CHAPTER 7

INTRODUCTION TO TRANSIENT FLOW

7.1 CAUSES OF TRANSIENTS

To this point we have emphasized steady flows, flows that do not change with time at any location in the pipeline system. In this brief chapter we will introduce two general categories of unsteady flow that we call *transient flow*. All transient flows are transitions, of long or short duration, from one steady flow state to another. Either of these end states may be the rest state. Each transient flow is a response of the fluid to some change in the hydraulic facilities that control and convey the fluid, or in the surrounding environment, that influences the flow.

The first type of transient, which we will refer to as *quasi-steady flow*, is characterized by the absence of inertial or elastic effects on the flow behavior. In such a flow the variation of discharges and pressures with time is gradual, and over short time intervals the flow appears to be steady. Typical examples are the drawdown of a reservoir, the draining of a large tank, or the variation in demand in a water distribution system over a 24-hour period. This type of transient was considered briefly in Chapter 6 and will be examined in more detail in Section 7.2.

The second kind of transient is known as *true transient flow*, in which the effect of the fluid inertia and/or the elasticity of the fluid and pipe is an essential factor in the flow behavior and must be considered. If inertial effects are significant but pipe and fluid compressibility effects are relatively minor or negligible, then we have a true transient flow which we will refer to as a *rigid-column flow*. If in addition we must retain the elastic effects of the fluid and pipe in order to obtain an accurate characterization of the transient, we will call this a *water hammer* condition. The distinction between rigid-column flow and water hammer is not easily categorized and depends, in a general way, on how rapidly events change in a system. For example, the oscillation of the water level in the surge chamber of a hydroelectric facility can be analyzed accurately as a rigid-column flow. In this case inertial effects must be considered, but elastic or compressibility effects clearly are minor. On the other hand, the sudden closure of a valve in a pipeline is a water hammer situation; to simulate accurately the resulting behavior would require the inclusion of the elasticity of both the pipe and the liquid in the analysis. When the valve is closed more slowly, however, uncertainty arises. If the closure time is sufficiently long, then a rigid-column flow analysis may represent the physics of the problem well and produce good results. If the analyst is in doubt, then a water hammer analysis should be used because it is a more complete and general characterization of the flow. The groundwork for the study of true transients will be laid in Section 7.3 where both rigid-column flow and water hammer will receive attention. The study of water hammer problems will build on this foundation with extensive coverage in Chapters 8 through 13. Chapter 12 will treat both rigid-column flow and water hammer analyses in pipe networks.

7.2 QUASI-STEADY FLOW

We begin by considering a large tank, or even a small reservoir, that is full of water. By large we mean that the depth of the water from the base to the top of the tank is large, and the area of the water surface at any particular level within the tank is also large in comparison with the dimensions of the discharge opening. We wish to drain the water from the tank, a task that can be accomplished in many ways. At one extreme the tank could be emptied by attaching to the base of the tank at point A a long pipe of small but constant cross-sectional area or diameter, with a control valve at the downstream end of the pipe at point B, as shown in Fig. 7.1. Almost irrespective of how fast the valve is opened at B, the fluid will drain relatively slowly from the tank if the tank dimensions are sufficiently larger than the pipe cross-sectional area. At any instant there will be almost no perceptible motion in the tank itself, and there will be only a gradual temporal acceleration (positive or negative) of the fluid in the pipe; thus inertial effects are insignificant. A very small region of local convective acceleration will be found at the pipe entrance. At any instant the flow processes largely appear to be no different than a truly steady flow. After some time has elapsed, it will be found that the water level in the tank is indeed dropping, and the tank will eventually be empty. This flow is a good example of a quasi-steady flow.

Figure 7.1 The draining of a large tank via a quasi-steady flow process.

Turning briefly to Fig. 7.2, which shows the same tank with a drain line of much different dimensions, it is immediately apparent that the tank will now empty very quickly, with all the fluid undergoing a significant acceleration during the process. This particular illustration is a deliberate caricature to emphasize the role that fluid acceleration plays in fluid transients. In true fluid transients at least one of two kinds of fluid acceleration, temporal or convective, will be a significant factor in any energy principle that is utilized in an analysis.

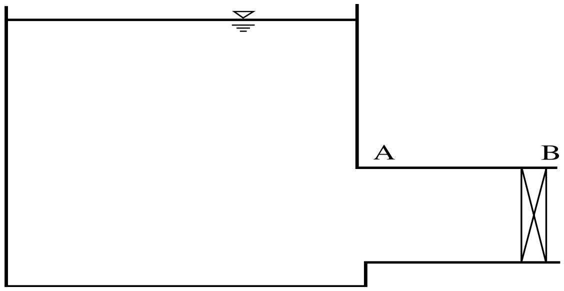


Figure 7.2 A change in dimensions leads to a flow which is not quasi-steady.

