJ_2 = 1.0$ ft^3/s. If $D_1 = 8.0$ in, then $Q_1 = 1.15$ ft^3/s, and we must have $Q_2 = -\ 0.15$ ft^3/s to satisfy continuity at the junction, which is inconsistent with the set of specified heads. If $Q_1 = 1.0$ ft^3/s, then $D_1 = 7.60$ in; hence D_1 must be less than 7.6 in for a solution to be possible.

These analyses illustrate the important fact that the outcome of a design depends directly upon the assumptions that go into that design. While cost has not yet been considered in these designs, the usual objective is to minimize the total cost of meeting a set of specified demands. We will include cost considerations later in the chapter.

Now let's examine a larger network of 30 pipes and 16 nodes, as shown in Fig. 5.15. For this network with 3 supply sources there are 16 junction continuity equations, and it is therefore possible to determine 16 pipe diameters if the heads are given at all 16 nodes. The input data to obtain a "design" solution by using NETWK is in the file FIG5_15.IN on the CD. The reader can list this file and use it as input to NETWK to obtain a solution. This input lists the pipe lengths and the nodal demands. If the option DESIGN=1 is given in the $SPECIF list, then (1) NETWK interprets a 0 for a diameter as one that is to be determined, and (2) the elevation of the HGL at each node must be listed after the elevation of the node under the NODES command. Thus for this network with DESIGN=1 we must assign 16 pipes a zero diameter in the input data. The example input data set has assigned diameters of 18 in and 15 in, respectively, to the two pipes from the source pumps, and pipes 10, 11, 12, 17, 18, 19, 24, 25, 26, 27, 28, and 29 have been given diameters of 6 inches. This problem is quite large for a hand solution, but the approach to a solution, if done by hand, could follow precisely the approach that was applied to the networks that have just been examined. First, the discharges in the pipes with specified diameters would be computed so that each pipe head loss matches the difference in head between its end nodes. These pipes would then be removed from the network, and the demands at their ends would be adjusted for their discharges. Next, from the junction continuity equations the discharges in the remaining pipes would be determined, and finally, with these discharges known, the diameters of the remaining pipes would be found.



Pump 1

Q, ft^3/s	8	15	22
h_p, ft	157	152	144

Pump 2

Q, ft^3/s	6	12	18
h_p, ft	152	147	139

Figure 5.15 A 30-pipe, 16-node network.

In the solution from NETWK we find after the design solution has computed the pipe diameters that NETWK then selects the closest standard diameter from its default list of standard diameters and performs an analysis of this system since the option NOMSOL=1 is included in the $SPECIF list. From the analysis the column giving HGL elevations will change from the initially specified values because the standard pipe sizes will not produce the same frictional head losses.

The program NETWEQST in the NETWK program package is intended specifically for design problems. It allows the user to specify the unknown variables and allows any of the variables in the network to be regarded as an unknown. The mechanics of this solution will be explained later in this chapter. By selecting all of the pipe discharges and a number of pipe diameters that is equal to the number of nodes in the network, this type of design problem can be solved. The input data file for NETWEQST for this problem follows.

```
Large Design Example
/*
$SPECIF IUNENT=4 $END
PIPES
1 0 2 500 18 .004 15
2 0 3 500 15 .004 11
3 2 1 800 12 .004 5
4 2 3 800 6  .004 1
5 3 4 800 12 .004 6
6 1 5 1800 6 .004 5
7 2 6 1800 12 .004 6
8 3 7 1800 12 .004 6
9 4 8 1800 12 .004 3
10 6 5 800 6  .004 .5
11 6 7 800 6  .004 .5
12 7 8 800 6  .004 .5
13 5 9 1600 12 .004 5
14 6 10 1600 12 .004 5
15 7 11 1600 12 .004 5
16 8 12 1600 12 .004 5
17 10 9 800 6 .004 1
18 10 11 800 6 .004 .2
19 11 12 800 6 .004 .2
20 9 13 1600 12 .004 4
21 10 14 1600 8 .004 1
22 11 15 1600 8 .004 2
23 12 16 1600 8 .004 1
24 14 13  800 6 .004 1
25 14 15  800 6 .004 1
26 15 16  800 6 .004 1
27 2 5 2500 6 .004 1
28 2 7 2500 6 .004 1
29 3 8 2500 6 .004 1
30 0 14 1000 10 .004 2.5
RESER
30 605
PUMPS
1 8 157 15 152 22 144 500
2 6 152 12 147 18 139 500
NODES
1 1.2 500 630
2 1.2 490 645
3 .8 485 640
4 1.6 480 632
5 1.4 495 618
6 1.2 494 620
7 1. 490 616
8 .8 483 613
9 2. 493 605
10 2 492 608
11 3.6 488 605
12 2.8 484 603
13 4. 480 595
14 2 478 600
15 1.8 475 594
16 2 470 586
RUN
```

Figure 5.16 Input for NETWEQST.

The option IUNENT=4 tells NETWEQST that the HGL-elevations at the nodes are listed as the last item after the NODES command, and the last item on each line following the PIPES command is an initial estimate of the discharge in that pipe, to be used to start the Newton solution method. The manual for NETWEQST is on the CD as file NETWEQST.DOC. To solve this problem with NETWEQST, the responses listed in Fig. 5.17 should be provided in response to the **bold** prompts from NETWEQST.

Pipes = 30, Nodes = 16, Sources = 3
46 unknowns must be given. Give no. of each:
1. HGLs at nodes 0
2. Nodal demands 0
3. Pipe discharges 30
4. Pipe diameters 16
Give 16 pipe diameter numbers 3-9 13-17 20-23 30

Figure 5.17 Prompts and responses for the 30-pipe example.

In obtaining solutions to these design problems, one must have considerable understand-ing of either the system performance or the sizes of the specified diameters; otherwise the corresponding set of specified heads can lead to an impossible situation. We have already seen how such a situation can occur for the 16-pipe network. So we cannot select arbitrari-ly all of the pipes that will be assigned a prescribed diameter. Since the pipes having specified diameters carry a fixed discharge, the network problem becomes in essence one with these pipes removed. The reduced network must still be able to satisfy all specified demands at the nodes. There are different combinations of circumstances that may make this impossible to do. First, if in creating the reduced network the original network has become divided into two or more separate networks, then each separate network must have at least one supply source. Second, in the reduced network the specified heads must allow the flow to move in the direction that is dictated by the demands. Furthermore, we know it will not be possible to prescribe a diameter for every supply pipe in the network, because the resulting set of computed discharges (that are fixed by the prescription of diameters and of heads at the end nodes) will generally not sum to the total demand in the network.

In the 30-pipe problem the diameters of the pipes connected to the source pumps were both given, but the reservoir pipe diameter was not given. If D_{30} is given, then either D_1 or D_2 must not be given. The heads may remain unchanged if D_{30} is given and D_1 is not given (but still with D_2 = 18 in). However, if D_2 is not given when D_{30} is given as 6 in, then D_1 must be given a diameter that is larger than 18 in, because with D_1 = 18 in the solution of the continuity equations produces a negative flow in pipe 4, but this is not possible for the heads that are given at the ends of that pipe.

Assigning diameters to pipes that connect to the source pumps fixes the discharge that these pumps can supply; therefore the discharge through the pipe from the reservoir must equal the difference between the sum of the demands on the network and the amount of the discharge from the two source pumps. With these restrictions it is difficult to create even one loop in the reduced network. Therefore, we must verify that a branched network is obtained when the pipes having specified diameters are removed from the network, and if the removal of these pipes separates the original network into two or more smaller networks, then each of these new networks must have a supply source. Nor can we specify the diameter of a dead end pipe because, with a specified diameter and a pair of given heads at the ends of the line, the computed discharge usually will not match the specified demand at the end of the line.

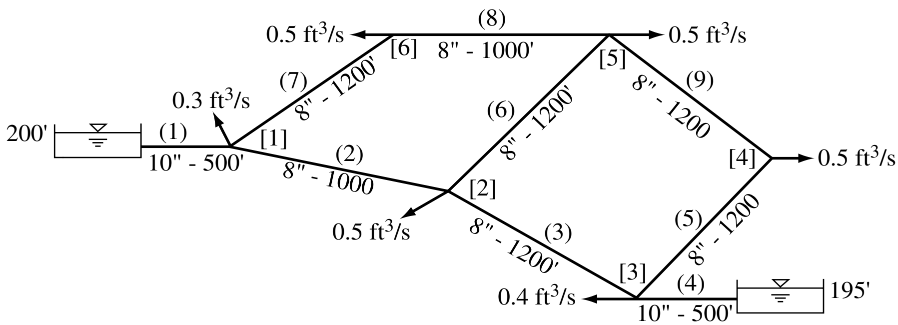
Less obvious impossible situations can develop. For example, if the diameter of one or more pipes that contain source pumps is specified to be too large so that the inflow to the network from this/these source/s exceeds the sum of the network demands, and if furthermore the reservoir water surface elevation were specified to be above H_{14}, then no way exists for the surplus inflow from the source pumps to leave the system. Since the specification of the pipe diameter and the head at the ends of this pipe fixes its discharge, this discharge value may not match the external demand at the end of the pipe. For example, if in the 30-pipe network the diameter of pipe 1 were set at 24 in rather than 18 in, such an impossible situation would be the result. On the other hand, if D_{30} is given and D_2 is not, then $D_1 = 18$ in would cause an impossible situation but $D_1 = 24$ in would not. When more than two pipes meet at a junction, the possibilities of creating impossible situations are more numerous and more complex. NETWK does detect the existence of impossible situations and then prints a brief message related to the problem. When this occurs (and it will occur frequently if one is not sufficiently careful and/or experienced in the specification of diameters and heads), it is important first to examine carefully the possible causes of this situation; then the values of the specified diameters and/or heads can be adjusted, or an alternate set of pipes can be selected for the specification of diameters. In making these adjustments, it may be helpful to sketch the reduced network after the pipes with specified diameters have been removed, and then to keep in mind the process that will be followed in obtaining this design.

Example Problem 5.4

Designs are to be obtained for the looped water distribution system below, given the heads at the nodes listed in the table. Since three independent loops (two real loops and one pseudo loop) exist here, the sizes of three pipes must be specified, and the diameters of the remaining six pipes are to be found. All specified pipe diameters are to be 6 inches. Determine whether the assignment of 6-inch diameters to the following combinations of pipes will allow a solution of the remainder of the branched system; if an impossible situation has been created, determine why this is the case. Otherwise solve the branched network.

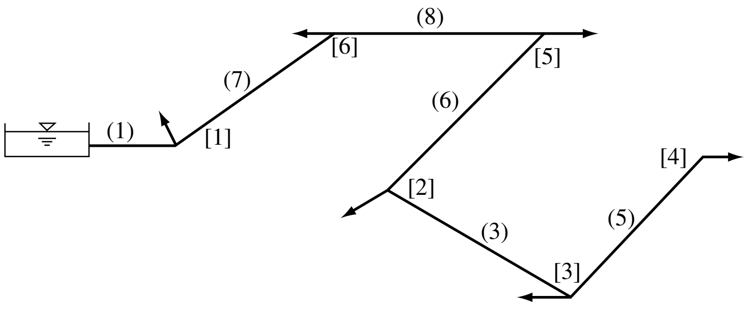
Node	Elev. ft.	H ft.
1	100	197
2	98	194
3	100	194.5
4	95	190
5	95	191
6	90	188

Case	Spec. Pipe
1	2, 4, 9
2	4, 6, 9
3	4, 8, 9
4	3, 8, 9
5	2, 8, 9
6	2, 6, 9

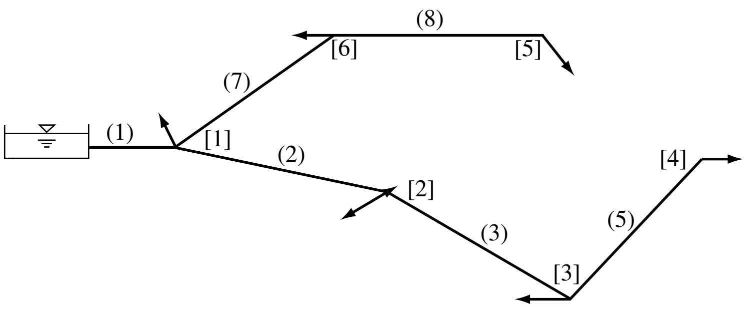


The first steps are to determine the discharges in the pipes with specified diameters and then reduce the network to the branched system. Sketches for these reduced branched systems will be presented.

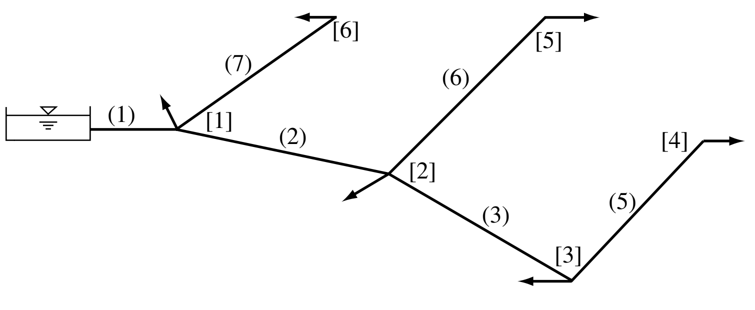
Case 1:
This case is not valid since $H_3 > H_2$, so the flow cannot pass through pipe 3 to meet downstream demands.



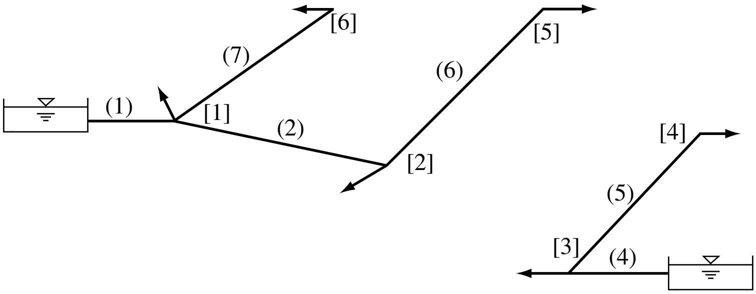
Case 2:
This case is not valid for the same reason as Case 1.



Case 3:
This case is not valid since $H_3 > H_2$; thus the flow cannot satisfy the demand at nodes 3 and 4.



Case 4:
This configuration is valid. We first compute the discharges in the 6-inch pipes.



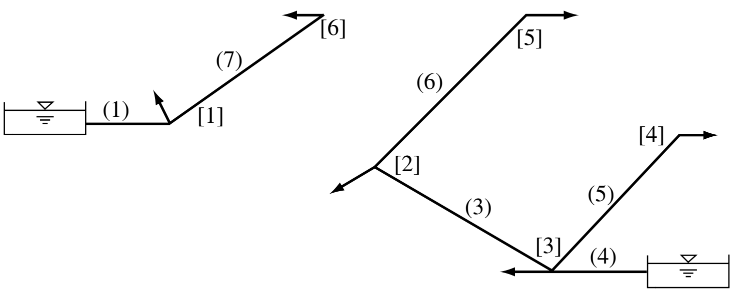
Pipe	Q ft^3/s	f	Changed QJ's ft^3/s
3	0.145	0.024	$QJ_3 = 0.545$, $QJ_2 = 0.355$
8	0.435	0.020	$QJ_5 = 0.935$, $QJ_6 = 0.065$
9	0.214	0.023	$QJ_5 = 1.149$, $QJ_4 = 0.286$

The solution for the reduced branched system provides the following discharges and pipe diameters:

Pipe	Q ft^3/s	Dia. in.
1	1.869	9.000
2	1.504	9.556
4	0.831	9.592
5	0.286	4.902
6	1.149	8.965
7	0.065	2.446

Case 5:

This case is valid. A solution can be obtained with NETWEQST. (Alternatives are to use NETWK or apply HYDEQS to obtain individual discharges and/or diameters.) The table of input data, the list of prompts and responses, and two tables of results follow:




```
Input Data
Example Problem 5.4
/* 1 .3 100 197
$SPECIF IUNENT=4 $END
PIPES
1 0 1 500 10 .001 2.4
2 1 2 1000 6 .001 1
3 3 2 1200 6 .001 .2
4 0 3 500  6 .001 1
5 3 4 1200 6 .001 .5
6 2 5 1200 6 .001 .5
7 1 6 1200 1 .001 1
8 5 6 1000 6 .001 .2
9 5 4 1200 6 .001 .2
NODES

2 .5 98 194
3 .4 100 194.5
4 .5 95 190
5 .5 95 191
6 .5 90 188
RESER
1 200
4 195
RUN
```

Pipes= 9, Nodes=6
15 Unknowns must be given. Give no. of each:
1. HGLs at nodes 0
2. Nodal demands 0
3. Pipe discharges 9
4. Pipe diameters 6
Give 6 pipe diameter numbers 1 3-7

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. in	e x10³ in	Q ft³/s	HEAD LOSS ft.
1	0	1	500	6.539	1.0	0.800	3.00
2	1	2	1000	6.000	1.0	0.435	3.00
3	3	2	1200	13.247	1.0	1.214	0.50
4	0	3	500	13.079	1.0	1.900	0.50
5	3	4	1200	4.902	1.0	0.286	4.50
6	2	5	1200	8.965	1.0	1.149	3.00
7	1	6	1200	2.445	1.0	0.065	9.27
8	5	6	1000	6.000	1.0	0.435	3.00
9	5	4	1200	6.000	1.0	0.214	1.00

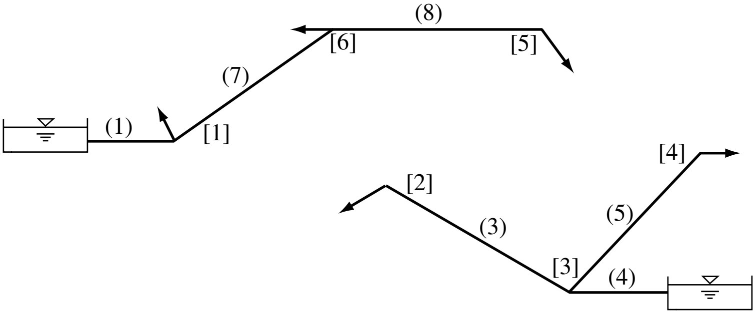
NODE DATA

NODE	DEMAND ft³/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in²	HGL ELEV. ft.
1	0.300	100	97.0	42.0	197.0
2	0.500	98	96.0	41.6	194.0
3	0.400	100	94.5	41.0	194.5
4	0.500	95	95.0	41.2	190.0
5	0.500	95	96.0	41.6	191.0
6	0.500	90	98.0	42.5	188.0

Case 6:

The case is valid. Using the same input as in case 5, the solution can be found with the following responses to the prompts from NETWEQST (Solution not given):

Pipes = 9, Nodes = 6
15 unknowns must be given. Give no. of each:
1. HGLs at nodes 0
2. Nodal demands 0
3. Pipe discharges 9
4. Pipe diameters 6
Give 6 pipe diameter numbers 1 2-5 7 8



* * *

5.4 DESIGNING SPECIAL COMPONENTS

Section 5.3 defined two types of design problems: (1) problems in which as many pipe diameters are sought as there are nodes in the network; and (2) problems in which an individual pipe diameter is sought so that some specified condition (e.g., a pressure) occurs at a prescribed node. That section examined the first problem category. This section considers the second problem type. Previously the solution of such problems involved a trading of known and unknown variables. For example, if a nodal pressure was specified, then a nodal demand (or pipe diameter or length, etc.) was placed in the list of unknowns. Now, however, it will not be necessary to swap a variable from the known to the unknown list. Instead a new unknown will be introduced into the network problem, and a new equation will be added to the list of equations, thus satisfying the requirement that the number of independent equations and the number of unknowns must match.

How is it possible to obtain another independent equation, one might ask. As was stated before, the basic network relations are

$$NP = NJ + NL \quad \text{if the network has two or more supply sources}$$

or

$$NP = (NJ - 1) + NL \quad \text{if the network has fewer than two supply sources.}$$

These relations apply to both branched and looped networks and cannot be changed. In a branched network $NP = NJ - 1$ and therefore $NL = 0$. The key in introducing an additional unknown is also to create another independent equation. This additional equation will enforce another condition, usually a nodal HGL (or pressure) that is required. This unknown will be called a differential head device, and in mathematical equations it will be given the symbol ΔH; it will represent a variety of devices such as a booster pump, a pressure-reducing station, a valve, a pipe with an unknown diameter, or a wall roughness. A differential head device will be something that creates a (positive or negative) head difference, other than the frictional head loss of the original pipe, between the ends of a pipe. If the pipe diameter or roughness is to be unknown, then the new pipe will produce a head loss that is the sum of the frictional head loss of the original pipe and the computed differential head. The equation for this additional unknown, ΔH, that will be added to the equation system will be an energy equation that is written between the node where the pressure (or pressure head, or HGL elevation) is specified and another point of known head in the network. This other point of known head will usually be a supply source such as a reservoir or source pump. However, if two or more differential head devices are introduced, then the added equation might be an energy equation between two nodes with specified pressures. The additional equation is a special pseudo loop that will generally be independent of the other loops because it imposes an additional condition on the network that requires a pipe to have a different head loss than that which is caused by fluid friction alone. The phase "generally independent" is used because, as described later, inappropriate specifications can cause the added equation not to be independent. This loop is called special because it consists of a continuous path along pipes from an internal pipe, one end of which has a specified HGL, to a supply source, whereas the usual pseudo loop follows a sequence of pipes between two supply sources. Thus this special loop is similar to a pseudo loop connecting the downstream end of a pressure-reducing valve (PRV) or the upstream end of a back-pressure valve (BPV) to a supply source or to another PRV or BPV where the HGL is specified.

To illustrate the concept and implementation of a differential head device, the small network consisting of 7 pipes and 5 nodes in Fig. 5.18 will be examined. In this network

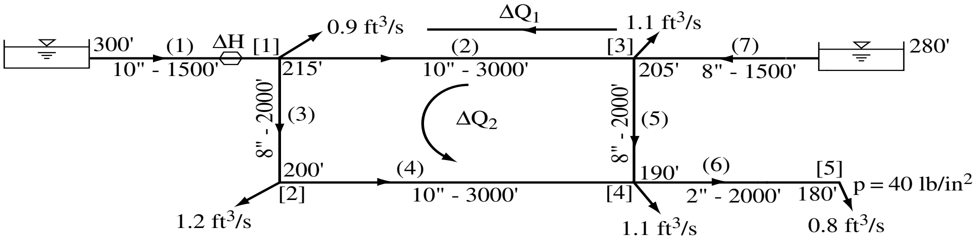


Figure 5.18 Small network with a differential head device.

the amount (magnitude and sign) of the differential head ΔH that is needed in pipe 1 is to be determined so that the pressure at node 5 is 40 lb/in^2. In addition to ΔH, the solution is to determine the discharges in all 7 pipes and the elevations of the HGL (and pressures) at the 4 internal nodes. (The pressure is specified at node 5, so this pressure can not be part of the solution.) In this problem ΔH in pipe 1 could be a booster pump if it is positive, or it could be a valve if it is negative. If it is negative, the differential head device may instead be a pressure-reducing valve; once ΔH is known, it is a simple computation to determine the pressure setting for a PRV which would produce this same additional head loss. Or if ΔH is positive but smaller in magnitude than the frictional head loss in the pipe containing it, then one could compute an "equivalent" pipe diameter that would produce the same head loss as the present frictional head loss and ΔH. Thus, depending upon what the solution produces for ΔH and what you want the differential head device to represent, it can be any of a variety of appurtenances.

If the ΔQ-system of equations is to model this network, the system may be written as

$$K_7(Q_{o7}+\Delta Q_1)^{n_7} - K_2(Q_{o2}-\Delta Q_1-\Delta Q_2)^{n_2} - K_1(Q_{o1}-\Delta Q_1)^{n_1} + \Delta H + 20 = 0 \quad (5.26)$$

$$K_3(Q_{o3}+\Delta Q_2)^{n_3} + K_4(Q_{o4}+\Delta Q_2)^{n_4} - K_5(Q_{o5}-\Delta Q_2)^{n_5} - K_2(Q_{o2}-\Delta Q_1-\Delta Q_2)^{n_2} = 0 \quad (5.27)$$

$$K_6Q_{o6}^{n_6} + K_5(Q_{o5}-\Delta Q_2)^{n_5} + K_7(Q_{o7}+\Delta Q_1)^{n_7} + (40(144)/62.4+180) - 280 = 0 \quad (5.28)$$

in which the vector of initial flows that satisfy all junction continuity equations might be

$$\{\boldsymbol{Q}_o\} = \begin{Bmatrix} Q_{o1} \\ Q_{o2} \\ Q_{o3} \\ Q_{o4} \\ Q_{o5} \\ Q_{o6} \\ Q_{o7} \end{Bmatrix} = \begin{Bmatrix} 2.5500 \\ -1.3533 \\ 3.0033 \\ 1.8033 \\ 0.0967 \\ 0.8000 \\ 2.5500 \end{Bmatrix} \quad (5.29)$$

The three unknowns are ΔQ_1, ΔQ_2, and ΔH. The solution of these three equations (using Newton's method with the above vector for $\{\mathbf{Q}_o\}$) yields

$$\begin{Bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta H \end{Bmatrix} = \begin{Bmatrix} -4.33 \\ -0.59 \\ 112.62 \end{Bmatrix} \tag{5.30}$$

To verify that the solution is correct, one must use the correct values for K and n for each pipe, which are listed in the following table:

Pipe No.	1	2	3	4	5	6	7
K	1.70	2.55	7.34	3.70	7.46	31.8	5.51
n	1.957	1.931	1.932	1.873	1.865	1.905	1.918

Any verification should also employ ΔQ_1, ΔQ_2, and ΔH to compute the HGL elevations and the pressure at each node.

It is much easier to let the computer do the arithmetic. Below is the input data file for NETWK to solve this problem, followed by the output. The line of input data after DHEAD consists of the following items: (1) the pipe containing ΔH; (2) an estimate of this ΔH; (3) the pressure that is being specified (the minus indicates that the pressure is in lb/in^2, rather than being specified as a HGL); (4) the designation (pipe since NODESP = 0) for a supply source, to use in forming the energy equation loop; and (5) the pressure in lb/in^2 at node 5. This solution file contains an extra table for differential head devices; it reports an INCREMENTAL HEAD of 112.62 ft and also states NO EQUIVALENT DIA. POSSIBLE. Had the value for HEAD LOSS minus the INCREMENTAL HEAD been negative, then an EQUIVALENT DIAMETER would have been reported in the last column of this extra table, as well as e and the head loss in the equivalent pipe.

```
Problem for Differential Head Device
/*
$SPECIF NPRINT=-3,COEFRO=.004 $END
PIPE-
1 10. 1500. 1 .9  215.
2 10. 3000. 1 3 1.1 205.
3 8. 2000. 1 2 1.2 200.
4 10. 3000. 2 4 1.1 190.
5 8. 2000. 3 4
6. 2000. 4 5 .8 180.
7 8. 1500. 3
RESER
1 300
7 280
DHEAD
1 40 -5 7 40.
RUN
```

Figure 5.19 Input data for NETWK for the differential head device problem of Fig. 5.18.

Problem for Differential Head Device
ALL DEMAND FLOWS ARE MULTIPLIED BY 1.0000
FLOW FROM PUMPS AND RESERVOIRS EQUALS 5.100

PIPE NO.	ORIG. DIA.	Q	INCR. HEAD	HEAD LOSS	TOTAL HEAD	e	EQUIV. DIA.
	in	ft^3/s	ft.	ft.	ft.		in
1	10.0	6.88	112.62	74.2	NO EQUIV.	DIA. POSSIBLE	

PIPE DATA

PIPE NO.	NODES FROM	TO	L	DIA.	e $x10^3$	Q	VEL.	HEAD LOSS	HLOSS /1000
			ft.	in		ft^3/s	ft/s	ft.	
1	0	1	1500	10	4.0	6.88	12.62	74.2	49.5
2	1	3	3000	10	4.0	3.58	6.56	41.6	13.9
3	1	2	2000	8	4.0	2.41	6.89	40.1	20.0
4	2	4	3000	10	4.0	1.21	2.21	5.25	1.8
5	3	4	2000	8	4.0	0.69	1.99	3.77	1.9
6	4	5	2000	6	4.0	0.80	4.07	20.8	10.4
7	3	0	1500	8	4.0	1.78	5.11	16.9	11.3

NODE DATA

NODE	DEMAND ft^3/s	DEMAND gal/min	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL. ELEV ft.
1	0.9	404	215	123.4	53.5	338.4
2	1.2	539	200	98.4	42.6	298.4
3	1.1	494	205	91.8	39.8	296.9
4	1.1	494	190	103.1	44.7	293.1
5	0.8	359	180	92.3	40.0	272.3

Figure 5.20. Solution using the input data listed in Fig. 5.19.

This problem might also be altered, for example, to specify a pressure of 40 lb/in^2 at node 2. This could be done by placing a special differential head device in pipe 3. We encourage you to modify the input file in Fig. 5.19, compare it with file FIG19.IN, obtain a solution and then compare it with file FIG19.OUT on the CD.

The foregoing example might cause a person to believe that it is possible to specify a pressure anywhere within a network and place a differential head device in any pipe. However, this is not the case; a problem can be specified for which there is no solution. An example is the specification of pressures at nodes 4 and 5 in this network without a differential head device in pipe 6. The discharge in pipe 6 must be 0.8 ft^3/s to satisfy the specified downstream demand, and this discharge dictates the head loss in pipe 6. Therefore the specification of pressures at both ends of pipe 6 without placing a differential head device in this pipe will result in an insoluble problem. Similar situations can be created.

In using the DHEAD command with NETWK, one must be relatively familiar with the performance and nature of the network if the specification of impossible situations is to be avoided. Should an impossible situation be specified, then NETWK will be unable to complete a solution. In some instances the iterative solution process will simply fail to converge; this condition becomes apparent when the number of iterations exceed the allowable maximum and the residual, reported as SUM or SUM OF DIFFERENCE, is not becoming smaller. Or NETWK will indicate that a singular matrix exists; then an examination of the system of equations should allow one to discover why the singular matrix exists. However, often it is easier simply to examine the network and the system specifications until it becomes apparent how to change the input data to allow a solution.

Aside from dead-end pipes with pressures specified at both ends, let us look further at some other conditions that can lead to an improperly posed problem. If we specify a larger HGL downstream from a node with a smaller HGL, then we must place a differential head device in one of the pipes between these two nodes. Or in a network with all of its supply sources in one subregion or at one end of the system, the specification of the HGL elevations must allow it to decrease continually in the downstream direction through the network, unless differential head devices have been placed in some of the intermediate pipes. And differential head devices that produce negative incremental heads will be needed in some pipes if HGL elevations are specified to decrease more rapidly in the downstream flow direction than can be caused by pipe friction alone. Experience shows it is difficult to avoid the creation of an impossible situation if the differential head devices are all located near the supply sources. In general, if HGL elevations are to be specified at several nodes, then the pipes containing differential head devices should also be near these nodes.

A network, diagrammed in Fig. 5.21, will illustrate some less easily recognized specifications that will cause an impossible situation. To receive maximum benefit from this description, the reader is encouraged to prepare input data for this network and actually obtain solutions etc. as the next few pages are read. This network is a skeletonized system for a small city. The supply for the network comes from a single source outside of town. A storage tank has been installed near the old main part of town to supply some of the demand during periods of above-average usage, and to receive water from the pump when demand is low. The town has grown, expanding into some areas with slightly higher elevations, especially to the west of the storage tank.

The present pump is not adequate; to begin the study we decide to seek a solution that will tell us the pump head that will meet the demand shown in Fig. 5.21 when the storage tank neither receives nor supplies any water. To set up the problem for NETWK, the DHEAD command can be used to indicate that pipe 1 contains the differential head device and that the elevation of the HGL at node 3 should be 580 ft., which is exactlythe elevation of the water surface in the storage tank.

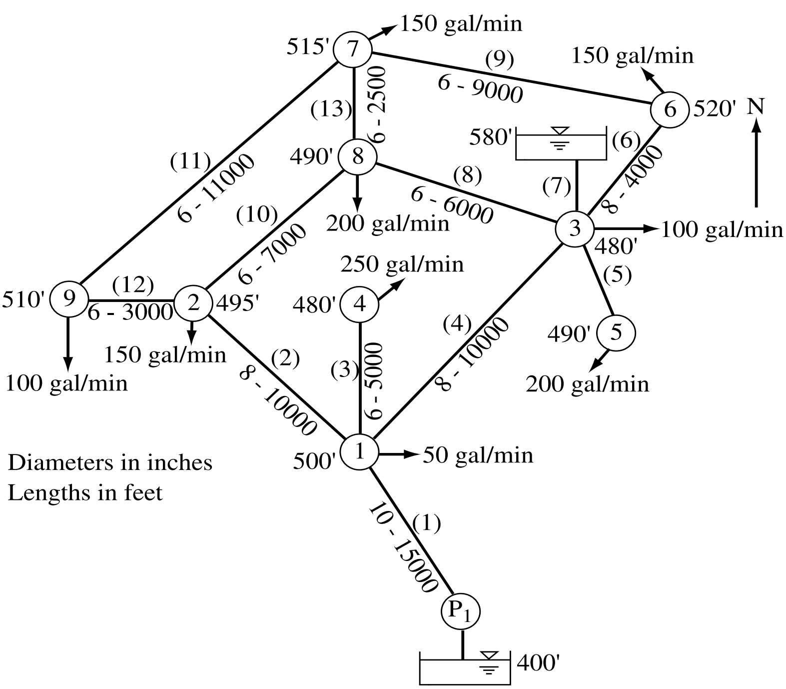


Figure 5.21 A skeletonized network for a small city.

We might remove the source pump, add up the demands and specify this sum as a negative demand at the node which replaces the source, i.e., 10. This problem might seem reasonable because the demands would be exactly satisfied by the flow from this new node, and the elevation of the HGL will be established throughout the system by retention of the connection between the reservoir and the network. However, we have created a problem for which there is no solution. The network has only one supply source. With a specified differential head device we must have a reservoir whose discharge is unknown in order to define the additional equation that is needed for a solution. We have created problem specifications that cause the flow from the reservoir to be known and equal to zero. The unknowns for this problem are the three corrective discharges ΔQ_1, ΔQ_2, and ΔQ_3, around the three loops of the system, plus the incremental head in pipe 1. Therefore four independent equations are required. Three of these are the energy equations for the three loops of the system. The fourth equation forces the head loss in pipe 7 to equal the dif-ference between the water surface elevation in the reservoir and the specified HGL at node 3; this equation is invalid because the flow in pipe 7 is not unknown. (We might run NETWK to attempt to solve this improperly-posed problem, using NPRINT=1 or larger, so the output could be studied.) From this problem we see that at least two supply sources must exist if a differential head device is to be used in a network.

The specification of the impossible might be avoided by treating the pump as a second supply source. It may be changed to a reservoir with a suitably chosen water surface elevation or the original pump could be retained. If the reservoir option is selected, the differential head reported by NETWK is the head that the pump should supply; if the existing pump is kept, the reported head will be the additional head needed by the new pump over that which is supplied by the existing pump. Figure 5.22 presents a suitable input data file for this problem. (The CD contains this file as FIG22.IN.)

```
SIZING A PUMP - RESERVOIR FLOW
SHOULD BE ZERO FOR DESIGN
/*
 $SPECIF NPRINT=10,NFLOW=1,NPGPM=1 $END
PIPES                                  NODES
 1 0 1 15000 10  .005                   1 50 500
 2 1 2 10000 8/                         2 150 495
 3 1 4 5000 6/                          3 100 480
 4 1 3 10000 8/                         4 250 480
 5 3 5 4000 6/                          5 200 490
 6 3 6 4000 8/                          6 150 520
 7 0 3 2000 8/                          7 150 515
 8 3 8 6000 8/                          8 200 490
 9 6 7 9000 6/                          9 100 510
 10 2 8 7000 6/                        RESER
 11 9 7 11000 6/                        7 580
 12 2 9 3000 6/                         1 400
 13 8 7 2500 6/                        DHEAD
                                        1 250 3 7 580
                                       RUN
```

Figure 5.22 NETWK input data file for the skeletonized network.

In this input data two reservoirs are given, the original storage tank with a water surface elevation of 580 ft. and the reservoir where the source pump really exists with a water surface elevation of 400 ft. The input line after the DHEAD command consists of (1) pipe 1 that contains the pump, (2) an estimate that this pump must supply about 250 ft of head, (3) the HGL is to be specified at node 3, (4) the source at the end of pipe 7 is to be a part of the additional equation containing the differential head, and (5) the specified HGL elevation.

To gain modeling experience with a differential head device, the following exercises are recommended:

(a) Extract from the CD the data listed in Fig. 5.22 and obtain a solution.

(b) Modify this data to designate supply sources as nodes.

(c) Modify the data from either (a) or (b) so that the original pump is now used in the problem specification. The original pump has the following pump characteristics:

Discharge, gal/min.	700	1200	1500
Head, ft.	370	350	280

(d) Add a second differential head device in pipe 9 that is to produce a HGL elevation of 605 ft. at node 9. In this analysis retain the requirement, as in (a), that the pump meet all of the demand.

Some study of the results from these four solutions will show the following: (a) The pump must develop a head of 391 ft to supply all of the flow. (b) An additional head of 70.2 ft above that produced by the present pump is needed, but if the diameter of pipe 1 is 11.64 in instead of the present 10 in, then the present pump would meet the requirements. (c) A booster pump that produces a head of 78.5 ft and a discharge of 3.01 ft^3/s is needed in pipe 9 to increase the HGL elevation at node 9 by 20 ft to 905 ft. (d) While the booster pump in pipe 9 does increase the pressure at several nodes where the pressure was low, it also decreased the pressure at node 6 to just under 16 lb/in^2. From these results the engineer must decide which improvements to propose for this water distribution system; proposals can then be examined further with appropriate analyses.

We turn now to a discussion of the twin questions of (1) how to select the pipes in which the pipe diameters will be specified, and (2) how to assign numerical values to these specified diameters. We begin by looking at the nine-pipe looped network in Fig. 5.23. Since this network has six nodes, diameters must be chosen for three pipes if all of the nodal demands and HGL elevations are given. The design demands are shown in the figure, and the target HGL elevations are listed in the table.

Node No.	**HGL Elev.** ft.
1	482
2	463
3	481
4	466
5	470
6	463

Pipe No.	**L** ft.
1	2000
2	2000
3	1200
4	1800
5	1200
6	2000
7	1300
8	2000
9	1300

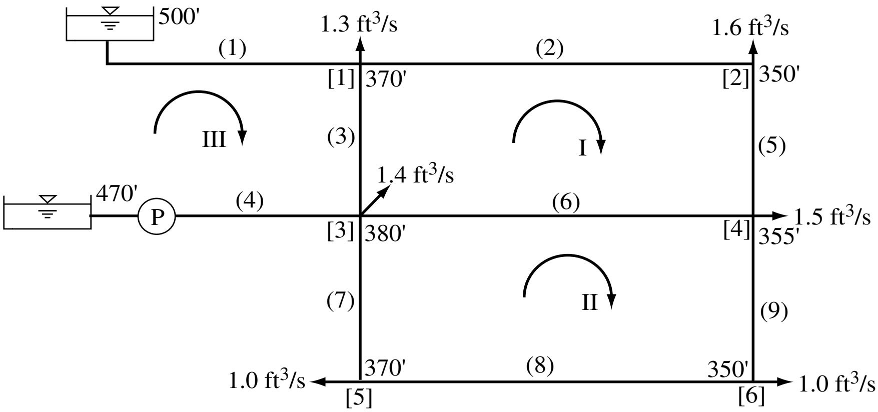


Figure 5.23 A nine-node pipe network.

In examining this network we note that the pipes to be deleted must prevent the existence of loops I, II, and III. The reduced network will be branched after three pipe diameters are specified. If pipes 5 and 9 are assigned diameters, then clearly loops I and II will not exist in the reduced network. But the effect of specifying the diameter of pipe 3 in order to eliminate the pseudo loop is not obvious; actually the network would be divided into two branched systems, but this step is permissible since each has a supply source. Thus pipes 3, 5, and 9 could be assigned diameters if the values for these diameters are suitably chosen. Many other combinations are also acceptable. If the diameter of pipe 3 is specified, then loops I and II can be broken by also specifying diameters for any of the following pairs of pipes: 6 and 9, 5 and 8, 5 and 7, or 5 and 6. And pipe 3 is not the only pipe whose removal would break pseudo loop III. Either pipe 1 or pipe 4 could replace the role of pipe 3. For these last two alternatives the reduced network would no longer be divided into two branched systems. Then the specification of an inflow into node 1 or node 3, respectively, would be required, because the assignment of a diameter to a pipe that connects a supply source to the network fixes the discharge from that source. Further thought will show for each of pipes 1, 3, and 4, that there exist five pairs of pipe numbers that could be chosen in order to arrive at a properly posed problem. For each triple of pipe numbers we can also choose reasonable pipe diameters. Table 5.7 lists the 15 combinations and reasonable sizes for these pipes. The reader will find it instructive to prepare input data and obtain, and study, the solutions from NETWK for several, if not all, of these combinations. File FIG5_23.IN contains the input for NETWK for the second combination which assigns diameters to pipes 3, 6, and 9.

Table 5.7 Possible pipe combinations whose diameters could be specified

Pipe Nos.	Sizes in	Pipe Nos.	Sizes in	Pipe Nos.	Sizes in
3, 5, 9	8, 6, 8	1, 5, 9	10, 6, 8	4, 5, 9	10, 6, 8
3, 6, 9	8, 10, 8	1, 6, 9	10, 10, 8	4, 6, 9	10, 10, 8
3, 5, 8	8, 6, 7	1, 5, 8	10, 6, 7	4, 5, 8	10, 6, 7
3, 5, 7	8, 6, 8	1, 5, 7	10, 6, 8	4, 5, 7	10, 6, 8
3, 5, 6	8, 6, 10	1, 5, 6	10, 6, 10	4, 5, 6	10, 6, 10

To investigate the appropriateness of values for diameters, suppose that pipe 6 had been assigned a diameter of 8 in instead of 10 in in the second combination. Since the head loss in pipe 6 is prescribed as 15 ft., the discharge must be Q_6 = 1.503 ft^3/s instead of a discharge of 2.784 ft^3/s that is obtained for the 10-in diameter. Since the diameter of pipe 5 is also specified, its flow is $Q_5 = 0.412$ ft^3/s from node 4. Since the energy line slopes from node 4 to node 6, the flow in pipe 9 must also be from node 4. The sum of the discharge in pipe 5 and the demand at node 4 is 1.912 ft^3/s; thus the flow into this node through pipe 6 must exceed 1.912 ft^3/s. However, since this is not the case, the specification $D_6 = 8$ in makes it impossible to find a consistent solution. To obtain a consistent solution the minimum diameter for pipe 6 can be computed by setting the discharge at 1.912 ft^3/s with a head loss of 15 ft in this pipe. This diameter is 8.765 in. If in this case the diameter of pipe 5 was a different prescribed value, it would cause us to compute a different minimum diameter for pipe 6. The specification of a diameter for a pipe must allow the junction continuity equations at both ends of it to be satisfied. In larger networks the satisfaction of this criterion at the ends of all pipes whose diameters are specified is often not an easy task.

5.5 DEVELOPING A SOLUTION FOR ANY VARIABLES

This section examines methods to determine any variable associated with a pipe network. The unknowns may be selected from the (1) hydraulic grade lines at nodes, (2) demands at nodes, (3) discharges in pipes, (4) pipe diameters, (5) pipe roughnesses, and (6) elevations of water supply surfaces. There are two restrictions: (1) the number of unknowns must equal the number of independent equations, i.e., NJ + NP, and (2) the knowns are such that an impossible flow situation is not created. Clearly the second restriction means that the specified variables must be appropriately configured and assigned values so that a solution for the unknowns will exist.