We turn now to a quasi-steady analysis of the flow from a generalization of the tank or reservoir in Fig. 7.1 to allow the cross-sectional area A of the tank to vary with elevation, as Fig. 7.3 depicts. A pipe with properties described by the Darcy friction factor f, length L, diameter D, and cross-sectional flow area A_p conveys a discharge Q

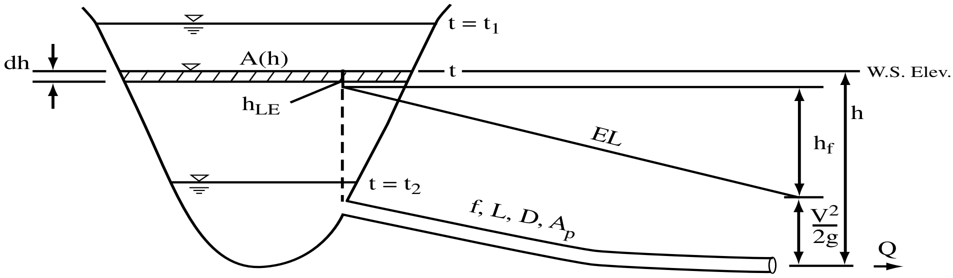


Figure 7.3 Quasi-steady flow from a tank of arbitrary cross-section.

from the tank. In the figure the EL is drawn for the time instant t. In passing from the tank to the atmosphere through the pipe, the fluid undergoes a local entrance loss h_{LE}, a Darcy pipe friction loss h_f, and exits with a mean velocity head $V^2/2g$. The sum of these terms is the instantaneous head h recorded in Eq. 7.1. The loss coefficient K_E for

$$h = \frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g} + K_E\frac{V^2}{2g} = \left[1 + K_E + f\frac{L}{D}\right]\frac{V^2}{2g} \tag{7.1}$$

the entrance loss is representative of any local loss, or sum of local losses, encountered by the flow between the tank and the exit; all such losses are treated identically. From Eq. 7.1 the exit velocity is

$$V = \frac{\sqrt{2gh}}{\left[1 + K_E + f\frac{L}{D}\right]^{1/2}} \tag{7.2}$$

and the discharge is

$$Q = VA_p = \frac{A_p\sqrt{2gh}}{\left[1 + K_E + f\frac{L}{D}\right]^{1/2}} \tag{7.3}$$

If we apply the continuity principle between the tank water surface and the pipe exit, we equate the fluid volume that exits from the pipe over the small time interval dt to the amount of fluid that is removed from the tank over that interval, and

$$Qdt = A(-dh) \tag{7.4}$$

The minus sign is needed because the quantity on the left side is intrinsically positive, but dh is itself negative as the water surface elevation drops with time. Thus we find the time interval $t_2 - t_1$ for the water surface elevation to change from h_1 to h_2 is

$$t_2 - t_1 = \int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2} \frac{Adh}{Q} \tag{7.5}$$

in which the area $A = A(h)$ is in general a function of the tank configuration, and the discharge $Q = Q(h)$ from the steady-flow work-energy principle, Eqs. 7.1-7.3. One common additional notational simplification is to let

$$\frac{1}{C} = \left[1 + K_E + f\frac{L}{D}\right]^{1/2} \tag{7.6}$$

so the discharge can be written in the form of the standard orifice equation

$$Q = CA_p\sqrt{2gh} \tag{7.7}$$

Now we apply these equations to determine the time that is required to drain partially a tank of constant cross-sectional area $A = A_o$. This situation includes the common case of a cylindrical tank of fixed diameter with a vertical centerline, and it also includes tanks having square and other cross-sectional shapes. In this case the time to drain the tank from level h_1 to level h_2 is

$$\Delta t = t_2 - t_1 = -\int_{h_1}^{h_2} \frac{Adh}{CA_p\sqrt{2gh}} = \frac{-A_o}{CA_p\sqrt{2g}}\int_{h_1}^{h_2} h^{-1/2}dh = \frac{-2A_o}{CA_p\sqrt{2g}}\left[\sqrt{h_2} - \sqrt{h_1}\right] \tag{7.8}$$

in which the removal of C from within the integral is only permissible when f is constant. If the top and bottom of this fraction are multiplied by the common factor $\left[\sqrt{h_2} + \sqrt{h_1}\right]$, an interesting practical interpretation of this result is obtained:

$$\Delta t = \frac{A_o(h_1 - h_2)}{\frac{1}{2}CA_p\sqrt{2g}\left[\sqrt{h_1} + \sqrt{h_2}\right]} = \frac{Volume}{Average\ Q} \tag{7.9}$$

In words, the elapsed time is the ratio of the tank volume that is emptied to the average of the discharges that occur at the beginning and end of the time period, a result that can aid computations and is intuitively appealing. For this result to be valid, however, the cross-sectional area and also the friction factor that is a part of C must remain constant throughout the draining process.

If either of the foregoing restrictions does not hold, the integral in Eqs. 7.5 and 7.8 will not simplify as it did in Eq. 7.8. For example, if the cylindrical tank is laid on its side, then A(h) no longer is constant. It is then possible (but not very practical) to evaluate the resulting expression as an elliptic integral (Byrd and Friedman, 1971), but it is normally more convenient just to evaluate Eq. 7.5 by use of some numerical integration procedure; the Trapezoidal rule or the more accurate Simpson's rule (Press et al., 1992) are just two of many possibilities. Closed-form solutions are also known to exist for certain area variations A(h) with a vertical centerline, specifically the cone, pyramid and paraboloid, but the form of these solutions is algebraically more complex and of limited utility.