In Chapter 4 the computer program EQUSOL1 was introduced and discussed. The subroutine FUNCT must be rewritten for each individual network when this program is used. (The use of MathCAD and TK-Solver is similar; the user supplies the equations to be solved and then identifies the unknown variables.) We now consider a computer program that does not require us to rewrite a subroutine for each different problem. This program will be restricted in its use to the solution of network problems, but the variables that are to be found will be specified in the input data. The program will accept differing numbers of each of the six types of unknown and known variables, so long as the total number of unknowns matches the number of independent equations. For example, an "analysis" problem could be specified, in which the discharges in all pipes and the HGL elevations at all nodes are determined and all pipe diameters, lengths, roughnesses, and nodal demands are prescribed. For analysis problems this program will not be as efficient as the programs in Chapter 4 that solved the Q-equations, the H-equations, or the ΔQ-equations because more equations are solved. The program will first read a description of the network so it will know how the pipes are connected; then it will define NJ junction continuity equations and NP pipe head loss equations; and finally it will solve these equations simultaneously for whichever variables that are identified as unknown. The input to this program must describe the network adequately in a manner that is common for pipe systems, i.e., giving data for each pipe and node in the network. Before reading further, print one of the versions of NETWEQS1 from the CD so you can study the listing as you read.

5.5.1. LOGIC AND USE OF NETWEQS1

Program NETWEQS1 will solve pipe-system problems for any of six types of unknowns, or any reasonable combination of them, so long as the number of unknowns equals the number of independent equations. The number of equations consists of the sum of the number of pipes NP and number of nodes or junctions NJ in the pipe system, or NEQS = NP + NJ. If no supply source is identified, then there exist only NJ - 1 independent equations from application of the continuity principle at the junctions.

The subroutine FUN defines these equations. The continuity equations are first evaluated in the `DO 10` loop, and then the head loss equations

$$H_i - H_j = \{ f(L/D)Q^2/(2gA^2) \}_k \tag{5.31}$$

follow in the remainder of this subroutine. The friction factor f is found by using Gauss-Seidel iteration if the flow is turbulent ($Re > 2100$) and by using $f = 64/Re$ if the flow is laminar, but if $Re < 160$, then $f = 0.4$ to prevent f from becoming unbounded whenever $Q \to 0$. (Note RE in the program is actually $Re/7.34347283$.)

The user must provide estimates for the unknowns, as well as values for the knowns, in the input file for NETWEQS1. When NETWEQS1 is executed, the user is asked to provide three input/output unit numbers, the acceleration of gravity g, and the kinematic viscosity VISC of the fluid. The default values for these parameters are IN2 = 2, IN5 = 5, IN4 = 4, g = 32.2 ft/s^2 and VISC = 0.00001417 ft^2/s. To accept all these defaults, simply give / in the Fortran program after the last value that has been entered. The meaning of the three input/output units is as follows: IN2 is the input unit for the majority of the data that describes the pipe system. If IN2 = 0 or IN2 = 5, then this data must be entered from the keyboard in the proper order without any prompts. When IN2 is not 0 or 5, then a prompt will request the name of the file that contains the input data. If the user is using MS-Fortran, an alternative is to give the input file name on the "command line" (or after typing NETWEQS1 to execute the program, leave a space and list the file name). If IN4 = 0 or 6, then the output will be written to the monitor; otherwise it will go to a file.

The input data for NETWEQS1 consists of two types. The first type describes the network, and this data is read by using logical unit IN2. The second type defines the unknowns, and it is read by using logical unit IN5. If this data is placed in a file (IN5 not equal to 0 or 5), then this file provides the data that defines the unknowns. The default IN5 = 5 indicates that these data are to come from the keyboard. If IN5 is 5 or 0, then NETWEQS1 will prompt for the input that is needed to define the unknowns. These data consist of the following 6 values (on separate lines):

1. The number of HGL elevations that are unknown at nodes.
2. The number of nodal demands that are unknown.
3. The number of unknown pipe discharges.
4. The number of pipe diameters that are unknown.
5. The number of pipe roughnesses that are unknown.
6. The number of unknown water surface elevations.

After these six lines that give the number of each type of unknown, the next lines give lists of node or pipe numbers that identify the individual unknowns. The number of these lines will match the number of categories (a maximum of six) that are given nonzero numbers. These lists of numbers can consist of individual values or a range of values separated by a minus sign (-). The subroutine RLINE will allow ranges of integers to be intermixed with single integers. The argument NUM returns the number of integers in the list in the Fortran program. For example, if a pipe system consists of 6 pipes and 5 nodes, and the unknowns are to be the discharges in all pipes and the HGL elevations at all nodes, then the input specifications should consist of the following numbers:

5; 0; 6; 0; 0; 0; 1 2 3 4 5; 1 2 3 4 5 6

In these files the semicolon (;) indicates that a new line should be used. If the keyboard is used to supply the input data, then a prompt appears in place of each semicolon and automatically separates the data. Alternatively this input could be the following:

5; 0; 6; 0; 0; 0; 1-5; 1-6

In place of the HGL elevation at node 5, if it were desirable to determine the demand at node 5, then the input file could be the following:

4; 1; 6; 0; 0; 0; 1 2 3 4; 5; 1 2 3 4 5 6

An alternative listing of this input might be this set:

4; 1; 6; 0; 0; 0; 1-4; 5; 1-6

To reiterate, if IN5 = 5 or 0, then NETWEQS1 will prompt the user for the next expected piece of information. If IN2 and IN5 are given the same value so both types of data are in the same file, then the data read under IN5 (the data that defines the unknowns) is given after the data read by IN2 which defines the configuration of the pipe system.

The unit defined by IN4 is the Fortran unit number that will write the problem solution as output data. This output will go to the terminal/monitor if IN4 = 0, 5, or 6. Otherwise it will be written to a file. A prompt will request the file name, unless it is included on the command line. The program calls subroutine SOLVEQ (see Appendix A) to solve the linear system of equations that is obtained by implementation of the Newton method. In this solution the elements of the Jacobian matrix of derivatives are evaluated numerically, as described in Chapter 4.

5.5.2. DATA TO DESCRIBE THE PIPE SYSTEM

Most of the data that describe the system is normally placed in a file that will be read on Fortran logical unit IN2. These data consist of the following:

Line 1: Four integers; number of pipes NP, number of nodes NJ, number of reservoirs NRES, number of pumps NPUMP.

Line 2: Pairs of values; each pair consists of the pipe number that connects a reservoir to the network and the water surface elevation of this reservoir. Each pair can be on a separate line.

Line 3: If pumps exist (NPUMP>0), then seven values are required for each pump: the number of the pipe containing the pump, followed by three pairs of discharge and pump head which define the pump characteristic curve. The data for each pump is a separate line.

Next NP lines: These lines contain the pipe data, six items per pipe. The pipes must be numbered consecutively from 1 through NP. There is one line per pipe, sequenced by pipe number, since the pipe number itself is omitted. Each line contains the following:

1. The upstream node.
2. The downstream node.
3. The pipe length.
4. The pipe diameter.
5. The pipe roughness.
6. The discharge in the pipe.

The pipe diameters and wall roughnesses must have the same units, e.g., feet for ES units or meters for SI units. These values are only estimates if the variable is an unknown, for they then become initial values for the Newton method in the solution process.

Next NJ lines: These lines contain the node data, three items per node. There is one line per node, sequenced as the nodes are numbered because the node number is not included. Each line consists of the following:

1. The demand at the node.
2. The HGL elevation at the node.
3. The ground elevation of the node.

All values must be listed in consistent units, e.g., ft^3/s and ft for ES units or m^3/s and m for SI units.

To see how this input scheme works, consider the small network in Fig. 5.24. The input data set would take the form shown in Fig. 5.25.

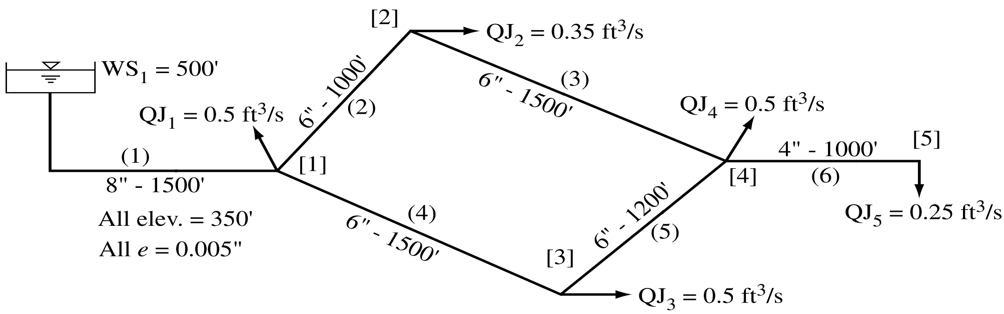


Figure 5.24 A small network to study with NETWEQS1.

```
6 5 1 0
1  500.
0  1  1500.  .667  .000417  2.1
1  2  1000.  .5   .000417   .82
2  4  1500.  .5   .000417   .47
1  3  1500.  .5   .000417   .78
3  4  1200.  .5   .000417   .28
4  5  1000.  .333   .000416667   .25
.5  476.05   350.
.35  464.8  350.
.5  460.7  350.
.5  458.9  350.
.25  452.8  350.
```

Figure 5.25 The input data set for the small network in Fig. 5.24.

Assume the data in Fig. 5.25 have been placed in a file FIG1.DAT, which is the file name for it on the CD. Upon execution of NETWEQS1, the following three values could be given from the keyboard in response to the first prompt: 2 5 4/ Next a file name is requested. The name FIG1.DAT would be given in reply. Since the second input logical unit has been given as 5, the user is then asked to define the number and types of the unknown variables. Upon supplying 5 0 6 0 0 0, the user is requested to give the numbers associated with items 1 and 3. The two responses could be 1-5 and 1-6. Next the output file name is requested. The solution in this output file consists of the following two tables:

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. in	e x10⁴ in	Q ft³/s	HEAD LOSS ft.
1	0	1	1500	0.667	4.17	2.100	23.95
2	1	2	1000	0.500	4.17	0.824	11.39
3	2	4	1500	0.500	4.17	0.474	5.98
4	1	3	1500	0.500	4.17	0.776	15.21
5	3	4	1200	0.500	4.17	0.276	2.17
6	4	5	1000	0.333	4.17	0.249	10.94

Figure 5.26 The output from NETWEQS1.

NODE DATA

NODE	DEMAND ft^3/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	0.500	350	126.0	54.6	476.0
2	0.350	350	114.7	49.7	464.7
3	0.500	350	110.8	48.0	460.8
4	0.500	350	108.7	47.1	458.7
5	0.250	350	97.7	42.4	447.7

Figure 5.26, concluded. The output from NETWEQS1.

5.5.3. COMBINATIONS THAT CAN NOT BE UNKNOWNS

We have noted that NP + NJ independent equations exist, and therefore we might regard this many variables as unknowns that we may seek to find. However, there are combinations of these variables that cannot be selected as unknowns because mathematical problems then arise, for which there is no solution. The difficulty is that the equations are mathematically inconsistent. If an impossible problem is specified, then either convergence to a solution will not occur or the Jacobian matrix that is used in implementing the Newton method will be singular. Let us look further at the elements that create a problem for which it is impossible to find a solution. It has become obvious to us for a single pipe that it is not possible to specify its diameter and roughness, the discharge in it and the heads at both ends. By specifying D, e, Q, H_i, and H_j, the relation between discharge and head loss is fully defined, and the need for a frictional head loss equation has been eliminated. When these variables are all prescribed, a problem is defined which has no solution, and the equations are inconsistent. Therefore, if any pipe in a network problem has D, e, and Q given and the heads at both ends are selected as known values, the problem has no solution. One of these variables must be unknown.

There are many less obvious combinations of known and unknown variables that can cause a problem to be impossible to solve. To illustrate some of the possibilities, consider the three-pipe looped network in Fig. 5.27. For this network there are five basic equations, two junction continuity equations and three pipe head loss equations. Assume all demands are known, and we decide to specify Q_1 in pipe 1. Now it is no longer possible to specify the heads at both ends of either pipe 2 or 3 along with their diameters and roughnesses, because the specification of Q_1 has also fixed Q_2 and Q_3, since $Q_2 = Q_1 - QJ_2$ and $Q_3 = QJ_3 - Q_2 = QJ_3 + QJ_2 - Q_1$. We would therefore be deceiving ourselves if we placed either Q_2 or Q_3 in the list of unknowns. Regardless of which

Node	Demand ft^3/s	HGL Elev. ft.
1	- 2.2	200
2	1.2	198
3	1.0	195

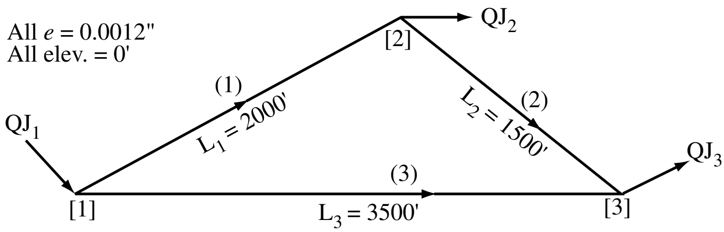


Figure 5.27 A three-pipe looped network.

discharge is specified, the other two have also been fixed if any two of the three demands are specified.

Table 5.8 lists 20 combinations of known and unknown variables and the solution for each combination that leads to a set of consistent simultaneous equations. The lengths of pipes are $L_1 = 2000$ ft., $L_2 = 1500$ ft. and $L_3 = 3500$ ft. To limit the entries in the table it is assumed that all roughnesses are the same, $e_1 = e_2 = e_3 = 0.0012$ in, and that the elevation of the hydraulic grade line at node 1 is $H_1 = 200$ ft. For cases 10, 13, and 14 no solution is possible, i.e., an impossible problem has been specified. These three

Table 5.8
Combinations of Variables as Unknowns, With the Remaining Variables Specified.

Case	Unknown Variables					Specified Variables				
1	H_2	H_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	QJ_2	QJ_3
	196.55	196.43	1.257	0.057	0.943	10	6	10	1.2	1.0
2	QJ_2	H_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	H_2	QJ_3
	1.636	195.30	1.537	- 0.096	1.097	10	6	10	195	1.0
3	H_2	QJ_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	QJ_2	H_3
	195.91	1.315	1.380	0.180	1.135	10	6	10	1.2	195
4	H_3	QJ_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	QJ_2	H_2
	192.15	1.797	1.539	0.339	1.792	10	6	10	1.2	195
5	QJ_2	QJ_3	Q_1	Q_2	Q_3	D_1	D_2	D_3	H_2	H_3
	0.584	1.484	0.933	0.0491	1.135	10	6	10	198	195
6	QJ_2	QJ_3	D_1	Q_2	Q_3	Q_1	D_2	D_3	H_2	H_3
	0.651	1.135	0.855	0.349	1.135	1.0	6	10	198	195
7	QJ_2	QJ_3	Q_2	D_2	Q_3	D_1	Q_2	D_3	H_2	H_3
	0.433	1.635	0.933	6.863	1.135	10	0.5	10	198	195
8	QJ_2	QJ_3	Q_1	Q_2	D_3	D_1	D_2	Q_3	H_2	H_3
	1.349	0.584	0.933	0.349	0.795	10	6	1.0	198	195
9	QJ_2	QJ_3	D_1	D_2	Q_3	Q_1	Q_2	D_3	H_2	H_3
	1.635	0.5	10.26	6.86	1.135	1.0	0.5	10	198	195
10	QJ_2	QJ_3	D_1	Q_2	D_2	Q_1	Q_3	D_3	H_2	H_3
	Inconsistent (no solution)					1.0	1.0	10	198	195
11	QJ_2	QJ_3	D_1	Q_2	D_3	Q_1	Q_3	D_2	H_2	H_3
	0.651	1.349	10.26	0.349	9.54	1.0	1.0	6	198	195
12	QJ_2	QJ_3	D_1	D_2	D_3	Q_1	Q_2	Q_3	H_2	H_3
	1.5	0.5	10.26	6.86	9.54	1.0	0.5	1.0	198	195
13	QJ_2	QJ_3	D_1	D_3	Q_3	Q_1	Q_2	D_2	H_2	H_3
	Inconsistent (no solution)					1.0	0.5	10	198	195
14	QJ_2	QJ_3	D_2	D_3	Q_3	Q_1	Q_2	D_1	H_2	H_3
	Inconsistent (no solution)					1.0	0.5	10	198	195
15	H_2	H_3	D_1	Q_2	Q_3	Q_1	D_2	D_3	QJ_2	QJ_3
	198.45	197.35	12.28	0.2	0.8	1.4	6	10	1.2	1.0
16	H_2	H_3	Q_1	D_2	Q_3	D_1	Q_2	D_3	QJ_2	QJ_3
	198.27	197.35	1.4	6.23	0.8	12	0.2	10	1.2	1.0
17	H_2	H_3	Q_1	Q_2	D_3	D_1	D_2	Q_3	QJ_2	QJ_3
	198.27	197.17	1.4	0.2	9.87	12	6	0.8	1.2	1.0
18	H_2	H_3	D_1	D_2	Q_3	Q_1	Q_2	D_3	QJ_2	QJ_3
	Inconsistent (no solution)					1.2	0.2	10	1.2	1.0
19	H_2	H_3	Q_1	D_2	D_3	Q_2	Q_3	D_1	QJ_2	QJ_3
	Inconsistent (no solution)					0.2	1.0	12	1.2	1.0
20	H_2	H_3	D_1	Q_2	D_3	Q_1	Q_3	D_2	QJ_2	QJ_3
	Inconsistent (no solution)					1.2	1.0	10	1.2	1.0

A note on units: All heads are in ft; all discharges are in ft^3/s; all diameters are in in.

inconsistent combinations are all created by specifying the head at both ends of one of the pipes while trying also to specify its diameter and discharge. In case 13 this over-specification is obvious. For pipe 2 Q_2, D_2 and its head loss $h_{f2} = H_2 - H_3$ are all known. In case 10 this overspecification is not so obvious until it is realized that, since $H_1 = 200$ ft. is given, then Q_3, D_3 and $h_{f3} = H_1 - H_3$ are in effect given. And for case 14 the same type of overspecification for pipe 1 occurs; the discharge Q_1, diameter D_1, and head loss $h_{f1} = H_1 - H_2$ are all specified. NETWEQST detects such inconsistencies by finding that the Jacobian matrix in the Newton iteration is singular; it reports this and then stops.

Let us examine cases 15, 16, and 17. For this particular network the specification of any one discharge and the demands QJ_2 and QJ_3 is equivalent to a specification of the other two discharges. This situation occurs because the continuity equation at node 2 requires, if Q_1 is given, that $Q_2 = Q_1 - QJ_2$ (or if Q_2 is given, then $Q_1 = Q_2 + QJ_2$). At node 2 $Q_2 + Q_3 = QJ_3$, so with Q_2 found from the continuity equation at node 2, we find that the continuity equation at node 3 then requires $Q_3 = Q_2 + QJ_3$. However, the fact that the specification of any one discharge also fixes the other two discharges (with QJ_2 and QJ_3 known) does not in itself result in an inconsistent problem because the junction continuity equations are part of the system of equations. It does mean that we are making the problem more computationally intensive than is necessary. In case 15 a mental computation with the continuity equations would give $Q_2 = 0.2$ ft^3/s and $Q_3 =$ 0.8 ft^3/s. Next the head losses in pipes 2 and 3 could be computed, from these H_2 and H_3 could be determined, and finally D_1 could be found. Similar steps requiring the solution of only one equation at a time (and the Colebrook-White equation) can be used for cases 16 and 17. For all other cases in the table two simplifications in the solution process also exist. Treating each consistent case as a system of NP + NJ simultaneous equations will always be successful, even if it leads to more arithmetic than is necessary.

Another selection of known variables that produces inconsistent equations is the specification of all of the pipe discharges that join at a node while simultaneously giving the demand at this node. By doing so, that junction continuity equation no longer defines a relation between the pipe discharges and the demand there. The inconsistency will occur whether or not the junction continuity equation is satisfied by the given discharges. Cases 18, 19, and 20 are problems for which no solutions exist because a junction continuity equation can not be used. In case 18 the overspecification is obvious because Q_1, Q_2, and QJ_2 are all knowns, and yet these three variables are the only variables in the continuity equation at node 2: $Q_1 - Q_2 = QJ_2$. (We have actually reduced the network to a one-pipe network.) That case 19 gives all variables in the junction continuity equation at node 3 is now obvious. However, case 20 is not quite so obvious. Since the inflow at node 1 (the magnitude of the negative demand there) must equal $QJ_2 + QJ_3$, we note that giving Q_1 and Q_3 along with QJ_2 and QJ_3 results in a specification of all variables in the junction continuity equation at node 1.

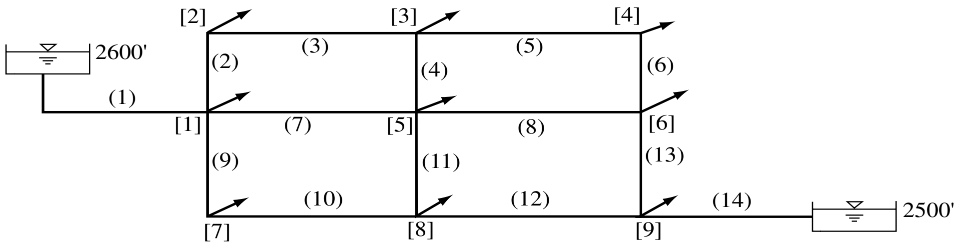
The foregoing situations will always result in a failure to find a solution. There are other specifications that will cause NETWEQS1 (or NETWEQST) to seek a solution but fail. Here are two more examples: (a) a situation requires a reservoir to supply a flow to the network, but the water surface elevation of the reservoir is given a value that is lower than the head at the other end of the connecting pipe; (b) consider a junction where two pipes join to meet a positive demand, but at the same time specify the heads at the opposite ends of the pipes so the flow must leave the junction. So we see that an unthinking specification of known values can, and often will, create a problem for which there is no solution, and the likelihood of this occurring increases with the size of the network because it then becomes increasingly difficult to identify situations for which there is no solution. An inconsistent problem will often become evident with NETWEQS1 when a message from the linear algebra solver SOLVEQ says that the Jacobian matrix is singular, which usually means that the linear system of equations, consisting of the Jacobian matrix and the equation vector as the known vector, is not an independent system of equations.