The flow defined in Fig. 7.3 can be made more general by allowing a nonzero constant inflow Q_o at the top of the tank. We will again write the outflow from the pipe in the form of Eq. 7.7. At first glance there appear to be two inflow cases, one with $Q_o > Q$ and the water surface in the tank rises, and the other with $Q_o < Q$ and the water surface falls. Such turns out not to be the case, for an individual consideration of each case leads to the restatement of Eq. 7.4 for both possibilities as

$$Adh = (Q_o - Q)dt \tag{7.10}$$

If we again assume that $A = A_o$ and f are constants, then Eqs. 7.7 and 7.10 lead to

$$dt = \frac{A}{Ca\sqrt{2g}} \cdot \frac{dh}{\frac{Q_o}{Ca\sqrt{2g}} - \sqrt{h}} \tag{7.11}$$

With integration between the same limits as in Eq. 7.8, we obtain

$$\Delta t = \frac{2A}{\left(Ca\sqrt{2g}\right)^2} \left\{ Q_1 - Q_2 - Q_o \, ln\left[\frac{Q_o - Q_2}{Q_o - Q_1} \right] \right\} \tag{7.12}$$

after some care in integration and several lines of algebra. This result, however, is only valid if Q_o is outside the discharge interval (Q_2, Q_1); otherwise Eq. 7.12 will lead to the logarithm of a negative number. The cause of this behavior is not difficult to understand. During the outflow process the discharge Q takes on all values between Q_1 and Q_2. If Q_o were one of these intermediate values, then an equilibrium between inflow and outflow in the tank would occur at that discharge, the unbalanced driving force for the transient would cease, and the process would not continue on to state 2. Moreover, if the inflow were to match either Q_1 or Q_2, then Eq. 7.12 predicts that the time interval that is required to reach the end state is infinite (i.e., a steady equilibrium is never quite reached, according to this representation of the flow).

One additional generalization that can be useful is to allow the inflow to be $Q_o(t)$, a time-varying inflow. A re-organization of Eq. 7.10 yields

$$\frac{dh}{dt} = \frac{Q_o(t) - Q(h)}{A(h)} = F(t, h) \tag{7.13}$$

in which $F(t, h)$ is simply a shorthand, functional representation of the formula that precedes it. Only a little effort is needed to convince oneself that this equation can not be integrated directly as a quadrature. Press et al. (1992) present a chapter on various alternatives in integrating ordinary differential equations, and others have written entire books; Appendix A on numerical methods presents the fourth-order Runge-Kutta formula (Section A.4.2) as one reliable way to solve this kind of problem. The formula in the appendix is described in terms of the variables $y(x)$ which replace the variables $h(t)$ here.

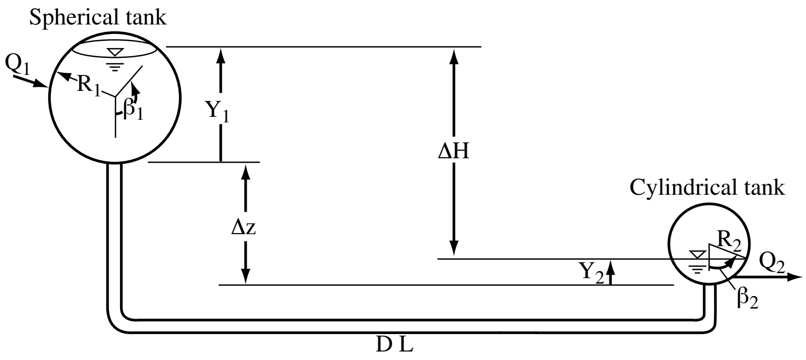
The concept of quasi-steady flow can be applied to a variety of system configurations, including some which are much more extensive than the cases discussed here, so long as it is correct to assume that no large accelerations are present in the transient. In fact, the extended-time simulations in Chapter 6 to determine long-term variations in network demand are quasi-steady flow applications. In such cases the time dependency will only enter the problem through the mass conservation statement.

Example Problem 7.1

A spherical tank of internal radius $R_1 = 20$ ft supplies water to a horizontal cylindrical tank of internal radius $R_2 = 15$ ft and length $L = 20$ ft through an 8-in-diameter pipe that is 500 ft long ($e = 0.004$ in and $\nu = 1.217 \times 10^{-5}$ ft^2/s). The base of the cylindrical tank is 25 ft lower than the bottom of the spherical tank. A constant inflow $Q_1 = 0.5$ ft^3/s enters the spherical tank, which has a small opening to admit atmospheric pressure (14.7 lb/in^2 absolute) to the top of the tank. The cylindrical tank is closed; at time $t = 0$ the pressure of the air over the water is 13 lb/in^2. In this problem assume air behaves isothermally with a temperature of 60^{o}F. The external discharge Q_2 leaving the cylindrical tank is described by data in the following table, which should be converted into a continuous function by using a cubic spline function.

Time, s	0	500	1000	1600	2000	2400	3000	3600
Q_2, ft^3/s	0.00	3.27	3.85	4.68	5.00	4.80	4.00	0.00

If at time $t = 0$ the water depth in the spherical tank is $Y_{10} = 30$ ft and in the cylindrical tank is $Y_{20} = 8$ ft, determine the discharge between the tanks and the water depths and volumes in each tank over one hour (3600 s) in increments of 30 seconds.



First we must establish some relationships between the volume and depth in each tank. In the spherical tank the differential volume element $d\forall$ is a thin circular slice which can be written as $d\forall = \pi r^2 dY = \pi R^2 \sin^2 \beta_1 \, dY$ with $\cos\beta_1 = 1 - Y/R$, from which we find $dY = R \sin\beta_1 \, d\beta_1$, and the differential volume becomes

$$d\forall = \pi R^3 \sin^3 \beta_1 \, d\beta_1$$

If this expression is integrated over the range of β_1 from 0 to π, we obtain the entire spherical volume $\forall = 4\pi R^3/3$. Partly full volumes can than be expressed as a function of β_1 by integrating from 0 to β_1 to obtain

$$\forall = (\pi R^3/3)[2 - \cos\beta_1 (\sin^2\beta_1 + 2)]$$

With the aid of the identity $\sin^2\beta + \cos^2\beta = 1$ we find the relation $\beta_1(Y)$ between angle and depth is $\sin^2\beta_1 = 2(Y/R) - (Y/R)^2$, which allows us to write the volume of water in the spherical tank directly as a function of depth:

$$\forall = (\pi R^3/3)[3(Y/R)^2 - (Y/R)^3] = \pi Y^2 (R - Y/3)$$

In a similar way the volume as a function of the angle β_2 can be shown to be

$$\forall = LR^2(\beta_2 - \cos\beta_2 \sin\beta_2)$$

with L being the length of the tank and $Y_2 = R_2(1 - \cos \beta_2)$.

Since $d\forall/dt$ represents the net discharge from a tank, the following two ODEs each describe the rate of change of water surface elevation in a tank:

$$\frac{dY_1}{dt} = \frac{Q_1 - Q}{\pi Y_1 \left(2R_1 - Y_1\right)} \qquad \frac{dY_2}{dt} = \frac{Q - Q_2}{2LR_2^2 \left(1 - \cos^2 \beta_2\right)}$$

Here Q_1 and Q_2 are respectively the prescribed inflow and outflow from tanks 1 and 2. The discharge Q in the pipe must also satisfy the hydraulic equation

$$F = \left(\frac{fL}{D} + \sum K_L\right)\frac{Q|Q|}{2gA^2} - Y_1 + \Delta z + Y_2 + \frac{M p_o}{\gamma \rho_o V_{air}} = 0$$

The last term is the pressure head created by the air pressure above the water surface in the tank (more on this topic can be found in Secs. 12.5 and 13.2), in which M is the air mass in the tank, γ is the specific weight of water, p_o is the initial absolute air pressure, and ρ_o is the corresponding air density, found from the perfect gas law.

Program SHPTANK solves this problem. To review the details of its structure, the reader should obtain a listing of it from the CD. Principally it calls two subroutines, SPLINESU to accomplish the cubic spline interpolation, and RUKUST to solve the two ODEs simultaneously. Within subroutine SLOPE that supplies the two derivatives dY_1/dt and dY_2/dt to RUKUST we will find that the hydraulic equation is solved there for the values of Y_1 and Y_2 by use of the Newton method. The input file to solve this problem is

```
20 15 20 30 8 500 0.6666667 0.00033333 1.217E-5 1.5 30 120 32.2 25 420
3556 0.5 8 0 0 500 3.27 1000 3.85 1600 4.68 2000 5 2400 4.8 3000 4 3600 0
```

A portion of the solution is listed in the following table:

Time sec	Y_1 ft	β_2 radians	Y_2 ft	V_1 ft^3	V_2 ft^3	Q ft^3/s
30	29.91	1.086	8.01	28186.7	3032.6	3.414
60	29.82	1.088	8.03	28099.5	3042.2	3.401
90	29.73	1.089	8.06	28012.7	3055.0	3.386
.						
.						
.						
1530	26.18	1.048	7.51	24274.7	2771.1	3.261
1560	26.11	1.045	7.48	24191.8	2752.3	3.266
1590	26.04	1.043	7.44	24108.7	2734.4	3.271
1620	25.96	1.037	7.37	24025.5	2697.0	3.286
1650	25.89	1.030	7.27	23941.5	2646.0	3.309
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3480	21.23	0.908	5.77	18294.4	1902.3	3.334
3510	21.16	0.907	5.76	18209.5	1896.6	3.332
3540	21.09	0.905	5.73	18124.5	1886.4	3.332
3570	21.02	0.902	5.70	18039.5	1872.0	3.334
3600	20.96	0.899	5.67	17954.5	1853.9	3.338

* * *

7.3 TRUE TRANSIENTS

The study of true transient flows must include fluid inertia and may also include, in addition, the elasticity or compressibility of the fluid and the conduit. The analysis of transient flows in either case requires the application of Newton's second law which leads to the Euler equation. In Section 7.3.1 the Euler equation is developed; in Section 7.3.2 it is employed to study several rigid-column flow problems where the fluid inertia must be considered but where elasticity is unimportant and can be omitted. In Section 7.3.3 we briefly investigate the effects of elasticity to demonstrate this additional factor requires a different approach to the solution of such problems.

7.3.1. THE EULER EQUATION

The Euler equation is derived by applying Newton's second law to a small cylindrical control volume of fluid at the pipe centerline, as shown in Fig. 7.4. The resulting equation will apply to one-dimensional flow along the pipeline when we disregard variations in fluid or flow properties across the cross section. Further, the equation will apply to flows of both constant and variable density, so it is valid for both rigid-column and water hammer flows.

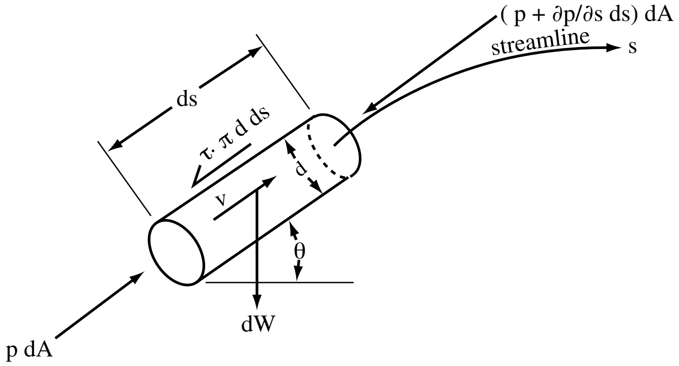


Figure 7.4 A cylindrical fluid element with all forces shown.