Example Problem 5.5

The pipe lengths and other data for a 14-pipe network supplied by two reservoirs are given in the file EXP5_5.IN on the CD. Obtain a solution from NETWK using this input, and then prepare input data for NETWEQS1 and obtain solutions therefrom for the following cases:

1. The heads at all nine nodes, as well as the discharges in all 14 pipes, will be regarded as unknown. (This is the problem solved by NETWK.)
2. At node 5 the demand QJ_5 will be considered unknown, and the head H_5 will be specified as $H_5 = 2504.3$ ft.
3. All of the demands are considered unknown, and the heads are as given in the input data to NETWEQS1.
4. The heads at all nine nodes and the discharges in all 14 pipes are considered unknown.
5. The heads at all nine nodes are unknown, and the diameters of pipes 2, 7, and 10 are unknown with the discharges in these three pipes specified.

Repeat the five cases with the reservoir attached to pipe 14 having a water surface elevation of 2450 ft. and with a pump in this pipe that has the following operating characteristic data pairs: $Q_1 = 1$ ft^3/s, $H_1 = 55$ ft.; $Q_2 = 2$ ft^3/s, $H_2 = 50$ ft.; $Q_3 = 3$ ft^3/s, $H_3 = 43$ ft.



The input data, to be read on logical unit IN5, are shown below for the five cases. For case 4 NETWEQS1 reports a "singular matrix," which indicates that at least one redundant equation was included in the system of equations. It should be clear that a solution to case 4 was not possible because we can not specify the heads at both ends of all pipes while simultaneously specifying all of the demands. These same cases are solved by using NETWEQS1 with a pump in line 14 and the water surface of this reservoir lowered to 2450 ft. These solutions follow those from NETWEQS.

<u>Data file for NETWEQS1 using input IN2 ≠ 5</u>

```
14 9 2 0
1 2600 14 2500
0 1 1500 1. .000166667 9.7
1 2 1000 .66666667 .000166667 2.9
2 3 2000 .66666667 .000166667 1.7
5 3 1000 .66666667 .000166667 0.2
3 4 2000 .5 .000166667 0.9
6 4 1000 .5 .000166667 0.5
1 5 2000 .66666667 .000166667 2.7
5 6 2000 .5 .000166667 0.8
1 7 1200 .66666667 .000166667 2.9
7 8 2000 .66666667 .000166667 1.7
5 8 2000 .66666667 .000166667 0.8
8 9 1200 .66666667 .000166667 1.4
9 6 1200 .66666667 .000166667 1.2
0 9 1500 .66666667 .000166667 1.3
1.3 2548.6 2410
1.2 2522.8 2405
1.0 2504.1 2400
1.4 2481.1 2340
0.9 2504.3 2405
1.5 2485.4 2350
1.2 2518.1 2405
1.0 2491.6 2400
1.5 2491.6 2370
```

Case 1 input, IN3 ≠ 5

```
9
0
14 14
0
0
0
1-9
1-14
```

Case 2 input, IN3 ≠ 5

```
8
1
14
0
0
0
1 2 3 4 6 7 8 9
5
1-14
```

Case 3 input, IN3 ≠ 5

```
0
9

0
0
0
1-9
1-14
```

Case 4 input, IN3 ≠ 5

```
9
0
0
14 3
0
0
1-9
1-14
```

Case 5 input, IN3 ≠ 5

```
9
0
11

0
0
1-9
1 3 4 5 6 8 9 11 12 13 14
2 7 10
```

To repeat the five cases with a pump inserted in line 14 and a specified reservoir water surface elevation, the input data file for NETWEQS1 above is modified by replacing the first two input lines with the following three input lines:

```
14 9 2 1
1 2600 14 2450
14 1 55 2 50 3 43
```

The remainder of the input file is unchanged.

The solutions in file EXP5_5.OUT on the CD were obtained for the 14-pipe network by using NETWEQS1 for the five cases. The reader should be able to obtain identical solutions. The solutions to the five cases which include a pump are in file EXP5_5.OU1 on the CD, which can be used to verify that your solutions are correct. In these solutions the pump is called device 1 when it is in pipe 14. The program output lists the change in head caused by each such device. In case 1 we find the following message:

```
Devices caused the following changes in heads:
Device 1 in pipe 14     Change in head = 53.45 ft.
```

* * *

Example Problem 5.6

For the small network below do the following:

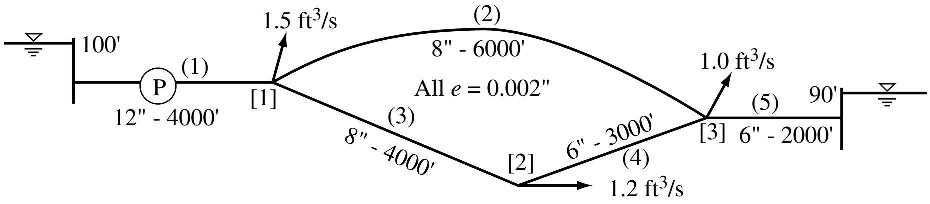
(a) Write the equations that describe the system.

(b) For the specified physical system, find the discharge in each pipe and the head at all nodes (duplicate this solution by preparing input data for NETWK).

(c) Determine the diameter of pipe 1 so the discharge through pipe 5 into the reservoir is $Q_5 = 0.5$ ft^3/s.

(d) Find the head that the pump must produce so that the discharge through pipe 5 into the downstream reservoir is $Q_5 = 1.0$ ft^3/s.

Q, ft^3/s	4.5	4.0	3.5
$\mathbf{h_p}$, ft.	54	50	44



(a) The equations are the following:

$$F_1 = 100 + h_p - H_1 - f_1(L_1/D_1)(Q_1/A_1)^2/(2g) = 0$$

$$F_2 = H_1 - H_3 - f_2(L_2/D_2)(Q_2/A_2)^2/(2g) = 0$$

$$F_3 = H_1 - H_2 - f_3(L_3/D_3)(Q_3/A_3)^2/(2g) = 0$$

$$F_4 = H_2 - H_3 - f_4(L_4/D_4)(Q_4/A_4)^2/(2g) = 0$$

$$F_5 = H_3 - 90 - f_5(L_5/D_5)(Q_5/A_5)^2/(2g) = 0$$

$$F_6 = Q_1 - Q_2 - Q_3 - QJ_1 = 0$$

$$F_7 = Q_2 + Q_4 - Q_5 - QJ_3 = 0$$

$$F_8 = Q_3 - Q_4 - QJ_2 = 0$$

with $$h_p = -4Q_1^2 + 42Q_1 - 54$$

(b) The 8 unknowns are Q_1, Q_2, Q_3, Q_4, Q_5, H_1, H_2, and H_3. Using NETWEQS1 to solve this problem would require the following input data:

From keyboard:

```
2 5 3/
```

In a file from logical unit 2:

```
5 3 2 1
1 100
5 90
1 4.5 54 4 50 3.5 44
0 1 4000 1.0 .000167 4.2
1 3 6000 .667 .000167 1.3
1 2 4000 .667 .000167 1.5
2 3 3000 .500 .000167 0.3
0 3 2000 .500 .000167 0.5
1.5 126 0.
1.2 98 0
1.0 95 0
```

Since the second logical unit was given as 5, the keyboard data for the unknowns is

```
3
0
5
0
0
```

and then

```
1-3
1-5
```

The solution from NETWEQS1 follows.

PIPE DATA

PIPE NO.	NODES FROM	NODES TO	L ft.	DIA. ft.	e x10^4 ft.	Q ft^3/s	HEAD LOSS ft.
1	- 1	1	4000	1.000	1.67	4.102	26.44
2	1	3	6000	0.667	1.67	1.191	29.17
3	1	2	4000	0.667	1.67	1.411	26.67
4	2	3	3000	0.500	1.67	0.211	2.49
5	- 2	3	2000	0.500	1.67	- 0.400	- 5.37

Devices caused the following changes in heads:
Device 1 in pipe 1 Change in head = 50.98 ft.

NODE DATA

NODE	DEMAND ft^3/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in^2	HGL ELEV. ft.
1	1.500	0	124.5	54.0	124.5
2	1.200	0	97.9	42.4	97.9
3	1.000	0	95.4	41.3	95.4

This can be regarded as the solution to an analysis problem since all of the physical features of the network are known, and the solution describes the performance of this existing network in response to the specified demands. We could verify that this solution is the solution from NETWK by supplying this input file to NETWK:

```
Example Problem
/*
 $SPECIF OUTPU1=2 $END
PIPES
1 0 1 4000 12 .002
2 1 3 6000 8
3 1 2 4000 8
4 2 3 3000 6
5 3 0 2000
NODES
1 1.5 0
2 1.2
3 1
RESER
5 90
PUMPS
1 4.5 54 4 50 35 44 100
RUN
```

(c) The equation set is unchanged from part (a). However, here the unknowns are different. The input data file in part (b) is again supplied to NETWEQS1. When we are asked to identify the unknowns, the following keyboard input will be supplied:

```
3
0
4
1
0
0
```

followed by:

```
1-3
2-5
1
```

(In parts (b) and (c) we supply a discharge of 0.5 ft^3/s for pipe 5 in the input data file. This was merely an estimate in part (b), but now this value is the specified discharge.)

The solution from NETWEQS1 is the following:

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. ft.	e x10⁴ ft.	Q ft³/s	HEAD LOSS ft.
1	- 1	1	4000	1.041	1.67	4.200	22.09
2	1	3	6000	0.667	1.67	1.246	31.72
3	1	2	4000	0.667	1.67	1.454	28.23
4	2	3	3000	0.500	1.67	0.254	3.50
5	- 2	3	2000	0.500	1.67	- 0.500	- 8.03

Devices caused the following changes in heads:
Device 1 in pipe 1 Change in head = 51.84 ft.

NODE DATA

NODE	DEMAND ft³/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in²	HGL ELEV. ft.
1	1.500	0	129.8	56.2	129.8
2	1.200	0	101.5	44.0	101.5
3	1.000	0	98.0	42.5	98.0

(d) One way to determine the required pump head so that $Q_5 = 1.0$ ft³/s is to replace the pump and its upstream reservoir with a node having a demand of - 4.7 ft³/s; this change will force the flow into the downstream reservoir to be 1.0 ft³/s. The input to NETWEQS1 is as follows:

```
5 4 1 0
5 90
4 1 4000 1 .000167 4.7
1 3 6000 .667 .000167 1.3
1 2 4000 .667 .000167 1.5
2 3 3000 .5 .000167 0.3
0 3 2000 .5 .000167 1.0
1.5 126 0.
1.2 98 0
1 95 0
- 4.7 130 0
```

and the solution from NETWEQS1 will show that the pump must supply a head of 200.3 - 100 = 100.3 ft.

PIPE DATA

PIPE NO.	NODES FROM	TO	L ft.	DIA. ft.	e x10⁴ ft.	Q ft³/s	HEAD LOSS ft.
1	4	1	4000	1.000	1.67	4.700	34.21
2	1	3	6000	0.667	1.67	1.536	46.90
3	1	2	4000	0.667	1.67	1.664	36.38
4	2	3	3000	0.500	1.67	0.464	10.52
5	- 1	3	2000	0.500	1.67	- 0.995	- 29.22

NODE DATA

NODE	DEMAND ft³/s	ELEV. ft.	HEAD ft.	PRESSURE lb/in²	HGL ELEV. ft.
1	1.500	0	166.1	72.0	166.1
2	1.200	0	129.7	56.2	129.7
3	1.000	0	119.2	51.7	119.2
4	- 4.700	0	200.3	86.8	200.3

* * *

Example Problem 5.7

Obtain solutions to Example Problems 5.5 and 5.6 using program NETWEQST. This program is one of the auxiliary programs in the NETWK package. It was developed to allow the user to specify the variables which are unknown, and it accepts the same input data as NETWK to define the physical features of the network. In other words it will solve the same problems as NETWEQS1 does, but it uses the same input files as NETWK. However, not all commands and options in NETWK are acceptable to NETWEQST. On the other hand it has additional options that provide the user some freedom in the way information is provided about the unknowns. If NETWEQST is to be used, then the following input files could be used, for example, to solve case 2 in Example Problem 5.5 and Example Problem 5.6, part (c), taking the default of being prompted for information that defines the unknowns. The prompts from NETWEQST are in bold type, and the responses are not.

For Example Problem 5.5, case 2, the portion of the input which defines the network is identical to that for NETWK in Example Problem 5.5 itself. Here the bold prompts from NETWKST are followed by the responses that define case 2:

Pipes = 14, Nodes = 9, Sources = 2
23 unknowns must be given. Give no. of each:
1. HGLs at nodes 8
2. Nodal demands 1
3. Pipe discharges 14
Give 8 HGLs at node numbers 1-4 6-9
Give 1 nodal demand numbers 5
Give number of nodal HGL-elevations provided 1
As pairs give 1 node number and the HGL
5 2504.3
Give number of pipe discharges provided 0

The data for Example Problem 5.6(c) is similar:

```
Example 5.6(c) using NETWEQST
/*
$SPECIF $END
PIPES
1 0 1 4000 12 .002
2 1 3 6000 8
3 1 2 4000
4 2 3 3000 6
5 3 0 2000
NODES
1 1.5 0
2 1.3
3 1
RESER
5 90
PUMPS
1 4.5 54 4 50 35 44 100
RUN
```

Pipes = 5, Nodes = 3, Sources = 2
8 unknowns must be given. Give no. of each:
1. HGLs at nodes 3
2. Nodal demands 0
3. Pipe discharges 4
4. Pipe diameters 1
Give 4 pipe discharge numbers 1-4
Give 1 pipe diameter numbers 1

Give number of nodal HGL-elevations provided 0
Give number of pipe discharges provided 1
As pairs give 1 pipe number and the discharge therein
5 - 0.5

* * *

5.6 HIGHER ORDER REPRESENTATIONS OF PUMP CURVES

The head produced by a pump has heretofore been defined as a function of the discharge by fitting a single second-order polynomial through three pairs of points. If the pump operation occurs within a relatively narrow discharge range, and these are near the normal capacity of the pump, then such a simple representation is adequate. When this is not the case, then more advanced procedures are needed to define well the pump's operating characteristics. Various interpolation procedures can be used for the mathematical representation of a pump curve. This section discusses how pump curves can be duplicated mathematically when equations are needed to define their operating characteristics.

5.6.1. WITHIN RANGE POLYNOMIAL INTERPOLATION

Any number of values might be used to define a pump characteristic curve, and a polynomial of any order might be used to interpolate the head corresponding to any given discharge if the range of the discharge values brackets the given discharge. A first-order polynomial is simply a straight line. To represent the pump head well with a first-order polynomial interpolation, we should first ensure that the smaller discharge Q_i is less than or equal to the given discharge Q, and that the larger discharge Q_{i+1} is greater than Q. The interpolating function for a first-order polynomial is

$$h_p = h_{pi} + \left(h_{pi+1} - h_{pi}\right)\left(Q - Q_i\right) / \left(Q_{i+1} - Q_i\right) \tag{5.32}$$

in which the quantities with subscripts i and $i+1$ are known, h_p is the interpolated head of the pump and $Q_i \le Q \le Q_{i+1}$. When Q becomes larger than Q_{i+1}, then the first point is dropped and the next point is added. The use of a higher-order polynomial requires more data. An nth-order polynomial requires at least $n+1$ pairs of data points since an nth-order polynomial passes through $n+1$ points, e.g., a second-order polynomial passes through three points, a third-order polynomial through four points etc. The Lagrange formula is a convenient interpolation formula to use for this purpose because the increment between consecutive values of the independent variable, the discharge Q in this case, need not be constant. Other formulas do require a constant increment of the independent variable. The Lagrange interpolation formula is

$$h_p = \sum_{i=1}^{n} F_i H_i \tag{5.33}$$

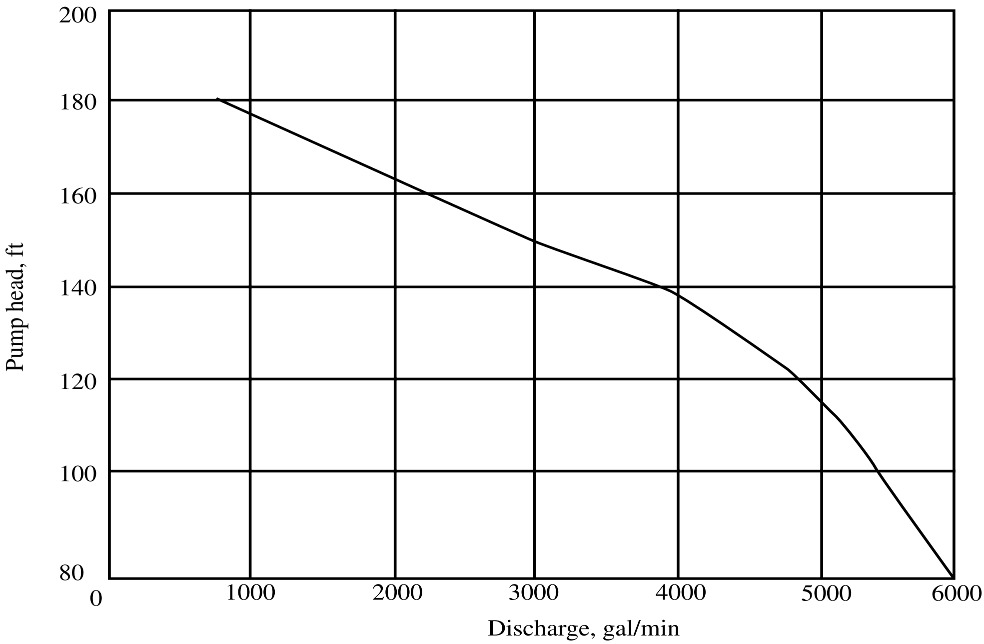
in which each H_i is the pump head at point i, and each F_i is the quotient of two products:

$$F_i = \prod_{\substack{j=1 \\ j \ne i}}^{n} \left(Q - Q_j\right) \Bigg/ \prod_{\substack{j=1 \\ j \ne i}}^{n} \left(Q_i - Q_j\right) \tag{5.34}$$

in which the two products Π include n - 1 terms, with the term $j = i$ omitted. To implement the Lagrange interpolation successfully in a computer program, two requirements must be met: (1) the discharge for which the head is wanted must lie within the range of the discharge data points (otherwise the process is extrapolation), and (2) Eqs. 5.33 and 5.34 must be properly written. The program LAGRANGE on the CD is designed to read n pairs of points for a pump curve and then provide the pump head for any specified discharge. The program can also be converted into a function subprogram which will pass (h_p, Q) pairs to the function from the main program, and an argument will specify the Q for which the head is to be determined.

Example Problem 5.8

A pump curve is shown below. Enter 10 pairs of points from this curve into a file, and then use Lagrange's formula with a third-order polynomial interpolation to obtain values of the pump head corresponding to specified discharges, i.e., find h_p for discharges of 850 gal/min, 5800 gal/min, 4200 gal/min, etc.



We start the solution by selecting 10 discharge values along the abscissa and reading the corresponding values of pump head to obtain the following:

Q gal/min	800	1600	2400	3200	4000	4500	4800	5200	5600	6000
$\mathbf{h_p}$ ft.	181.5	170.0	160.0	148.5	138.6	128.0	120.4	109.0	95.0	80.0

These data pairs now must be entered into a file that can be read by program LAGRANGE. The input from the keyboard will be 10 2 3, followed by the filename. Then provide the discharges 850, 5800, 4200, etc. in response to the prompt **Give discharge (minus to terminate)**. The heads returned by the program are the following: $Q = 850$ gave $h_p = 181.51$, $Q = 5800$ gave $h_p = 87.53$, $Q = 4200$ gave $h_p = 135.16$.

* * *

5.6.2. SPLINE FUNCTION INTERPOLATION

One disadvantage of using Lagrange interpolation is seen when the interpolation interval shifts to continue to bracket the discharge; then the first derivative, which is needed in the Newton method, is not continuous. An alternative is to use spline interpolation. An essential difference between spline and piecewise polynomial interpolation is that, although a given spline function interpolates only between two consecutive points, both the spline function and one or more of its derivatives are continuous across these points. We will only discuss cubic splines here, since they require roughly the same computational effort as quadratic splines and have both continuous second and first derivatives across the data points.

Cubic splines develop a third-order polynomial between each pair of consecutive points as the interpolating function, or

$$y^{(i)} = a_i x^3 + b_i x^2 + c_i x + d_i \tag{5.35}$$

in which superscript i refers to the segment of the curve before point i, the dependent variable y plays the role of the pump head h_p, and x replaces Q. (For notational simplicity let H represent h_p in the remainder of this section.) For example, if we use four (H_j, Q_j) pairs, there will be three interpolating equations of the form of Eq. 5.35. In this case the total number of unknown (a, b, c, d) coefficients is $4(n - 1)$, or for our example $4 \times 3 = 12$ unknowns. Thus $4(n - 1)$ equations are needed. By substituting the known (H, Q) pairs at points j and $j + 1$ at the ends of segment i, we obtain $2(n - 1)$ of these equations. Another $(n - 2)$ equations are developed by equating the first derivatives of the two interpolating equations that apply at each data point, and an additional $(n - 2)$ equations result from equating second derivatives at these same points. The last two required equations come from boundary or end conditions. There are two commonly used kinds of boundary conditions. One sets the second derivatives at the beginning and/or end of the global interval to zero; that is, $(d^2y/dx^2)_1 = H_1'' = 0$ and/or $(d^2y/dx^2)_n = H_n'' = 0$. These are called natural cubic splines. The other sets y_1' and/or y_n' to values calculated by assigning values to the first derivatives.