Along the streamline direction s, Newton's second law gives

$$\sum F_s = ma_s = m\frac{dv}{dt} \tag{7.14}$$

where m is the fluid mass in the cylindrical fluid parcel. The term dv/dt is in general the total or substantial derivative of the fluid velocity. Substituting the applied forces into this equation and writing the mass in terms of density and volume results in

$$pdA - \left(p + \frac{\partial p}{\partial s}ds\right)dA - dW\sin\theta - \tau\,\pi d(ds) = \frac{dW}{g}\frac{dv}{dt} \tag{7.15}$$

If we divide by dW to produce a non-dimensional equation that is written per unit weight, and the local pipe slope is expressed in terms of distance and elevation along the pipe, we arrive at the one-dimensional Euler equation

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{4\tau}{\gamma d} = \frac{1}{g}\frac{dv}{dt} \tag{7.16}$$

If we now expand the cross-sectional area of the parcel to fill the pipe cross section and introduce the average velocity V, we obtain a more useful equation:

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{4\tau_o}{\gamma D} = \frac{1}{g}\frac{dV}{dt} \tag{7.17}$$

Here τ_o is the shear stress at the wall. Because the wall shear stress is usually not of primary interest and because we will be working almost entirely with cylindrical pipes, we prefer to express the shear stress in terms of the Darcy-Weisbach friction factor f as

$$\tau_o = \frac{1}{8} f \rho V|V| \tag{7.18}$$

The form of the velocity representation in this equation is desirable because it preserves the proper direction of the shear force whenever the flow reverses direction.

With the substitution of Eq. 7.18 into Eq. 7.17 and the assumption that the local elevation of the pipe can be described solely as a function of location s, we obtain the Euler equation of motion

$$\frac{1}{g}\frac{dV}{dt} + \frac{1}{\gamma}\frac{\partial p}{\partial s} + \frac{dz}{ds} + \frac{f}{D}\frac{V|V|}{2g} = 0 \tag{7.19}$$

If we also introduce the piezometric head H via $p = \rho g(H - z)$ and expand the total derivative in the form

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} \tag{7.20}$$

then the Euler equation can be written in the alternative form

$$\frac{1}{g}\frac{\partial V}{\partial t} + \frac{\partial}{\partial s}\left(H + \frac{V^2}{2g}\right) + \frac{f}{D}\frac{V|V|}{2g} = 0 \tag{7.21}$$

since $V\partial V/\partial s = \partial(V^2/2)/\partial s$. The sum of piezometric head and velocity head that appears in the middle term is the sum that is usually displayed in diagrams as the Energy Line.

7.3.2. RIGID-COLUMN FLOW IN CONSTANT-DIAMETER PIPES

The neglect of the elasticity of the pipe and fluid in a pipe of constant diameter forces any change in velocity, in theory, to occur instantaneously throughout the entire pipe. In addition, the steady form of the mass conservation equation applies throughout the pipe so that the velocity everywhere in the pipe is the same at any given time. This section will only examine such flow in single pipes in order to emphasize basic principles; similar transient flows in pipe networks are studied in Chapter 12.

There are relatively few closed-form solutions of Eqs. 7.19 or 7.21, even with these restrictions. One of these solutions describes the development or establishment of flow from rest through a horizontal pipe from a constant-head reservoir, as shown in Fig. 7.5.

In the simpler version of the flow establishment problem we assume that the fluid is inviscid, i.e. without friction, so the last term in Eq. 7.21 is dropped and only the inertia of the fluid is important. First we must choose the limits of integration, with respect to the distance s, to begin the solution of Eq. 7.21; we select section 1 at the upstream end of the pipe (even though the pressure is changing rapidly and the velocity is indeed nonuniform over this section; see Street et al., 1996, pp. 362, 367) and section 2 to be

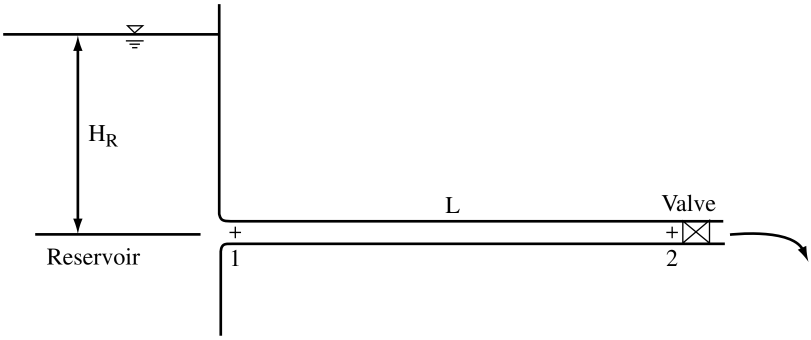


Figure 7.5 Constant-head reservoir with valve at downstream end of horizontal pipe.

just upstream from the valve. Now we formally integrate with respect to the distance s from the reservoir, point 1, to the valve, point 2:

$$\frac{1}{g}\int_{1-2}\frac{\partial V}{\partial t}\,ds = -\int_{1-2}\frac{\partial}{\partial s}\left(H+\frac{V^2}{2g}\right)ds \tag{7.22}$$