In detail, the equations for the 4-point example are the following:

$$H_1 = a_1 Q_1^3 + b_1 Q_1^2 + c_1 Q_1 + d_1 \tag{5.36}$$

$$H_2 = a_1 Q_2^3 + b_1 Q_2^2 + c_1 Q_2 + d_1 \tag{5.37}$$

$$H_2 = a_2 Q_2^3 + b_2 Q_2^2 + c_2 Q_2 + d_2 \tag{5.38}$$

$$H_3 = a_2 Q_3^3 + b_2 Q_3^2 + c_2 Q_3 + d_2 \tag{5.39}$$

$$H_3 = a_3 Q_3^3 + b_3 Q_3^2 + c_3 Q_3 + d_3 \tag{5.40}$$

$$H_4 = a_3 Q_4^3 + b_3 Q_4^2 + c_3 Q_4 + d_3 \tag{5.41}$$

$$\left(\frac{dy^{(1)}}{dx}\right)_2 = \left(\frac{dy^{(2)}}{dx}\right)_2 \quad \Rightarrow \quad 3a_1 Q_2^2 + 2b_1 Q_2 + c_1 = 3a_2 Q_2^2 + 2b_2 Q_2 + c_2 \tag{5.42}$$

$$\left(\frac{dy^{(2)}}{dx}\right)_2 = \left(\frac{dy^{(3)}}{dx}\right)_3 \quad \Rightarrow \quad 3a_2Q_3^2 + 2b_2Q_3 + c_2 = 3a_3Q_3^2 + 2b_3Q_3 + c_3 \tag{5.43}$$

$$\left(\frac{d^2y^{(1)}}{dx^2}\right)_2 = \left(\frac{d^2y^{(2)}}{dx^2}\right)_2 \quad \Rightarrow \quad 6a_1Q_2 + 2b_1 = 6a_2Q_2 + 2b_2 \tag{5.44}$$

$$\left(\frac{d^2y^{(2)}}{dx^2}\right)_3 = \left(\frac{d^2y^{(3)}}{dx^2}\right)_3 \quad \Rightarrow \quad 6a_2Q_3 + 2b_2 = 6a_3Q_3 + 2b_3 \tag{5.45}$$

The boundary conditions are either

$$\left(\frac{d^2y^{(1)}}{dx^2}\right)_1 = H_1'' = 0 \tag{5.46a}$$

$$\left(\frac{d^2y^{(3)}}{dx^2}\right)_4 = H_4'' = 0 \tag{5.46b}$$

or

$$\left(\frac{dy^{(1)}}{dx}\right)_1 = specified = 3a_1Q_1^2 + 2b_1Q_1 + c_1 \tag{5.47a}$$

$$\left(\frac{dy^{(3)}}{dx}\right)_4 = specified = 3a_3Q_4^2 + 2b_3Q_4 + c_3 \tag{5.47b}$$

In these equations y has been used as the continuous dependent variable. For use in interpolating a point from a pump curve, the dependent variable will be called the pump head H, and in subsequent equations it will be used in place of y.

One obvious continuation is simply to solve the above equations for a_i, b_i, c_i and d_i, $i = 1, 2, 3$ and then use the appropriate equation to compute H for a given Q. However, an alternative that requires less arithmetic is the following interpolation equation:

$$H^{(i)} = a_iH_j + b_iH_{j+1} + c_iH_j'' + d_iH_{j+1}'' \tag{5.48}$$

The coefficients $a, b, c,$ and d are now obviously different than before. The coefficients a and b are weighting functions that are applied to the dependent variable H at points j and $j + 1$, and c and d are weighting functions applied to the second derivatives at these same points. In the finite element method a and b are the shape, basis or interpolation functions that are associated with a linear one-dimensional element. It can easily be shown that $a_i = (Q_{j+1} - Q)/(Q_{j+1} - Q_j)$ and $b_i = (Q - Q_j)/(Q_{j+1} - Q_j)$ with $a_i + b_i = 1$. We see that a_i and b_i are linear functions of Q. For simplicity the subscripts and superscripts will be deleted in many of the following equations; just keep in mind that the interpolating function provides values of Q within the interval $[Q_j, Q_{j+1}]$. Since c and d are functions of a and b, the number of additional unknowns that are introduced with each new segment is two rather than four. Thus the total number of equations for n intervals will be $2(n - 1)$ rather than $4(n - 1)$. Since $b = 1 - a$, only one new unknown appears for each new data point, so the number of required equations is only n - 1. The relations between c and d and a and b are

$$c = (a^3 - a)(Q_{j+1} - Q_j)^2/6 \tag{5.49a}$$

and

$$d = (b^3 - b)(Q_{j+1} - Q_j)^2/6 \tag{5.49b}$$

Thus the dependence of the interpolating equation on Q is entirely through the linear Q-dependence of a and b. Since the derivatives are also weighted by c and d (depending on a and b), a cubic interpolating polynomial exists over the closed interval $[Q_j, \ Q_{j+1}]$. To verify these statements, we note first that c and d contain terms involving Q^3 and Q^2 since the definitions of c and d contain a^3 and b^3. Thus the interpolating equation is a third-order polynomial. And we see also that $da/dQ = -1/(Q_{j+1} - Q_j)$ and $db/dQ = 1/(Q_{j+1} - Q_j) = -da/dQ$. Now we compute the derivative of H itself to obtain

$$H' = \frac{dH}{dQ} = \frac{H_{j+1} - H_j}{Q_{j+1} - Q_j} - \frac{3a^2 - 1}{6}\left(Q_{j+1} - Q_j\right)H_j'' + \frac{3b^2 - 1}{6}\left(Q_{j+1} - Q_j\right)H_{j+1}'' \tag{5.50}$$

and the second derivative is

$$H'' = aH_j'' + bH_{j+1}'' \tag{5.51}$$

Since $a = 1$ at Q_j and $a = 0$ at Q_{j+1}, and also $b = 0$ at Q_j and $b = 1$ at Q_{j+1}, we have verified that the relations between c and a and between d and b are valid.

To apply Eq. 5.48 in practice, we must first determine numerical values for the second-derivative terms that appear in that equation. The required equations, ones that allow us to evaluate those terms, can be obtained by evaluating the first derivative at points 2, 3, ... , n - 2 and equating pairs from adjacent segments. We do not need the original equations or the equations that are obtained by equating second derivatives, since these are already satisfied by the interpolating polynomial. Equating first derivatives at the data points yields

$$\begin{aligned}\frac{H_j - H_{j-1}}{Q_j - Q_{j-1}} &+ \frac{Q_j - Q_{j-1}}{6}H_{j-1}'' + \frac{Q_j - Q_{j-1}}{3}H_j'' \\ &= \frac{H_{j+1} - H_j}{Q_{j+1} - Q_j} - \frac{Q_{j+1} - Q_j}{3}H_j'' - \frac{Q_{j+1} - Q_j}{6}H_{j+1}''\end{aligned} \tag{5.52}$$

This equation comes directly from Eq. 5.50 with $a = 0$, $b = 1$ at Q_j for the derivative on the left side, and $a = 1$, $b = 0$ at Q_j on the right side of point j. This equation (i.e., these equations, since j is incremented) can be rewritten to display better the linear relation between the second derivatives of H, with known values on the right side, as

$$\begin{aligned}\left(Q_j - Q_{j-1}\right)H_{j-1}'' &+ 2\left(Q_{j+1} - Q_{j-1}\right)H_j'' - \left(Q_{j+1} - Q_j\right)H_{j+1}'' \\ &= 6\left\{\frac{H_{j+1} - H_j}{Q_{j+1} - Q_j} - \frac{H_j - H_{j-1}}{Q_j - Q_{j-1}}\right\}\end{aligned} \tag{5.53}$$

Written in matrix notation, Eq. 5.53 consists of a coefficient matrix $[A]$ multiplied by the vector of unknown second derivatives $\{H''\}$, which equals the known vector $\{B\}$, or

$$[A]\{H''\} = \{B\} \qquad (5.54)$$

To make the system complete, boundary conditions must supply the first and last values. If the natural condition is used, then H_1'' and H_n'' are given zero values, which in effect starts the system at point 2 and ends the system at point $n - 1$. If first derivatives are specified, then these values provide the first and last equations in the system of equations. We note that only three consecutive values of the second derivatives are linked together in this system of equations, regardless of the choice of boundary conditions. This tridiagonal system of equations is very common, and it can be solved readily by decomposition or elimination methods. Since only one element exists in front of the diagonal, a single forward elimination pass through the rows of the matrix can convert the matrix into an upper triangular matrix with only two nonzero elements. Then a back substitution can obtain the solution for the second derivatives.

We have just seen that this alternative to the use of cubic spline interpolation requires first the solution of a tridiagonal equation system to determine numerical values for the second derivative of H at each of the points where (H_j, Q_j) pairs are given. By then applying the other interpolation relations, the head H can be found directly for any Q in the overall range of the interpolation.

The program SPLINECU implements this process. A listing of it can be obtained from the CD for study. This program is designed to read N pairs of values for (H_j, Q_j) and then determine H at M uniformly spaced values of Q, starting with Q_1 and ending with Q_n, instead of finding H for a specified Q. That is, it produces a full table of values for H. The third column in this table provides values of dH/dQ; when we compute elements of the Jacobian matrix in applying the Newton method to the solution of a network problem, this table of values is useful. The program could be modified to function in the same way as the Lagrange interpolation program, or to allow the user to provide a list of Q values for which heads are desired, and this list could be provided from a file or given individually from the keyboard. Or it could be converted into a function subprogram to supply the head for any specified discharge in solving a network problem involving a pump. Such a subprogram is on the CD under the name SPLINESU. Actually the program first reads Q and then H for each data pair, then forms and solves the tridiagonal equation system, and finally develops the new table with M entries.

Example Problem 5.9

Use program SPLINECU to obtain values of the pump head H and the derivative dH/dQ with an increment ΔQ = 100 gal/min. between 800 gal/min. and 6000 gal/min. As input data use the 10 points that are listed in the output table of Example Problem 5.8.

Solution: The following values should first be entered from the keyboard: 2 3 10 53 0. The first three and last three lines of output should be the following:

800.0	181.50	- 0.01508
900.0	179.99	- 0.01505
1000.0	178.49	- 0.01495
5800.0	87.53	- 0.03755
5900.0	83.77	- 0.03766
6000.0	80.00	- 0.03770

* * *

Example Problem 5.10

A pump having the characteristic curve of head vs. discharge given in Example Problem 5.9 is operated over six hours, as described by the following (assumed smooth) data:

Time hr.	0	1	1.5	2.0	2.8	3.8	4.9	5.2	6.0
Q gal/min	800	2500	3500	4700	5500	4900	3500	2900	2500

The pump efficiencies corresponding to the discharges in Example Problem 5.9 are

Q gal/min	800	1600	2400	3200	4000	4500	4800	5200	5600	6000
e	0.40	0.50	0.65	0.75	0.825	0.847	0.845	0.81	0.79	0.74
H ft.	181.5	170.0	160.0	148.5	138.6	128.0	120.4	109.0	95.0	80.0

Use cubic splines to define from the data pairs the relations that are needed, and determine the energy used by the pump during the six-hour period.

There are three relations that must be established by spline functions:

1. the discharge Q as a function of time t,
2. the pump head H as a function of the discharge Q, and
3. the efficiency e as a function of the discharge Q.

When these relationships have been determined, the amount of energy that is consumed can be found by numerically integrating the equation

$$Energy = \gamma \int (QH/e)dt$$

To complete this solution, it is convenient to convert the program SPLINECU into a subroutine to find the second derivatives for these relations; then a numerical integration subroutine will be used to obtain the energy. A program ELECECG to accomplish these tasks can be listed from the CD for further study as the rest of this example is read. It calls on SIMPR to complete the integration after the newly created subroutine SPLINESU has been called three times to determine the second derivatives. The arguments of SPLINESU are as follows: N = the number of data pairs, X = an array of N values for the independent variable, Y = an array of N values for the dependent variable, D2Y = an array of second derivatives returned by SPLINESU, D = a work array having N values, ITY = 0 for natural boundary conditions or ITY = 1 for prescribed first derivatives at the ends of the domain.

The program has three parts: (1) the main program that calls SPLINESU three times to obtain three sets of second derivatives and then calls the numerical integration routine SIMPR; (2) a block data subprogram to enter the data pairs rather than reading them from a file; and (3) the function subprogram EQUAT that defines the equation to be integrated. The second derivatives and sets of three data pairs are passed to EQUAT by means of the block common statements. EQUAT contains the logic that will determine, from the time, which two instants in time are to be used so that cubic spline interpolations can provide the values of the discharge as QQ, pump head as HH and efficiency as EE, and then the argument of the numerical integration *QH/e* is returned as EQUAT. The main program supplies the constant for γ and converts gal/min. to ft^3/s and energy in ft-lb to energy in kilowatt-hours.

The answer is Energy = $1.915\text{x}10^9$ ft-lb or 2,600,000 kWh.

* * *

5.7 SENSITIVITY ANALYSES

We now turn to a third major type of network design. So far we have explored two design categories: the first sought to determine the size of as many pipes as possible (NJ of them since the equations would permit no more), and the second sought to determine the size of individually chosen components by considering each of them as a device that created a differential head at its location in the network. The first design category is encountered when a new network is being designed. The second type is more relevant to an existing system, for example, one in which we must determine the capacity and head of a pump to achieve a desired pressure at some point in response to some specified demands. The third design category seeks to identify the components of the network to upgrade, improve, or replace in order to increase the level of network performance most efficiently. The actual determination of unit sizes might be accomplished later, according to procedures used in the second type of design. In a sense this section describes methods that can be used to decide which system elements are most important to the improvement of system performance. For example, as a city's water use increases, the pressures may become too low during peak demand periods. Which of several pumps should be replaced by a larger one? An excellent quantitative means for making such a decision is to perform an appropriate sensitivity analysis and replace the pump with the largest pressure sensitivity. This section describes the determination of the magnitude of the sensitivity of one variable with respect to another variable in the network.

The quantification of sensitivity, which is how much one variable changes in response to a change in another variable or several variables, provides the designer a deeper understanding of network performance. Here we usually apply sensitivity analysis to identify the best component to change or replace to overcome a deficiency in the present performance of a network. A natural question is how these deficiencies can "best" or "most economically" be remedied. The answer may require a change in one or several pipe diameters, an increase in the head produced by existing pumps, an increase in the elevation of storage tanks (reservoirs), or the addition of pumps or pressure-reducing valves, etc. Normally there are a host of possible ways to correct inadequate performance. Some possibilities will be discussed in this section, but these should be regarded only as examples to stimulate thinking about alternatives. The sensitivity of one variable to another variable can be expressed by the partial derivative of the first variable with respect to the second variable. The variable(s) whose sensitivity is sought is (are) the dependent variable(s), and the variable that is the candidate for change to improve the network performance is the independent variable. There are usually several independent variables which are candidates for change. There may also be more than one dependent variable, but often one variable will be selected.

Generally it is not possible to define algebraically the partial derivative of any particular dependent variable with respect to another chosen independent variable when dealing with piping systems (there are exceptions), but these derivatives can be defined approximately by numerical methods. The mathematical definition of a partial derivative is

$$\frac{\partial f}{\partial x} = \underset{h \to 0}{limit} \frac{f(x+h, y, z) - f(x, y, z)}{h} \tag{5.55}$$

or, in more practical terms with $h = \Delta x$, $\partial f / \partial x \approx \Delta f / \Delta x$ when the other variables are unchanging. Thus, as x is changed by some small amount Δx, the corresponding change in the dependent variable (equation, system, or process) Δf is determined, and this latter difference, when it is divided by the change in the independent variable, produces an approximation of the derivative. Under conditions near those for which f is evaluated (assuming all other parameters remain constant), the sensitivity of the dependent variable is quantified as this derivative.

As this derivative becomes larger, the dependent variable f is more strongly affected by a change in the independent variable x, or the more sensitive f is to x. A negative derivative indicates that one variable decreases as the other increases. In a pipeline system there are many derivatives, or sensitivities, that can be determined and whose magnitudes provide useful information about the most effective way to change system performance. A few of many examples follow: (1) Low pressure can be corrected best by enlarging the pipe diameter that creates the largest $\partial p/\partial D$; (2) Low pressure can be corrected best by increasing the head of the pump with the largest $\partial p/\partial h_p$; (3) Too small a flow into a storage tank can be best corrected by increasing the power at the pumping station with the largest $\partial Q_{res}/\partial P$; (4) A fire demand at a node can best be augmented by the pump that has the largest $\partial QJ/\partial Q_p$; (5) Too large a pressure can best be reduced by a PRV in the pipe whose downstream head H produces the largest negative magnitude of $\partial p/\partial H$; etc.

The magnitudes of these sensitivities are generally not constant but change with problem specifications, such a peaking factors, and the largest may come from a different component (independent variable) as the total demand or demand pattern changes, or conditions under which the network is to perform change. The selection of sensitivities to compute will depend on the particular focus of each network performance study. Often several different sensitivities will provide nearly the same information.

Consider now the network shown in Fig. 5.28 to become acquainted with some of the possibilities. All of the water that is consumed in a typical daily operation must come from the two pumping stations. The tank (reservoir) at the end of pipe 11 is large and should receive water during periods of low demand so it can supply water when the demands are larger. To simplify the analyses assume the water level in the storage tank is constant at 200 m. The demands in the diagram are those that typically occur during the high de-mand period of a day. These demands are larger than those which existed when the system was designed, and now the tank does not fill sufficiently during low demand periods; the power to one of the pump stations must be increased. Which station should be upgraded (power input increased)?

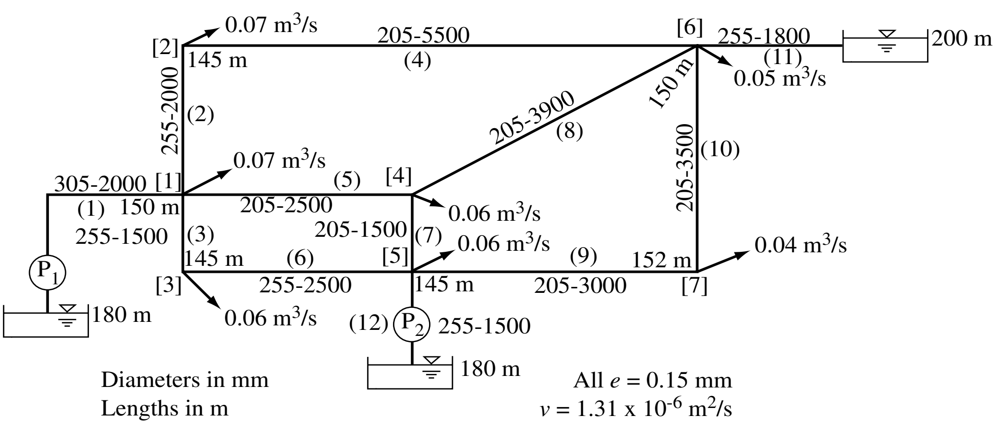


Figure 5.28 A network for sensitivity analysis.

To obtain more information on network performance, a series of solutions was obtained for peaking factors from 0.5 to 1.2. If NETWK is used to obtain these solutions, a convenient way to do this is to use the CHANGE command. The input data file to obtain such a series of solutions is presented in Fig. 5.29. (The option NETPLT = 3 in the $SPECIF list tells NETWK to write a file that can be used by program SENSITV.) The discharges in the two pipes from the pumping stations and the pipe that connects the storage tank to the network have been plotted as a function of the peaking factor and are

```
Illustration of sensitivities
/*
$SPECIF NFLOW=3,NPGPM=3,NUNIT=4,PEAKF=.5,
 NPRINT=-3,NETPLT=3 $END
PIPES
1 0 1 2000 305 .15
2 1 2 2000 255
3 1 3 1500
4 2 6 5500 205
5 1 4 2500
6 5 3 2500 255
7 5 4 1500 205
8 4 6 3900
9 5 7 3000
10 7 6 3500
11 0 6 1800 255
12 0 5 1500 255
NODES
1 .07 150
2 .07 145
3 .06 145
4 .06 140
5 .05 145
6 .05 150
7 .04 152
RESER
11 200
PUMPS
1 .15 50 .25 43 .35 35 180
12 .1 48 .15 43.25 .25 33.0 180
RUN
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
CHANGE
DFRAC
1.1
END
```

Figure 5.29 The input data file for the analysis of flow in the network in Fig. 5.28.

shown in Fig. 5.30. In this plot a negative flow in pipe 11 indicates a flow into the tank. This tank is seen to supply water whenever the peaking factor exceeds 0.58; the

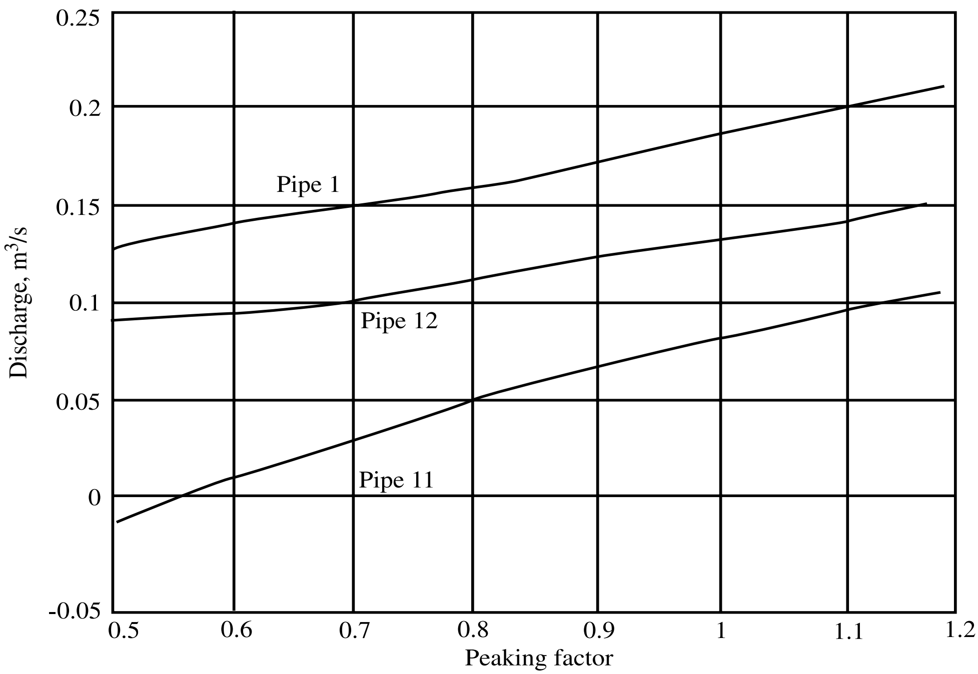


Figure 5.30 Discharge as a function of peaking factor for three pipes.

reason for the problem is clear. To choose which pumping station to upgrade, it would be useful to determine the sensitivity of the discharge from (into) the reservoir as a function of the power consumed at each of the two pumping stations, i.e., $\partial Q_{11}/\partial P_1$ and $\partial Q_{11}/\partial P_2$. (The cost associated with pumping is linearly related to the power consumption, so a properly chosen multiplier of each sensitivity will provide the increase in reservoir discharge per unit cost.) The pumping station with the larger sensitivity is the one to upgrade and is the lower cost alternative. Maximization of the sensitivity of discharge to power is the same as minimization of the cost of obtaining a desired discharge or volume of water relative to the cost of energy to pump this water.