Since continuity assures us that all of the fluid in the pipe must undergo the same acceleration, this leads to

$$\frac{L}{g}\frac{\partial V}{\partial t} = -\left(H+\frac{V^2}{2g}\right)_2 + \left(H+\frac{V^2}{2g}\right)_1 \tag{7.23}$$

From Eq. 7.23 we see that the acceleration term is the difference in fluid energy per unit weight, which is also the difference in energy grade line values, between the two end points of the integration. The valve is instantaneously opened fully at $t = 0$, and flow develops thereafter. When $t = 0^+$ at section 2, H_2 drops to zero and remains so, and $V_2 = V$ will grow with time from 0 to the steady state velocity V_0. If a separate Bernoulli equation is now written between the reservoir and section 1, assuming no energy loss, then the sum of the two terms at section 1 is simply the reservoir energy per unit weight or head H_R, a constant. Thus

$$\frac{L}{g}\frac{dV}{dt} = H_R - \frac{V^2}{2g} \tag{7.24}$$

in which the remaining derivative is a function of time only, an ordinary derivative. After the flow has become established, i.e. steady, the left term becomes zero, and the steady velocity V_0 can be found from the remainder of Eq. 7.24 to be

$$V_0 = \left(2gH_R\right)^{1/2} \tag{7.25}$$

To determine the discharge behavior as a function of time during the establishment time interval, we solve Eq. 7.24 for dt and integrate the resulting expression as

$$\int_0^t dt = \frac{L}{g}\int_0^V \frac{dV}{H_R - \frac{V^2}{2g}} \tag{7.26}$$

or

$$t = 2L\int_0^V \frac{dV}{V_0^2 - V^2} \tag{7.27}$$

We can either use partial fractions or a table of integrals to evaluate this expression. Upon some additional algebra, we find the final result as

$$t = \frac{L}{V_0} ln\left[\frac{V_0 + V}{V_0 - V}\right] \tag{7.28}$$

From Eq. 7.28 we learn that, strictly speaking, the flow-establishment time is infinite, since the logarithm does not remain bounded as V approaches V_0. Since this result is not a practical one, it is usual to declare the flow to be steady when $V = 0.99V_0$.

It is more realistic to consider the establishment of flow in the presence of fluid friction and local losses, so we now re-examine this problem. We assume for simplicity that the friction factor remains constant. In this case the integration of Eq. 7.21 with the inclusion of pipe friction yields

$$\frac{L}{g}\frac{\partial V}{\partial t} = -\left(H + \frac{V^2}{2g}\right)_2 + \left(H + \frac{V^2}{2g}\right)_1 - \frac{fL}{D}\frac{V^2}{2g} \tag{7.29}$$

so long as V is always positive. The result is similar to Eq. 7.24, but the evaluation of the energy at sections 1 and 2 now changes to account for the local losses at the entrance and through the valve, respectively. Writing an energy equation between section 1 and the reservoir now produces $H_R - K_E V^2/2g$ as the sum of terms at section 1; in other words, the energy level in the reservoir is now reduced by the local head loss of the entrance as the flow moves to section 1. Between the downstream exit and section 2 a local energy loss occurs at the valve, causing the head to be $H_2 = K_V V^2/2g$ above the datum. With these substitutions and some algebraic rearrangement, we find

$$\frac{L}{g}\frac{dV}{dt} = H_R - \left(1 + K_E + K_V + \frac{fL}{D}\right)\frac{V^2}{2g} = H_R - C_1\frac{V^2}{2g} \tag{7.30}$$

defining C_1 in Eq. 7.30 to shorten subsequent algebra. The steady-state velocity V_0 is in this case found to be

$$V_0 = \left(2gH_R / C_1\right)^{1/2} \tag{7.31}$$

The integration of Eq. 7.30 closely follows the procedure for integrating Eqs. 7.26 and 7.27, but with $2L/C_1$ replacing $2L$ in Eq. 7.27, assuming C_l is constant. The solution is

$$t = \frac{L}{V_0 C_1} ln\left[\frac{V_0 + V}{V_0 - V}\right] \tag{7.32}$$

We see that the time to reach steady flow remains infinite, but the steady-state velocity itself has been reduced by the effects of pipe friction and the local losses. We also see that the deletion of these real-fluid effects leads to $C_1 = 1$ and the previous solution.

Example Problem 7.2

A horizontal pipe 24 inches in diameter and 10,000 ft long leaves a reservoir 100 ft below its surface and ends at a valve. The steady-state friction factor is 0.018 and is assumed to remain constant during the acceleration process.

(a) If the valve is suddenly opened completely, what is the time that is required to attain 99% of the steady-state velocity? Neglect the frictional loss and local losses in this part.

(b) Solve the problem again, including pipe friction but omitting local losses.

(c) Solve the problem again, including pipe friction and using loss coefficients of 0.5 and 5.0 for the entrance and valve, respectively.

(d) Plot the results of (c) to show how the velocity approaches the steady state.

(e) Repeat (c) but allow f to vary, selecting e to produce $f = 0.018$ at steady state.