The first two solutions from the input in Fig. 5.29 to NETWK (which can be obtained from NETWK, or another program in Chapter 4, or from NETWEQS1, to verify the values) provide the data in columns 2, 3, and 4 in Table 5.9. The first solution is for a peaking factor of 0.5 (since PEAKF = 0.5), and the second is for PF = 0.5(1.1) = 0.55 (since DFRAC under CHANGE is 1.1). Column 5 is the difference in discharge in pipe 11 (into the tank) for these two solutions. The difference in power $P = \gamma Q h_p$ from pump station 1 is given in column 6, and the difference from station 2 is in column 7. The sensitivities of the tank discharge to the power at the pumping stations are in columns 8 and 9. When the demands are 0.5 times those that are listed on the network diagram, it is best to augment the pumping at station 2 because $\partial Q_{11}/\partial P_2 = 0.0062$ is larger than $\partial Q_{11}/\partial P_1 = 0.0042$.

Table 5.9

PF (1)	Q_{11} m³/s (2)	Power₁ kW (3)	Power₂ kW (4)	ΔQ_{11} (5)	ΔP_1 (6)	ΔP_2 (7)	$\Delta Q_{11}/\Delta P_1$ (8)	$\Delta Q_{11}/\Delta P_2$ (9)
0.50	- 0.0154	64.74	42.35	0.0102	2.41	1.64	0.0042	0.0062
0.55	- 0.0053	66.65	43.99					

The same sensitivities were computed from the other paired consecutive solutions requested by the CHANGE command, with the results shown in Table 5.10. Over the entire range of peaking factors the sensitivity $\Delta Q_{11}/\Delta P_2 > \Delta Q_{11}/\Delta P_1$, and therefore the clear choice is to increase the input power to pump station 2.

The solutions that were used to obtain the sensitivities of the reservoir discharge Q_{11} to pump power did not directly require any of these variables to be changed from solution to solution. Instead the peaking factor was changed, which in turn caused these variables to change from solution to solution. An alternative was to obtain one series of solutions

Table 5.10

PF	0.50-0.55	0.550-0.605	0.605-0.666	0.666-0.732	0.732-0.805	0.805-0.886	0.886-0.974	0.974-1.072	1.072-1.179
$\Delta Q_{11}/\Delta P_1$ x10³	4.2	5.7	5.3	4.8	4.1	3.6	3.3	3.1	3.0
$\Delta Q_{11}/\Delta P_2$ x10³	6.2	8.6	7.9	6.9	5.8	5.3	4.9	4.5	4.4

in which P_1 was changed, and another in which P_2 was changed, but this would have required more effort. The fact that specifying a change in one variable (or parameter) causes changes in all other variables associated with network performance allows us to obtain many sensitivities from one series of solutions. The program SENSITV in the NETWK package is designed to allow the user to generate tables of sensitivities. Table 5.11 is the first portion of the output from SENSITV, in which the demand QJ_1 at node 1 (which is linearly related to the PF) was selected as the independent variable, and the discharge in pipe 11, or reservoir discharge Q_r, was selected as the dependent variable. In obtaining this table, the option to place the independent variables in the output table was selected. Table 5.13 then presents the final results from 10 solutions in a simpler format.

Table 5.11
Sensitivity of Discharge in Reservoir 1 to Changes in Demand at Node 1

Res.	Independent Variable at 1, Comparison between Solutions 2 and 1						
	QJ	QJ	Diff.	Q_r	Q_r	Diff.	Ratio
1	0.0350	0.0385	0.0035	- 0.0154	- 0.0053	0.0102	2.91
	Independent Variable at 1, Comparison between Solutions 3 and 2						
	QJ	QJ	Diff.	Q_r	Q_r	Diff.	Ratio
	0.0385	0.0424	0.0039	- 0.0053	0.0075	0.0128	3.32

Using SENSITV to obtain the sensitivities $\Delta Q_{11}/\Delta P_1$ and $\Delta Q_{11}/\Delta P_2$, we obtain the results that are listed in output Table 5.12. This time we chose to have only the ratios written to the output table. In this output from SENSITV the first independent variable is P_1, and the second independent variable is P_2.

Table 5.12

Sensitivity Comparison of Flow from Reservoir at Reservoir 1

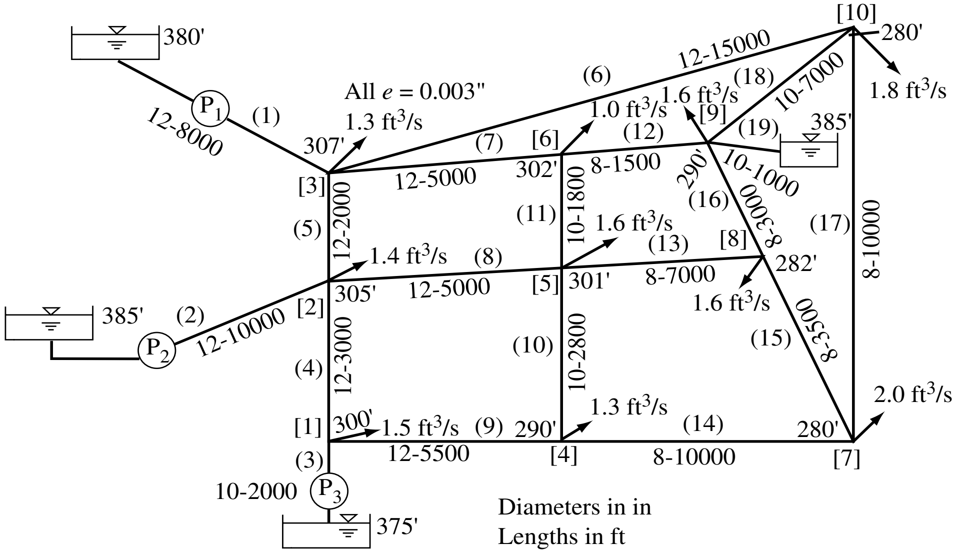
Solution: Reservoir	1-2 1	2-3 1	3-4 1	4- 5 1	5-6 1	6-7 1	7-8 1	8-9 1	9-10 1
1	0.004	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.003
2	0.006	0.009	0.008	0.007	0.006	0.005	0.005	0.005	0.004

Table 5.13

Sol.	**Indep. Variable** QJ_1	ΔQJ_1	**Dep. Variable** $Q_r = Q_{11}$	ΔQ_r	**Ratio** $\Delta Q_r/\Delta QJ_1$
1	0.03500		- 0.0154		
		0.00350		0.0101	2.89
2	0.03850		- 0.0053		
		0.00385		0.0128	3.32
3	0.04235		0.0075		
		0.00424		0.0135	3.18
4	0.04659		0.0210		
		0.00465		0.0139	2.99
5	0.05124		0.0349		
		0.00513		0.0139	2.71
6	0.05637		0.0488		
		0.00563		0.0141	2.50
7	0.06200		0.0629		
		0.00621		0.0144	2.32
8	0.06821		0.0773		
		0.00682		0.0145	2.13
9	0.07503		0.0918		
		0.00750		0.0150	2.00
10	0.08253		0.1068		

Example Problem 5.11

In the network shown below the pressures at some of the nodes near node 7 are less than desirable. Which of the three pumps should be enlarged? Use additional sensitivities to understand more completely the performance of this network.



Pump 1

Q_1, ft³/s	h_{p1}, ft
3.0	80
5.0	75
8.0	65

Pump 2

Q_2, ft³/s	h_{p2}, ft
3.0	80
5.0	75
8.0	65

Pump 3

Q_3, ft³/s	h_{p3}, ft
3.0	83
5.0	78
8.0	68

We want to determine which pump will most increase the pressure at node 7 for a given increase in the head of that pump. Since all three pumps are far from the node with the deficient pressure, it is difficult to guess which pump will most influence the pressure at that node. The table below provides a partial summary of several solutions that were obtained by using the CHANGE capability in NETWK. The input data for these solu-tions is on the CD as file EXP5_11.IN. We will find in the input file that the original solution is obtained with the pump curves that accompany the network diagram; the sec-ond solution is obtained by increasing the head of pump 1 by 10 ft; the third solution has the head of pump 2 increased by 10 ft with the head of pump 1 reset to the original value; the fourth solution has the head of pump 3 increased by 10 ft; and the fifth solu-tion is obtained by increasing the head of the reservoir by three feet. The last solution, in which the water surface elevation in the reservoir was changed by three feet, is in a different category than others in which the pumps heads were changed, since water must

Node	Sol. 1	Solution 2 Pump 1, Δh_p=10'		Solution 3 Pump 2, Δh_p=10'		Solution 4 Pump 3, Δh_p=10'		Solution 5 Res. 1, ΔH= 3'	
	Head ft	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h_p$	Head ft	ΔH/Δh
7	360.2	361.4	0.112	361.2	0.096	361.6	0.138	362.1	0.633
10	371.2	372.2	0.104	372.0	0.084	372.3	0.115	373.2	0.680
4	390.5	392.2	0.168	392.0	0.148	392.7	0.224	391.8	0.447
5	390.2	391.9	0.168	391.7	0.147	392.4	0.212	391.6	0.460
6	390.2	391.9	0.167	391.7	0.145	392.3	0.206	391.6	0.467

be supplied by the pumps to fill the reservoir. Increasing the head of pump 3 (solution 4) is the most effective way to increase the heads at all of the nodes in the table because the derivatives $(\Delta H/\Delta h_p)_3$ are larger than these derivatives for the other two pumps. However, since all values of $\Delta H/\Delta h_p$ are not vastly different, it would be more effective to increase the head of all three pumps, particularly if the heads are deficient by more than a small amount.

Other network components have an influence on the sensitivity of dependent variables to a change in the independent variables. The table below summarizes a set of sensitivity analyses that mimic the prior table, with the one exception that pipe 3 was changed in diameter from 10 in to 8 in before obtaining the series of solutions. Now pump 3, which previously produced the largest head increments, gives the smallest head increments.

Node	Sol. 1	Solution 2 Pump 1, ΔH=10'		Solution 3 Pump 2, ΔH=10'		Solution 4 Pump 3, ΔH=10'		Solution 5 Res. 1, ΔH= 3'	
	Head ft	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h_p$	Head ft	$\Delta H/\Delta h$
7	352.1	353.5	0.137	353.3	0.119	353.1	0.099	354.0	0.633
10	364.0	365.3	0.128	365.1	0.110	364.9	0.089	366.0	0.657
4	378.8	380.6	0.182	380.4	0.159	380.1	0.133	380.3	0.513
5	378.8	380.6	0.181	380.4	0.159	380.1	0.132	380.4	0.633
6	379.1	380.8	0.175	380.6	0.152	380.3	0.125	380.7	0.657

The reasons for this change in effectiveness are relatively clear. The 8-in pipe that contains pump 3 is just too small for this pump to cause the greatest increases in head at the downstream nodes; to increase the head, the pump must supply a larger portion of the total flow, and the head loss in the 8-in pipe increases too much as the discharge increases. We see that the interactions of network components can be complex and interwoven, and the only effective means of determining the sensitivity of selected variables with respect to others is to develop an appropriate series of solutions so these sensitivities can be estimated. These solutions must consider demands etc. that are near those for which the sensitivities are to be determined.

Consider the use of sensitivities from another perspective. We might ask which pump can be enlarged at the least cost in order to increase the head at certain nodes by a specified amount. The answer to this question is already embedded in the previous solutions. But now the independent variable is not the incremental head added by a pump but rather the power (which can be substituted for cost when only the magnitudes of the sensitivities are compared, since the cost will be in dollars per kilowatt-hour) that a pump delivers to the network. Tables containing the power consumption of each pump as three independent variables are given below. In these tables the sensitivities are in units of head per kilowatt instead of head/head, as it was in the previous tables. Each of the previous two tables is now replaced by two tables for clarity. The first of each pair of tables lists the power requirement and the incremental difference in power between a subsequent solution and the first solution. The second table of each pair divides the change in head at the listed node by the incremental power to obtain $\Delta H/\Delta P_i$ with subscript i being the pump number and P being power in kilowatts. The negative values for these sensitivities occur because the incremental power between solutions is negative, i.e., the power produced by that pump (when the head of another pump increases) is less than that of the original solution. If the negative derivatives are ignored, then the conclusion is unchanged; pump 3 will produce a larger incremental head at these nodes for a given cost than either pump 1 or pump 2 can supply if the line serving pump 3 has a 10-in diameter. This situation occurs because the positive values of $\Delta H/\Delta P_3$ are larger than either $\Delta H/\Delta P_1$ or $\Delta H/\Delta P_2$. However, if the supply line for pump 3 has an 8-in diameter, then the most cost-effective pump for increasing the head at the listed nodes is pump 1, since the second pair of tables shows that the positive values of $\Delta H/\Delta P_1$ are larger than the values of either $\Delta H/\Delta P_2$ or $\Delta H/\Delta P_3$.

Sensitivity of Nodal Head to Pump Power (Pipe 3 is 10-in dia.)

Pump	Sol. 1	Solution 2		Solution 3		Solution 4	
	P	P	ΔP	P	ΔP	P	ΔP
1	26.84	28.34	1.50	26.49	- 0.35	26.39	- 0.45
2	25.19	24.83	- 0.36	26.59	1.40	24.75	- 0.44
3	32.44	32.00	- 0.44	32.03	- 0.41	34.08	1.64
Sum			0.70		0.64		0.75

Node	Solution 1	Solution 2			
	Head, ft	ΔH	$\Delta H/\Delta P_1$	$\Delta H/\Delta P_2$	$\Delta H/\Delta P_3$
7	360.23	1.12	0.75	- 3.11	- 2.55
10	371.15	1.03	0.69	- 2.86	- 2.34
4	390.48	1.68	1.12	- 4.64	- 3.85
5	390.24	1.68	1.12	- 4.67	- 3.82
6	390.22	1.67	1.11	- 4.64	- 3.80
		Solution 3			
7		0.96	- 2.74	0.69	- 2.34
10		0.84	- 2.40	0.60	- 2.05
4		1.48	- 4.23	1.06	- 3.61
5		1.47	- 4.23	1.05	- 3.59
6		1.45	- 4.14	1.04	- 3.54
		Solution 4			
7		1.38	- 3.07	- 3.14	0.84
10		1.15	- 2.56	- 2.56	0.70
4		2.24	- 4.98	- 5.09	1.37
5		2.12	- 4.71	- 4.82	1.29
6		2.06	- 4.58	- 4.68	1.26

While the negative sensitivities were ignored above, they do present valuable information related to the network's performance, particularly if total power (or cost) is considered. In fact, to neglect negative values is to ignore potential savings. For example, when pipe 3 has a 10-in diameter, we find in the first table from the second solution that the incremental sensitivities for pumps 2 and 3 are - 0.36 and - 0.44 kW, respectively; these values indicate that the power requirements for these two pumps decrease as the power requirement for pump 1 increases by 1.50 kW. The net increase in required power is only 1.50 - 0.36 - 0.44 = 0.70 kW. Similarly, if the head across Pump 2 (see solution 3) is increased by 10 ft, then the net increase in power is slightly less, or 0.64 kW. Sometimes it is better to examine sums of differences (or just differences) rather than one difference divided by another difference, which is how we first defined "sensitivity." In this example it probably makes most sense to use a difference divided by a difference, but the difference in the denominator (or the independent variable) should be the sum of power differences. This sensitivity represents the change in head that is caused by the change in the overall or total power consumption P_t (or cost). If these are the important sensitivities, then the values in the following table should be used to decide which alternative will be the most cost-effective and/or best.

Sensitivity of Nodal Head to Pump Power (Pipe 3 is 8-in dia.)

Pump	Sol. 1	Solution 2		Solution 3		Solution 4	
	P	P	ΔP	P	ΔP	P	ΔP
1	29.23	30.55	1.32	28.90	- 0.33	28.96	- 0.27
2	27.46	27.12	- 0.34	28.70	1.24	27.22	- 0.24
3	22.59	22.28	- 0.31	22.31	- 0.28	23.78	1.94
Sum			0.67		0.63		1.43

Node	Solution 1	Solution 2			
	Head, ft	ΔH	$\Delta H/\Delta P_1$	$\Delta H/\Delta P_2$	$\Delta H/\Delta P_3$
7	352.14	1.37	1.04	- 4.03	- 4.42
10	364.00	1.28	0.97	- 3.97	- 4.13
4	378.80	1.82	1.38	- 5.35	- 5.87
5	378.81	1.81	1.37	- 5.32	- 3.61
6	379.08	1.75	1.33	- 5.15	- 5.65
		Solution 3			
7		1.19	- 3.61	0.96	- 4.25
10		1.10	- 3.33	0.89	- 3.93
4		1.59	- 4.82	1.28	- 5.68
5		1.59	- 4.82	1.28	- 5.68
6		1.52	- 4.68	1.17	- 5.18
		Solution 4			
7		0.99	- 3.67	- 4.13	0.83
10		0.89	- 3.23	- 3.74	0.75
4		1.33	- 4.93	- 5.59	1.11
5		1.32	- 4.89	- 5.50	1.11
6		1.25	- 4.63	- 5.21	1.05

We see there are many possibilities, and which is best depends upon the objective, coupled with the judgment of the engineer who is responsible for making the decision. And we must keep in mind that the magnitude of each sensitivity (and difference, or sum of differences) is not a constant but can take on quite different values as demands and other conditions change.

Node	Pipe 3, Dia. = 10 in			Pipe 3, Dia. = 8 in		
	Sol. 2	Sol. 3	Sol. 4	Sol. 2	Sol. 3	Sol. 4
	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$	$\Delta H/\Delta P_t$
7	1.60	1.50	1.84	2.04	1.89	0.69
10	1.47	1.31	1.53	1.91	1.75	0.69
4	1.43	2.31	2.99	2.72	2.52	0.93
5	2.40	2.30	2.83	2.70	2.52	0.92
6	2.39	2.26	2.75	2.61	2.30	0.87

Another goal might be the maintenance of as large a volume of water in the storage tank (reservoir) as possible. If so, the sensitivities that should be examined are the difference in discharge in pipe 19 (which connects the reservoir to the network) divided by the sum of the pump power consumptions; rather than seek the largest value as we did before, the smallest sensitivity (the one with the largest negative magnitude) is the one we want. The reason is that our desire is to maximize $|\Delta Q_{19}|$ (the numerator) while minimizing the increase in overall pump power consumption ΔP_t (the denominator). These tables of sensitivities follow:

Item	Pipe 19, Dia. = 10 in	Pipe 19, Dia. = 8 in
Flow Q_{19} from Sol. 1, original conditions	2.30 ft^3/s	3.13 ft^3/s
Flow Q_{19} from Sol. 2, $\Delta h_p = 10$ ft at pump 1	2.17 ft^3/s	3.00 ft^3/s
ΔQ_{19}	- 0.13 ft^3/s	- 0.13 ft^3/s
$\Delta Q_{19}/\Delta P_t$	- 0.19 $ft^3/s/kW$	- 0.19 $ft^3/s/kW$
Flow Q_{19} from Sol. 3, $\Delta h_p = 10$ ft at pump 2	2.19 ft^3/s	3.01 ft^3/s
ΔQ_{19}	- 0.11 ft^3/s	- 0.12 ft^3/s
$\Delta Q_{19}/\Delta P_t$	- 0.17 $ft^3/s/kW$	- 0.19 $ft^3/s/kW$
Flow Q_{19} from Sol. 4, $\Delta h_p = 10$ ft at pump 3	2.15 ft^3/s	3.04 ft^3/s
ΔQ_{19}	- 0.15 ft^3/s	- 0.09 ft^3/s
$\Delta Q_{19}/\Delta P_t$	- 0.20 $ft^3/s/kW$	- 0.06 $ft^3/s/kW$

* * *

To compute sensitivities, we must have two solutions available in which the independent variable x has changed and the change in the dependent variable f can be obtained. Thus a numerical approximation to $\partial f/\partial x$ is obtained by dividing the change in the dependent variable Δf by the change in the independent variable Δx, or $\partial f/\partial x \approx \Delta f/\Delta x$. These paired solutions were previously obtained from NETWK by using the CHANGE command. An alternative, and for some networks a more effective, way to obtain such a series of solutions is to obtain an "Extended Time Simulation." This is a time-varying or quasi-steady solution that ignores most fluid transient effects. Extended Time Simulations, as Chapter 6 will describe further, consist of a series of steady-state solutions with different prescribed demands, water surface elevations at reservoirs, and head-discharge relations at pumps that depend upon a demand function or flow rule, storage functions, and pump rules, etc. The NETWK code allows the results from such solutions to be written in tables with time in the first column and discharges or head losses for selected pipes, and/or pressure at selected nodes, to be listed in subsequent columns. Alternative tables giving reservoir water surface elevations as a function of time can also be obtained. These tables can be used to obtain most sensitivities that may be wanted, especially if the specifications for the Extended Time Simulation dictate that some other variable is linearly related to time. The time can be used as the independent variable for the sensitivities.

Example Problem 5.12

Use the Extended Time Simulation capability of NETWK to obtain a series of steady state solutions and from these obtain the sensitivities for the 12 pipe, 7 node network diagramed in Fig. 5.28. Express the peaking factor as a linear function of time. After verifying some of the sensitivities that have already been presented, allow the elevation of the water surface in the tank to vary so its level is 198 m at time $t = 0$ (when PF = 0.5). The tank is circular with a diameter of 30 m, and its bottom elevation is 195 m, i.e., at this level there is no more water in the tank. Plot as a function of peaking factor the discharge from the two pumping stations and the discharge into and out of the reservoir.

The input file to NETWK to obtain this solution is listed on the next page. In it the linear relationship between PF and time is dictated by the DEMAND FUNCTION which applies to all nodes. The output tables are not given here but can be developed by the reader. After they are obtained, we could either use SENSITV or import the tables into a spreadsheet and then generate the sensitivities.

Illustration of sensitivities using Ext. Time Simulation

```
/*
$SPECIF NFLOW=3,NPGPM=3,NUNIT=4,PEAKF=.5,
NPRINT=-3,NODESP=0,ISIML=1,NETPLT=3,COEFRO=.15 $END
PIPE-
1 305. 2000. 1 .07 150.
2 255. 2000. 1 2 .07 145.
3 255. 1500. 1 3 .06 145.
4 205 5500. 2 6 .05 150.
5 205. 2500. 1 4 .06 140.
6 255. 2500. 5 .05 145. 3
7 205. 1500. 5 4
8 205. 3900. 4 6
9 205. 3000. 5 7 .04 152.
10 205. 3500. 7 6
11 255. 1800. 6
12 255. 1500. 5
RESER
11 200
PUMPS
1 .15 50 .25 43 .35 35 180
12 .1 48 .15 43.25 .25 33.0 180
RUN
$TDATA PRINTT=-3,HTIME=24,INCHRP=1
LINEAR=1,ISUNIT=0 $END
PIPE TABLE
ALL
NODE TABLE
ALL
RESER. TABLE
11/
END TABLES
DEMAND FUNCTION
1 0 1. 12 1.6789738 24 2.35794769/
1-7/
STORAGE FUNCTION
1 195 0 198 2120.6 205 7069/
11/
END SIML
```

To use an Extended Time Simulation to produce solutions that portray the flow at the reservoir, the size of the storage tank at the end of pipe 11, and its water surface elevation, are included in the input file by prescribing a STORAGE FUNCTION. Since the tank is circular with a diameter of 30 m, the area is $A = \pi D^2/4 = 707$ m^2; with its base at 195 m the tank will have a starting water surface elevation of 198 m when PF = 0.5. We must first change elevation 200 to 198 under the RESER command, and then we add ISUNIT=0 to the $TDATA list and finally add STORAGE FUNCTION and two lines of data before END SIML. The following (partial) tables will then be obtained:

A negative flow in pipe 11 indicates that the storage tank is filling. From the middle table of the set it can be seen between hours 2 and 3 (when the peaking factor PF is between 0.5x[1 + 1.358x(2/24)] = 0.557 and 0.585) that the tank changes from filling to supplying the network. Shortly after hour 22, when the peaking factor PF is slightly larger than 0.5x[1 + 1.358x(22/24)] = 1.122, the tank has emptied. (The tank base is at 195 m, at which its volume becomes 0 m^3 in the storage function.) Thereafter, all of the demand must be met by the pumps, even as the PF increases, and this is shown in the negative pressure in the last two lines of the pressure table. Obviously these pressures are

PRESSURES (kPa) AT DESIGNATED NODES AS A FUNCTION OF TIME

TIME	NODE NUMBERS						
hrs.	1	2	3	4	5	6	7
0.0	620.79	599.96	658.84	661.46	662.26	497.17	487.94
1.0	609.88	587.35	647.43	649.93	651.75	493.83	481.35
2.0	599.55	575.78	636.57	639.30	641.86	491.76	475.94
3.0	590.05	565.61	626.55	629.88	632.93	491.03	472.67
4.0	580.08	554.89	616.02	619.76	623.56	488.25	468.68
.	.	.	.	.	.	.	.
21.0	311.87	247.25	323.15	328.72	353.57	235.90	178.93
22.0	287.71	217.97	297.98	302.10	330.01	217.86	151.05
23.0	- 56.82	- 287.95	- 47.95	- 175.37	- 20.87	- 442.91	- 453.93
24.0	- 98.43	- 343.07	- 91.34	- 227.28	- 62.87	- 502.91	- 513.54

DISCHARGES IN DESIGNATED PIPES AS A FUNCTION OF TIME

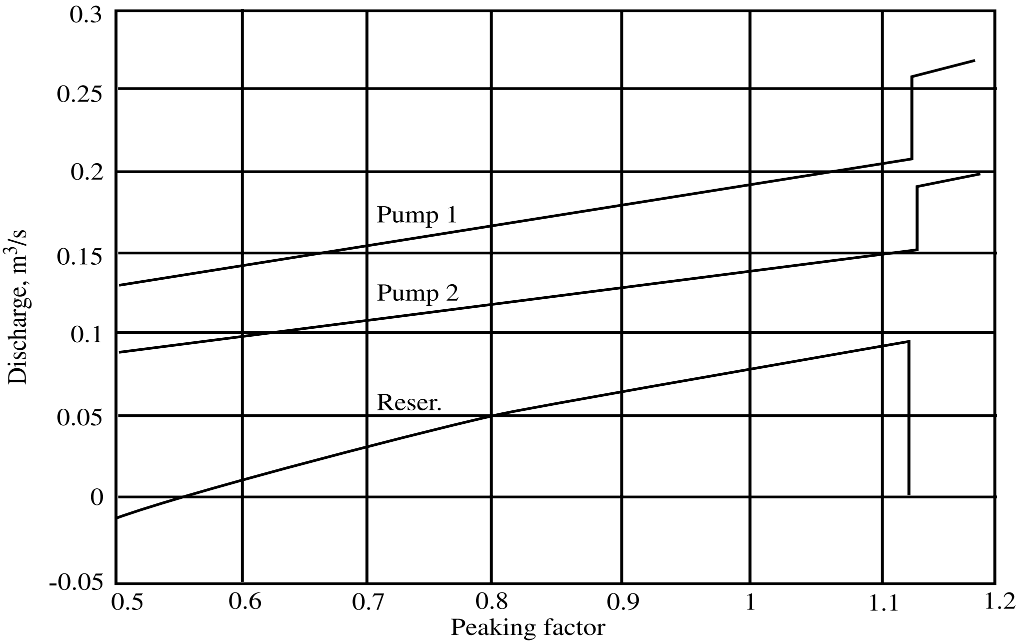
HR.	PIPE NUMBER											
	1	2	3	4	5	6	7	8	9	10	11	12
0.0	.127	.049	.021	.014	.022	.009	.027	.019	.028	.008	-.015	.088
1.0	.130	.050	.022	.013	.022	.010	.027	.018	.027	.006	-.010	.090
2.0	.134	.050	.023	.011	.022	.011	.027	.016	.027	.004	-.004	.093
3.0	.136	.050	.023	.009	.022	.012	.027	.015	.026	.002	.003	.094
4.0	.139	.051	.024	.008	.022	.013	.028	.013	.025	.000	.010	.098
.	.	.	.	.	.	.	.	.	.	.	.	.
21.0	.202	.063	.038	-.013	.025	.028	.033	-.008	.028	-.016	.092	.143
22.0	.207	.065	.039	-.014	.026	.029	.034	-.008	.028	-.017	.095	.147
23.0	.268	.101	.043	.020	.044	.026	.056	.031	.053	.007	.000	.192
24.0	.275	.103	.044	.021	.045	.027	.057	.032	.054	.007	.000	.197

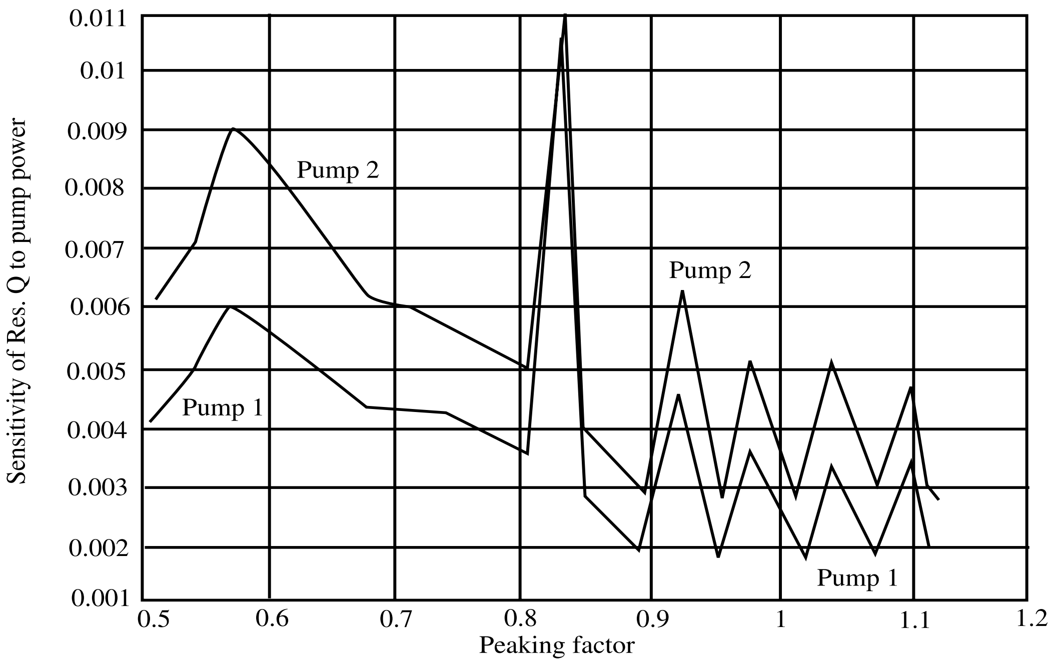
WATER SURFACE ELEVATION IN RESERVOIR 11

TIME hrs.	ELEVATION ft.
0.0	200.00
1.0	200.08
2.0	200.13
3.0	200.15
.	.
21.0	195.60
22.0	195.13
23.0	195.00
24.0	195.00

not real; this network cannot meet the demands with an empty tank. To prepare a plot of the discharge from the pump stations, the discharge table can be imported into a spreadsheet. The first column, which lists the time, can be changed to represent the PF by noting that time 0.0 corresponds to PF = 0.5 and time 24.0 corresponds to PF = 1.179. The plot shows the discharges in pipes 1, 12, and 11 as a function of peaking factor. We see the reservoir filling with PF < 0.58; when PF = 1.13 the reservoir has emptied. Since it can now supply no flow, the discharge from each pump station must sharply increase to satisfy the demand.

To obtain the sensitivities $\Delta Q_{11}/\Delta P_1$ and $\Delta Q_{11}/\Delta P_2$, columns in the table can be created for the power at each of the two pump stations with $P_1 = 9.806Q_1(h_{p1}) = 9.806Q_1(58.6 - 50Q_1 - 50Q_1^2)$ and $P_2 = 9.806Q_{12}(56.8 - 82.5Q_{12} - 50Q_{12}^2)$. The difference of P_1 and P_2 between separate entries (rows) is the divisor of the differences in the discharge Q_{11} to obtain the sensitivity of the reservoir flow to the pump power. These sensitivities are presented in the next plot. The curves are not smooth largely because of the limited accuracy in computing the discharge in pipe 11, since the sensitivities are dependent entirely upon these values. However, the conclusion is the same as when the level of the reservoir was constant at 200 m; it is better to increase the power at pump station 2.


0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
Discharge, m^3/s
Pump 1
Pump 2
Reser.
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Peaking factor


0.011
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
Sensitivity of Res. Q to pump power
Pump 2
Pump 1
Pump 2
Pump 1
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Peaking factor

*   *   *

5.8 PROBLEMS

5.1 The table below contains several pipes. Using the Darcy-Weisbach and the Hazen-Williams equations, compute the diameters of the pipes that are needed to convey the given discharge with the given head loss.

Pipe	Q	L	h_L	ex10^3	C_{HW}	Darcy-Weisbach		Hazen-Williams
No.	ft^3/s	ft	ft	in		f	D, in	D, in
1	1.0	2500	30	0.05	150			
2	2.0	400	20	20.0	95			
3	3.0	10000	105	5.0	138			

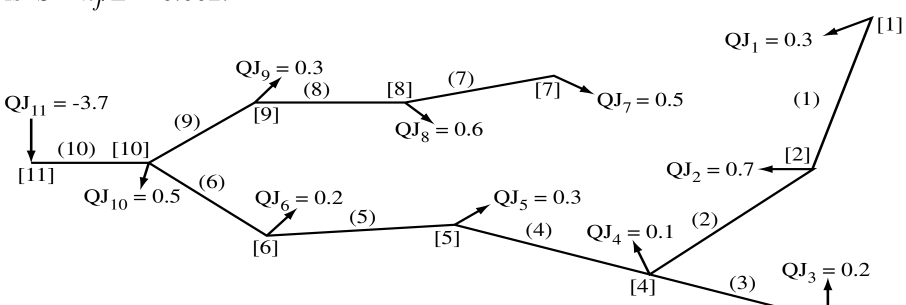
5.2 The table below contains several pipes. Using the Darcy-Weisbach and the Hazen-Williams equations, compute the diameters of the pipes that are needed to convey the given discharge with the given head loss.

Pipe	Q	L	h_L	ex10^3	C_{HW}	Darcy-Weisbach		Hazen-Williams
No.	m^3/s	m	m	cm		f	D, m	D, m
1	0.25	1500	20	0.08	150			
2	0.50	600	20	80.0	95			
3	1.50	4000	55	9.0	140			

5.3 Modify program DIAPIP so algebraic derivatives are used to evaluate the elements of the Jacobian in place of the numerical evaluation in the original listings.

5.4 Modify program DIAPIP so the two unknown variables are f and D rather than $SF = 1/\sqrt{f}$ and D.

5.5 The program SOLBRAN was used to determine the pipe diameters in a 10-pipe branched system; then the nodes and pipes were numbered by starting at the upstream end. This same branched system is shown below, but now the numbering proceeds from the downstream end. Prepare the input data for program SOLBRAN (or your own program) using this numbering, and obtain the solution. The slope of the energy line for all pipes is $S = h_f/L = 0.002$.



5.6 Retain the node numbers as in Problem 5.5, but begin the pipe numbering with 1 at the upstream end, prepare the input data for SOLBRAN and obtain the solution.

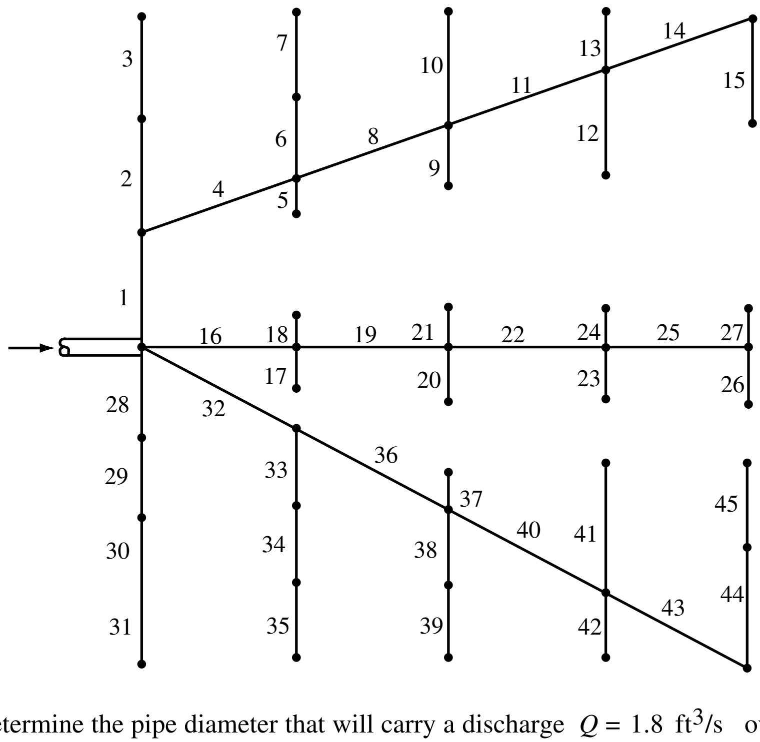
5.7 Use NETWK to solve Problem 5.5.

5.8 Develop a computer program that can determine the diameter of a pipe if the discharge, head loss, pipe length, and wall roughness are known. This program should be able to use either the Darcy-Weisbach equation (including the Colebrook-White equation) or the Hazen-Williams equation.

5.9 Modify program SOLBRAN to solve a branched system in which laminar flow exists in all pipes. Write this program so it reads from the input file for all pipes either the head losses or the diameters, and it determines either the pipe diameters or the head losses (i.e., it finds the variable that is not given) and the pipe discharges that will satisfy the specified demands.

5.10 Using the program from Problem 5.9 (or a slight modification of it), find the diameters of the tubing for the drip irrigation system shown below if each emitter (solid circle) is to supply 2 gal/min. The slope of the HGL is 0.008.

Pipe	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
L, ft	25	25	25	42	10	15	25	42	15	25	42	25	15	42	25
Pipe	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
L, ft	40	10	10	40	10	10	40	10	10	40	10	10	20	20	20
Pipe	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
L, ft	20	45	20	20	20	45	10	10	20	45	25	10	45	25	25



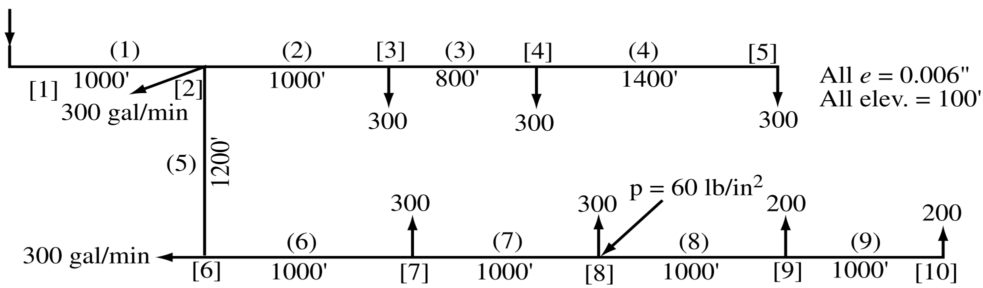
5.11 Determine the pipe diameter that will carry a discharge $Q = 1.8$ ft^3/s over a length of 4000 ft if the difference in head between the beginning and end of the line is to be 65 feet. The wall roughness for this pipe is $e = 0.005$ in.

5.12 Find the pipe diameter in Problem 5.11 by using the Hazen-Williams equation with $C_{HW} = 145$.

5.13 A 3000 ft long pipeline carries a discharge of 2.0 ft^3/s over 2000 ft of its length, at which point an unknown amount of water is withdrawn. The drop in head from the beginning to the end of the pipe is 30 ft. The pipe is 8-inch-diameter PVC pipe, and the kinematic viscosity of the water is $\nu = 1.2 \times 10^{-5}$ ft^2/s. Determine the amount of the demand at the intermediate point in the pipeline.

5.14 Determine all pipe diameters in the branched piping system in the sketch below so that the slope of the HGL is 0.008. All pipes have a roughness $e = 0.006$ inches. Also determine the pressure, pressure head, and elevation of the HGL at each node of this network, so that the pressure at node 8 is 60 lb/in^2. Then select the closest standard pipe diameter for each pipe from the list below and again obtain a solution for the pressure, pressure head, and elevation of the HGL at all nodes. What head should a pump in pipe 1 produce if its supply water surface elevation is 100 ft? If the combined motor-pump efficiency is 73 percent, what is the cost per day to pump continuously if electricity costs $0.10/kWh? Also determine the cost of the pipe. The standard pipe sizes and costs per unit length follow:

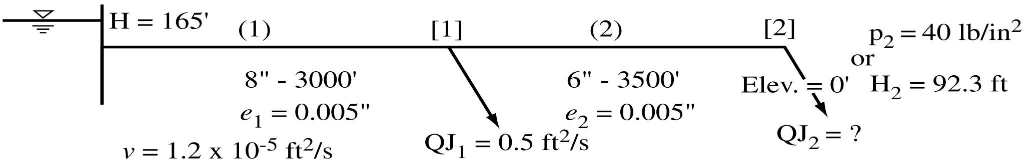
Dia. in.	4	6	8	10	12	15
Cost $/ft	3.67	5.33	7.67	10.67	16.67	24.00
Dia. in.	18	20	24	30	42	
Cost $/ft	43.33	56.67	80.00	100.00	145.00	



5.15 Develop a spreadsheet solution for the branched piping system in Problem 5.14. Use the closest standard pipe diameters that you determined in that problem and give the pressure and head at every node. In developing the spread sheet solution use the Hazen-Williams equation with $C_{HW} = 150$.

5.16 Modify program SOLBRAN so different HGL slopes can be specified for individual pipes or groups of pipes, and use it to solve Example Problem 5.3.

5.17 In the pipeline system shown atop the next page the pressure at the downstream node has been measured as $p_2 = 40$ lb/in^2. Compute the demand at this node twice by using the Darcy-Weisbach equation and the Hazen-Williams equation. Assume $C_{HW} =$ 145 for the Hazen-Williams roughness coefficient.



5.18 In the piping system of Problem 5.17, determine the diameter of pipe 2 so the discharge to node 2 is 0.6 ft^3/s.

5.19 Solve Problem 5.18 using the Hazen-Williams equation with $C_{HW} = 145$.

5.20 Analyze the 16-pipe, 9-node network shown in Fig. 5.6 and modified in Fig. 5.13. In the paragraph which follows Fig. 5.13, a design solution determines a diameter for all pipes except pipes 1, 3, and 16; adjust those diameters to the nearest standard pipe sizes, and assign a diameter of 150 mm to pipes 1, 3, and 16. To obtain the solution, you must first select appropriate pump characteristic curves and the number of pumps that should be in parallel and/or series. The first analysis should be based on the demands that were used in determining the pipe sizes, namely twice the average demand. Also obtain an analysis based on the average demands, and then obtain a third solution for which the demands are half of the average demands. Under this last demand condition, what discharge will be entering the storage tank when it is half full, i.e., when the water surface elevation in the tank is 119.5 m? (For these analyses assume the high-cost water from pipes 1 and 3 is shut off. Assume a fire flow of 0.08 m^3/s is needed at node 4 during the time of the largest hourly demand and both pipes 1 and 3 are open. What pressure will exist at node 4 to fight the fire, and how much flow will come from the four supply sources using the pump chosen earlier?

5.21 The 16-pipe, 9-node network was converted into a branched network by omitting the 7 pipes numbered 1, 3, 9, 10, 12, 13, and 16, as shown in Fig. 5.13. If pipe 16 were included and pipe 8 were omitted, would a branched system be formed? Since pipe 16 is the pipe to the storage tank, it generally would be considered to be part of the main transmission system. In fact, if pipes 2, 7, 12, and 16 are retained, the most direct path between the pump and storage tank exists to fill the tank during periods of low demand. Delete other pipes so this path exists, and determine the size of each pipe.

5.22 In Fig. 5.15 pipe 1 was given a diameter of 18 in, and pipe 2 was given a diameter of 15 in. For the pump characteristics given with this network, and for elevations of the HGL at nodes 2 and 3 of $H_2 = 645$ ft and $H_3 = 640$ ft, respectively, compute the discharges that the two pumps will supply. What discharge must the reservoir therefore supply? Verify your results by comparing them with the NETWK solution.