For part (a) we first use Eq. 7.25 to find the steady-state velocity:

$$V_0 = \left(2gH_R\right)^{1/2} = [2(32.2)(100)]^{1/2} = 80.2 \ ft/s$$

From Eq. 7.28 the time to reach 99% of this velocity is

$$t = \frac{L}{V_0} ln\left[\frac{V_0 + V}{V_0 - V}\right] = \frac{10,000}{80.2} ln\left[\frac{V_0 + 0.99V_0}{V_0 - 0.99V_0}\right] = \frac{10,000}{80.2} ln\left[199\right] = 660 \ s$$

For part (b) we begin by computing V_0 from Eqs. 7.30 and 7.31 with $K_E = K_V = 0$:

$$C_1 = 1 + K_E + K_V + \frac{fL}{D} = 1 + 0 + 0 + \frac{0.018(10,000)}{24/12} = 91.0$$

$$V_0 = \left[2(32.2)(100.0)/91.0\right]^{1/2} = 8.41 \ ft/s$$

From Eq. 7.32 we find

$$t = \frac{L}{V_0 C_1} ln\left[\frac{V_0 + V}{V_0 - V}\right] = \frac{10,000}{(8.41)(91.0)} ln(199) = 69.2 \ s$$

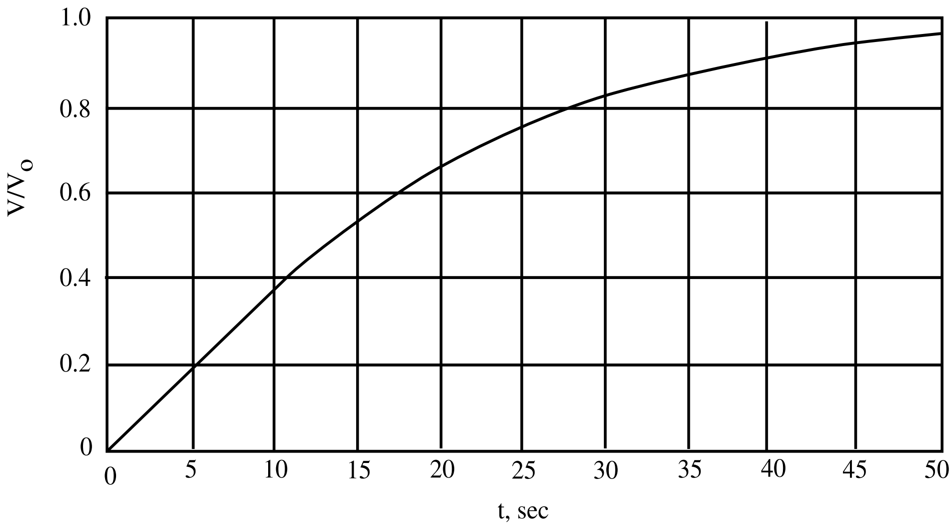
In part (c) we repeat the part (b) calculation with $K_E = 0.5$ and $K_V = 5.0$, leading to $C_1 = 96.5$, $V_0 = 8.17$ ft/s, and a time $t = 67.2$ s for the flow to become established.

A short computer program should be written to solve part (e) with f varying. To determine the correct pipe roughness e for the simulation, Eq. 7.30 with $dV/dt = 0$ is solved simultaneously with the Colebrook-White equation; the results are $e = 0.00129$ ft and $V_0 = 8.169$ ft/s. Then Eq. 7.30 is re-arranged to separate the variables, and a program is written to perform the numerical integration. A program to accomplish the integration is listed below; for the integration the program calls SIMPR, a subroutine that uses Simpson's rule (see Appendix A). The friction factor will be determined by equations in Table 2.2 with a few small changes: if $Re < 100$, then $f = 0.64$; the transitional Colebrook-White formula will be used whenever $Re > 2100$ rather than 4000. Some care must be taken to assure that the right side of Eq. 7.30 never becomes zero or negative;

hence the range of integration for V is from 0 to $0.99V_0 = 8.087$ ft/s. In this instance the time to steady flow is 66.4 s.

```
      EXTERNAL EQUAT
      COMMON SF
      SF=7.5
      CALL SIMPR(EQUAT, 0.0, 8.09, TIME, 1.0E-04, 30)
      WRITE(*,*)  ' Time = ', TIME
      END
C
      FUNCTION EQUAT(V)
      COMMON SF
      RE = 1.6433854E5*V
      IF (RE.LT.100.) THEN
      FR = 0.64
      ELSEIF (RE.LT.2100) THEN
      FR = 64.0/RE
      ELSE
    1 SF1 = SF
      SF = 1.14-2.0*ALOG10(6.3625E-4 + 9.35*SF1/RE)
      IF(ABS(SF-SF1) .GT. 1.0E-06) GO TO 1
      FR = 1.0/SF/SF
      ENDIF
      DEM = 100.0 - (6.5 + 5000.0*FR)*V**2/64.4
      IF(DEM .LT. 0.01) THEN
      EQUAT = 31055.9
      ELSE
      EQUAT = 310.559/DEM
      ENDIF
      RETURN
      END
```

The plot of the results from part (c), requested as part (d), shows the velocity increases rather rapidly until it reaches 80 to 90% of the steady-state value. By that time the acceleration has decreased noticeably, and steady state is approached asymptotically. As part (e) shows, only a short program is needed to add more accuracy to the computation, but in this particular example the difference in time to steady state is only four percent.



From these computations we see that pipe friction is the dominant factor in the flow establishment process when the pipe is sufficiently long. Overlooking this factor would be a severe error, part (a), but for very long pipes the effect of local losses is truly a minor effect, with the the steady-state velocities and flow-establishment times only differing by a few percent in this problem.