5.23 If the diameter of pipe 1 in the 30-pipe, 16-node network is changed from 18 in to 24 in, compute as in Problem 5.22 the discharge supplied by the two pumps. Why does this change create an impossible situation? What specification(s) could be changed to allow a solution?

5.24 In the 30-pipe, 16-node network assign to pipe 6 a diameter $D_6 = 6$ in but find the diameter D_{10} of pipe 10. To obtain this solution, use NETWK by appropriately modifying the input given in file FIG5_15.IN.

5.25 In the 30-pipe, 16-node network give pipes 6 and 9 the diameters $D_6 = 6$ in and $D_9 = 6$ in but compute the diameters D_{10} and D_{12} of pipes 10 and 12. To obtain this solution, use NETWK by modifying the input given in file FIG5_15.IN.

5.26 In the 30-pipe, 16-node network assign a diameter $D_{30} = 6$ in to pipe 30 that connects the reservoir to the network but determine the diameter D_1 of pipe 1 through which source pump 1 supplies the network.

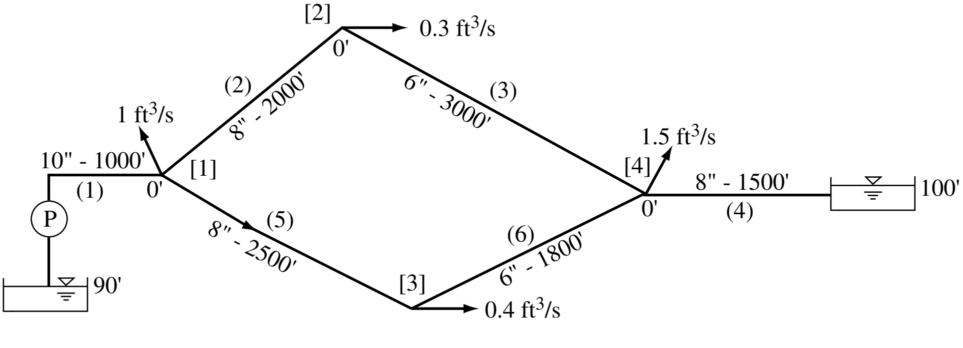
5.27 In the 30-pipe, 16-node network specify a diameter $D_{30} = 6$ in for pipe 30 that connects the reservoir to the network but compute the diameter D_2 of pipe 2 through which source pump 2 supplies the network. Initially retain 18 in for the diameter of pipe 1. Why is a solution not possible? Increase the diameter of pipe 1 to 24 in and obtain a solution.

5.28 The pressure can not become negative anywhere in a network, even though the mathematics of solving a network problem can produce negative pressures. Often 40 lb/ft^2 is the lowest pressure that is permitted. Determine the water surface elevation of the reservoir that supplies the 30-pipe, 16-node network via pipe 30 so the pressure at node 16 is 40 lb/ft^2 if the pipe diameters are determined by solving Problem 5.27 with D_1 = 24 in. First obtain this solution with $D_{30} = 6$ in, and then increase the diameter to $D_{30} = 12$ in. What feature is not realistic in the use of 6 in for D_{30}?. (Hint: use a differential head device in pipe 30.)

5.29 In the 9-pipe, 6-node network of Example Problem 5.4, indicate whether a solution is possible, or why a solution is not possible, for the following combinations of three pipes with their diameters specified as 6 in. If a solution is possible, solve the problem for the remaining six pipe diameters. Use the heads given in Example Problem 5.4, but in the last case modify the head at node 1 to $H_1 = 97$ ft.

Case	Pipe Numbers with Specified Diameters
1	1, 2, 5
2	1, 5, 7
3	3, 8, 9
4	1, 6, 7

5.30 In this small network you are to determine the head and discharge of the pump in pipe 1 so no flow will enter or leave the reservoir that is connected to the network by pipe 4 in response to the nodal demands shown on the diagram.



5.31 Retaining the head that was determined for the pump in Problem 5.30, but not the same discharge, determine the discharge that must be supplied by the reservoir if the demands are all increased to 1.5 times the values shown on the figure. In solving this problem, replace the pump by a DHEAD device of type 1, i.e., one that produces the specified differential head.

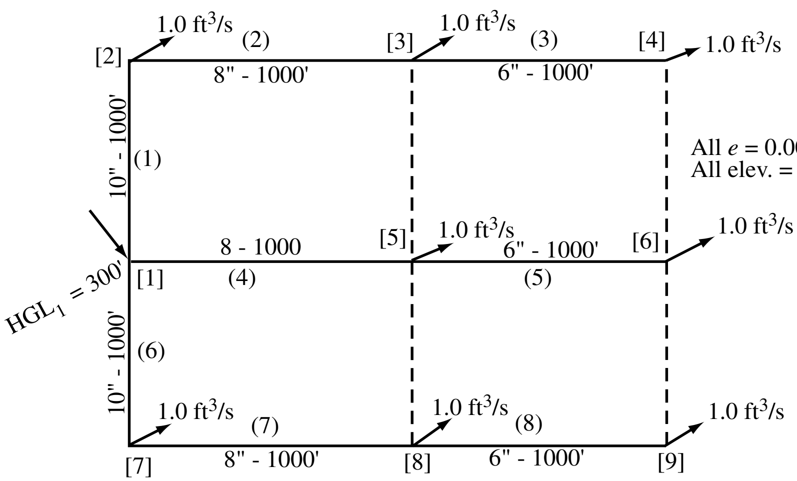
5.32 Rework Problem 5.31 with demands that are 0.8 times those in the diagram; in this case determine the discharge into the reservoir. Are the equations for this problem different from those of Problem 5.31? If so, what changes?

5.33 The 8-pipe network shown below was built to supply demands of 1.0 ft^3/s at each of eight nodes. Over the years the demands have doubled, and the network is now unable to supply 2.0 ft^3/s at these nodes. The 10-in pipes, numbers 1 and 6, are to be replaced by 12-in pipes, and the network is to be looped by adding the 4 pipes listed in the table:

Pipe	Node 1	Node 2
9	5	3
10	5	8
11	6	4
12	6	9

First analyze the original network for the original demands. Next analyze the same network again, but with the eight nodal demands each increased to 2.0 ft^3/s. At how many nodes is the present network unable to supply a pressure of at least 40 lb/in^2? Obtain a design solution for this network to determine the sizes of the four additional pipes; since eight diameters must be found in such a solution, also determine the sizes of pipes 2, 4, 5, and 7. The nodal HGL elevations that might be specified are listed in this table:

Node	1	2	3	4	5	6	7	8	9
H, ft.	300	291	276	264	290	280	291	276	264



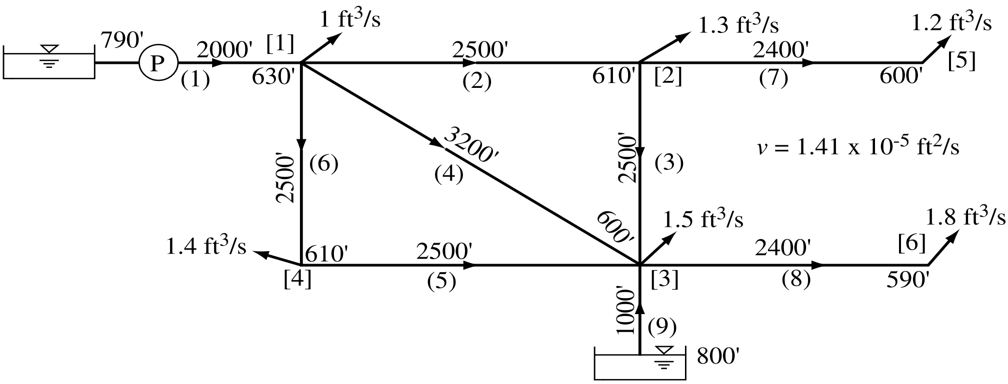
5.34 In the network shown in Fig. 5.23 a total of 15 combinations of three pipes exist and are candidates to have their diameters specified. Obtain a solution for each of these groupings using NETWK. In obtaining these solutions, also obtain an analysis solution for each case by using the nearest standard pipe sizes. Try specifying an 8-in diameter for pipe 6 while prescribing the diameters of pipes 3, 5, and 6, and note the message that the program returns to inform the user that inappropriate specifications have been made. Rather than increasing the diameter of pipe 6, adjust the nodal HGL-elevation specifications to specify a problem for which a solution is possible.

5.35 In Problem 5.33 each of eight nodal demands was 2 ft^3/s. Solve the same problem under the assumption that the new nodal demands are each 2.5 ft^3/s. Before you seek a design solution to this network, select appropriate diameters for pipes 1 and 2 (and all other pipes having specified diameters).

5.36 Design the looped network shown below. The target HGL-elevations at the nodes should be near those given in the head table for the demands shown on the sketch. Assume $e = 8.0\times10^{-6}$ in for pipes 1, 2, and 3 and $e = 6.0\times10^{-6}$ in for the other pipes.

Node No.	1	2	3	4	5	6
HGL, ft	832	805	798	815	795	785

Q ft^3/s	**Head** ft.
6	65
9	61
12	55

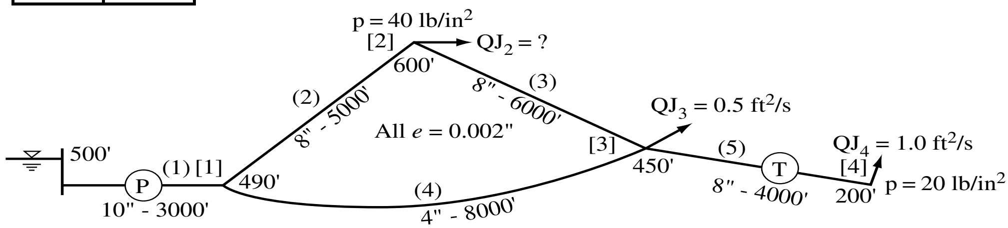


To complete this design, do the following: (1) Assign diameters to pipes 1, 3, and 6 as 13 in, 6 in, and 8.5 in, respectively, and determine the six diameters D_2, D_4, D_5, D_7, D_8, and D_9 to produce the specified HGLs. Obtain this design solution using NETWK. (2) Verify the results from NETWK with hand calculations by first finding the discharges in the three pipes using the specified diameters. Then find Q_3 and Q_6 from the Darcy-Weisbach and Colebrook-White equations. Next fit the given data to determine the polynomial for the pump curve and solve for the three unknowns Q_1, f_1, and h_p. With Q_1, Q_3, and Q_6 known, reduce the network and determine the other discharges and head losses. (3) Identify other pipes that are candidates to have their diameters specified, and identify specifications that would make a solution impossible.

5.37 Water is pumped from a reservoir with a water surface elevation of 500 ft over a hill crest of elevation 600 ft by means of the piping system shown in the next figure. The primary questions that need to be answered are: (a) what demand QJ_2 can be supplied at the top of the hill with a pressure of 40 lb/in^2, and (b) how much power can be extracted by the turbine in pipe 5 if 1.0 ft^3/s at 20 lb/in^2 is to be delivered at node 4? Write and then solve the system of equations that will provide these answers.

Pump

Q ft^3/s	h_p ft
2.0	290
2.5	285
3.0	272



5.38 Two pumps, pump a and pump b, have the operating characteristics given by the three (Q, h_p) pairs listed in the two tables below. At what rotational speed ratios $N_{ra} = (N_2/N_1)_a$ and $N_{rb} = (N_2/N_1)_b$ should each of these pumps be operated if the required combined discharge is $Q_{tot} = 3.5$ ft^3/s? Since the required discharge is well beyond the values in the tables for either pump, the two pumps must be placed in parallel. Assume that the middle point in each table represents the normal operating condition for each pump, and at their new rotational speeds the pumps should be operating at their maximum efficiencies.

Pump a (N_{a1} = 800 rev/min)

Q_a ft^3/s	h_{pa} ft.
0.75	43.00
1.10	38.75
1.50	32.20

Pump b (N_{b1} = 1000 rev/min)

Q_b ft^3/s	h_{pb} ft.
1.5	44.00
2.0	38.25
2.5	30.00

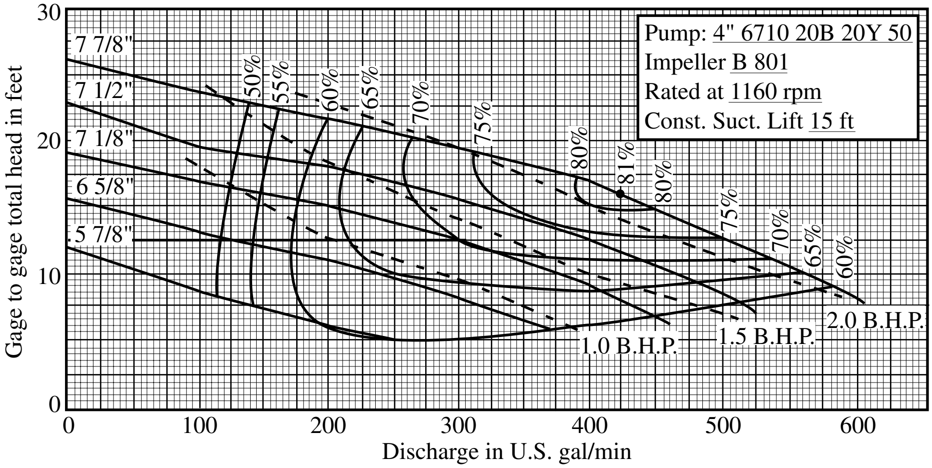
5.39 Modify program SPLINECU so that natural boundary condition is always used; it is desired not to give the user the option of specifying either natural boundary conditions (second derivatives set to zero at the ends of the domain) or the end slopes.

5.40 Program SPLINECU fits pairs of head vs. discharge data with a cubic spline and provides M interpolated values with equal increments. Convert this main program into a subroutine (function) that (1) receives one pair of values as arguments from the main program, and (2) provides to the main program the values of the second derivatives so that the main program can carry out cubic spline interpolations.

5.41 Modify program SPLINECU so it acts in the same way as program LAGRANGE, i.e., it provides the interpolated value for the pump head for any value of discharge that is supplied to it.

5.42 Modify program ELECENG so it computes energy consumption over any period of time. The program should perform the following tasks: (1) read the number of pairs of Q vs. time data, and then read these pairs; (2) read the number of pairs of Q vs. head H and Q vs. efficiency η; and then read these two sets of data pairs; and (4) compute the energy consumed by integrating the pump power over time.

5.43 The operation of a pump with a 7 7/8 in diameter impeller is described by the pump characteristic curves given below. Take 7 pairs of points from this curve, starting with 0 gal/min and ending with 600 gal/min in increments of 100 gal/min, and use a second-order polynomial between three consecutive pairs of points to interpolate values. Using this interpolation, obtain values of pump head for the following discharges, and compare the interpolated values with the corresponding values that are read from the pump curve itself: 50 gal/min, 120 gal/min, 190 gal/min, 250 gal/min, 330 gal/min, 410 gal/min, 460 gal/min, 550 gal/min.



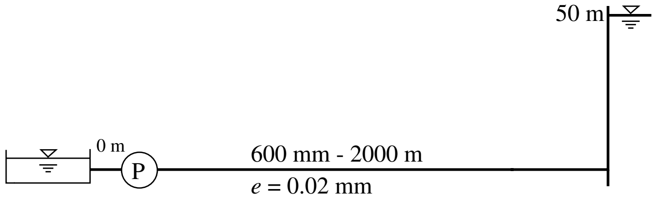
5.44 Repeat Problem 5.43 but use a cubic spline in place of the second-order polynomial for the interpolation.

5.45 For the pump whose characteristic curve is given in Problem 5.43, obtain the energy consumed by the motor when the discharge varies over a 24-hour period as the table describes:

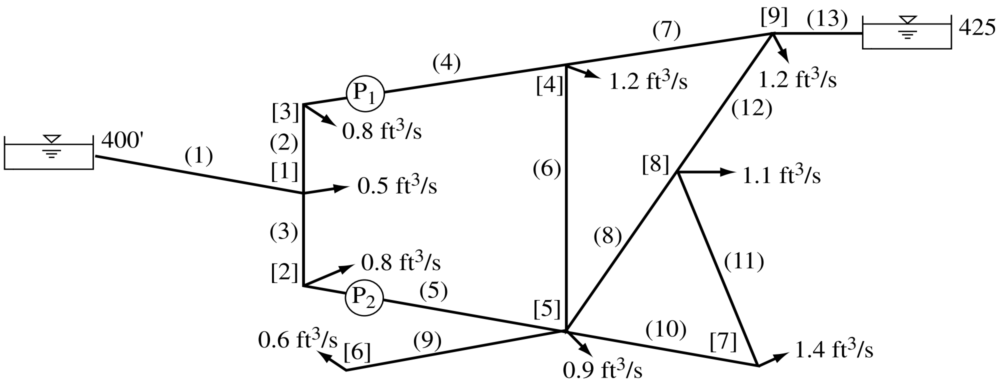
Time hr.	Q ft^3/s
0.0	0.20
2.0	0.40
4.5	0.70
6.0	1.10
8.2	1.40
10.3	1.50
12.3	1.60

Time hr.	Q ft^3/s
14.0	1.50
16.5	1.30
18.0	1.10
20.1	0.80
21.0	0.60
22.5	0.40
24.0	0.15

5.46 A pump is attached to a pipeline that has a length of 2000 m and a diameter of 600 mm (with $e = 0.02$ mm). The downstream reservoir has a water surface elevation that is 50 m above the supply reservoir water surface elevation. The pump characteristic curves show that the efficiency variation is essentially linear between Q = 0.0 m^3/s and Q = 1.2 m^3/s. At Q = 0.0 m^3/s the efficiency is zero, and at Q = 1.2 m^3/s the efficiency is 85%. And as the discharge increases from 1.2 m^3/s to 2.1 m^3/s, the efficiency varies linearly with discharge from 85% to 30%. Plot the power supplied by the pump to the fluid, and the power required by the pump from its motor, for discharges from 0.2 m^3/s to 2.1 m^3/s.



5.47 In the network shown below two booster pumps supply all of the water for the system, and this water must come from the reservoir on the left, which is extremely large. The reservoir on the right is a storage tank that receives water during periods of low demand and supplies some water during periods of higher demand. The pipe sizes, their lengths, etc., are defined in the input data file for NETWK. It has been decided to increase the head of one of the pumps so that pressures are larger at the downstream end of the network and larger flows enter the reservoir during periods of low demand. For the demands in the diagram, determine the increases in pressure and the discharge into the tank if the head of pump 1, or the head of pump 2, were increased by 5 ft. Which solution is more cost effective? Why is this the case? List some other options in improving the cost effectiveness of the system.

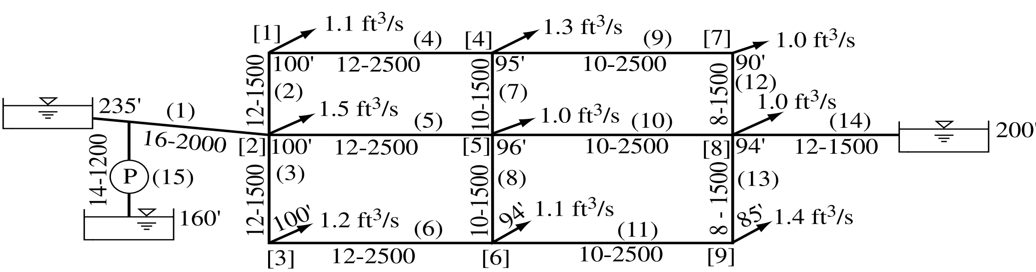



```
Chapter 5, Problem 5.47.
/*
$SPECIF OUTPU1=2,NPSERI=0 $END
PIPES
1 0 1 1000 14 .0008
2 1 3 700 12
3 1 2
4 3 4 1500 8
5 2 5 1500 12
6 5 4 1600
7 4 9 1400 8
8 5 8 1000
9 5 6 1200 6
10 5 7 1000 8
11 7 8
12 8 9 1200
13 0 9 500
NODES
1 .5 293
2 .8 285
3 .8 290
4 1.2 340
5 .9 345
6 .6 340
7 1.3 338
8 1.1 335
9 1.2 335
RESER
1 400
13 425
BOOSTER
4 1.5 55 3. 50 4.5 42/
5 3 63 5 60 7 55/
RUN
```

5.48 The network shown below has a pump in pipe 15 that obtains its water supply from ground water with a constant water surface elevation of 160 ft, and it pumps into a circular tank with a diameter of 185 ft. The bottom of the tank is at elevation 225 ft, and its top is at elevation 245 ft. The demands on the sketch are average values. The reservoir that is connected to the network by pipe 14 is water that is bought from an outside water agency for $ 0.35 per thousand cubic feet, and it is received from a conduit under a constant pressure that produces a HGL of 200 ft. The costs of the pump, well, tank and the connection to the outside water agency have been fully paid, so they should no longer be considered in economic analyses. Do the following:

1. Obtain a series of solutions in which the peaking factor (demand function) varies for all nodes from 1.5 to 0.5. In this series of solutions assume that the water surface elevation of the tank is at 235 ft when PF = 1.5; start with this PF and assume it decreases linearly over a 24 hr time period to 0.5.
2. Plot the discharge variation in pipes 1, 14, and 15 with the demand function.
3. Compute the cost of pumping the water from the well. For these costs assume that the combined pump-motor efficiency can be defined by a second-order polynomial function of the discharge, with the efficiencies related to the discharges in the pump characteristic table as follows: 0.70, 0.75, and 0.58. The cost of electrical energy is $ 0.08/kWh. Show how the pumping cost varies as the peaking factor changes, and how the average cost of pumping compares with the price of water purchased from the agency.
4. Show that the cost of water is a constant times the reciprocal of the sensitivity of the discharge to the pump power, i.e. the cost is equivalent to a constant times the sensitivity of power to discharge, which is $\Delta P/\Delta Q$.
5. Compute and plot the sensitivities of the pressure at nodes 4, 5, and 6 to the discharge in pipes 1 and 14.

Q ft^3/s	**h_p** ft
8.5	105.0
9.5	95.6
10.5	85.0



Dia. in in $\quad$ $\nu = 1.217 \times 10^{-5}$ ft^2/s
Lengths in ft