* * *

The simulation of flow shutdown by use of Eq. 7.30 for the physical problem depicted in Fig. 7.5 is actually a more difficult problem than the startup problem. The principal difficulty is in representing correctly the loss coefficient K_V for the valve, because it is incorrect to model this coefficient as a constant in this problem. Instead we now have a continually increasing head loss, and loss coefficient, across the valve which with pipe friction (and, to a minor extent, local losses) causes the flow to decelerate and eventually stop. We assume that all loss coefficients under unsteady-flow conditions are unchanged from steady-flow conditions at the same velocity. The head loss in the system will be the pipe friction loss described by the Darcy-Weisbach equation, the local entrance loss, and the valve head loss, as the governing equation, Eq. 7.30, shows. Since K_V varies with the valve setting, which in turn changes in some predetermined manner with time, a closed-form solution of this ODE is not possible. Thus we must solve this nonlinear equation by numerical methods.

Here we choose the fourth-order Runge-Kutta method, described in Appendix A.4.2, as the numerical solution technique for this problem. Equation 7.30 is rewritten in the form

$$\frac{L}{g}\frac{dV}{dt} = H_R - \left(1 + K_E + K_V + \frac{fL}{D}\right)\frac{V^2}{2g} = F(t, V) \tag{7.33}$$

Now the Runge-Kutta method can be applied directly, once the details of computing K_V and $F(t, V)$ are set. To complete the setup, we must know the valve operating schedule (percent open P vs. time) and the relation between K_V and percent open P.

If in addition we wish to know the maximum pressure head to occur (probably at the valve) as time progresses, we can insert the computed velocity in $h_L = K_V V^2/2g$ to find this head. However, one complicating factor occurs at the instant of closure; this loss coefficient becomes infinite as the velocity approaches zero, creating an indeterminate pressure head. Even under the best of circumstances, any numerical procedure will produce unreliable results at this point. Fortunately, the maximum pressure usually occurs somewhat before complete valve closure, so the numerical analysis will be terminated a fraction of a second before complete closure. Example Problem 7.3 presents the solution process for the reservoir-pipe system of Fig. 7.5.

Example Problem 7.3

The reservoir head on the pipeline in Fig. 7.5 is 60 ft. The 12 in-diameter line is 3000 ft long with an equivalent roughness $e = 0.012$ in. Since the valve has been fully open for a long time, the flow of water is steady.

(a) Calculate the steady-state velocity in the line assuming there is no loss at the valve. Then compute the maximum pressure in the line if the valve closes so that the rate of decrease in velocity is linear in time from its steady-state value to zero in 20 sec.

(b) Now assume the valve at the downstream end is a GA Industries 12-in globe valve whose loss characteristics are given in Appendix C.1 as a function of valve opening. Compute again the steady-state velocity with the valve fully open. Assuming the valve closes in 20 sec at a rate that is linear in time, find the maximum pressure in the line.

(c) Repeat part (b) but employ a cubic spline interpolation to represent the valve data.

To begin part (a) and find the steady-state velocity, we can apply Eq. 7.30 directly with $dV/dt = 0$ and $K_V = 0$:

$$H_R = \left(1 + K_E + K_V + \frac{fL}{D}\right)\frac{V_0^2}{2g}$$

$$60 = \left(1 + 0.5 + 0 + \frac{f(3000)}{1.0}\right)\frac{V_0^2}{64.4}$$

If we solve this equation with the Colebrook-White equation, we obtain V_0 = 7.91 ft/s and $f = 0.0201$. The linearly decreasing velocity creates a constant deceleration so that

$$\frac{L}{g}\frac{dV}{dt} = \frac{3000}{32.2}\frac{(-7.91)}{20} = -36.8\ ft$$

Now we can apply Eq. 7.29 from section 1 to 2. Since $(H + V^2/2g)_1 = H_R - K_E V^2/2g$ and $H_2 = z_2 + p_2/\gamma = 0.0 + p_2/\gamma$, we have

$$\frac{L}{g}\frac{dV}{dt} = -\left(\frac{p_2}{\gamma} + \frac{V^2}{2g}\right) + \left(H_R - K_E\frac{V^2}{2g}\right) - \frac{fL}{D}\frac{V^2}{2g}$$

$$\frac{L}{g}\frac{dV}{dt} = -\left(\frac{p_2}{\gamma}\right) + H_R - \left(1 + K_E + \frac{fL}{D}\right)\frac{V^2}{2g}$$

$$-36.8 = -\left(\frac{p_2}{\gamma}\right) + 60 - \left(1 + 0.5 + \frac{f(3000)}{1.0}\right)\frac{V^2}{64.4}$$

or

$$\frac{p_2}{\gamma} = 96.8 - (1.5 + 3000f)\frac{V^2}{64.4}$$

From this equation we see clearly that the pressure head at the valve increases as the velocity decreases, reaching a maximum at the instant of closure. We conclude that

$$\left(\frac{p_2}{\gamma}\right)_{max} = 96.8\ ft \qquad or \qquad (p_2)_{max} = 96.8\left(\frac{62.4}{144}\right) = 42.0\ lb/in^2$$

In part (b) we repeat the sequence of computations in part (a) but represent the hydraulic behavior of the valve more accurately. We start by again computing the steady-state velocity, but this time we include the actual valve loss in the computation. Following Appendix C.1, we write

$$K_L = K_V = 890\frac{D^4}{C_V^2}$$

and for the 12-inch globe valve we find in the second table of this appendix C_V = 1750, leading to

$$K_V = 890 \frac{12^4}{1750^2} = 6.0$$

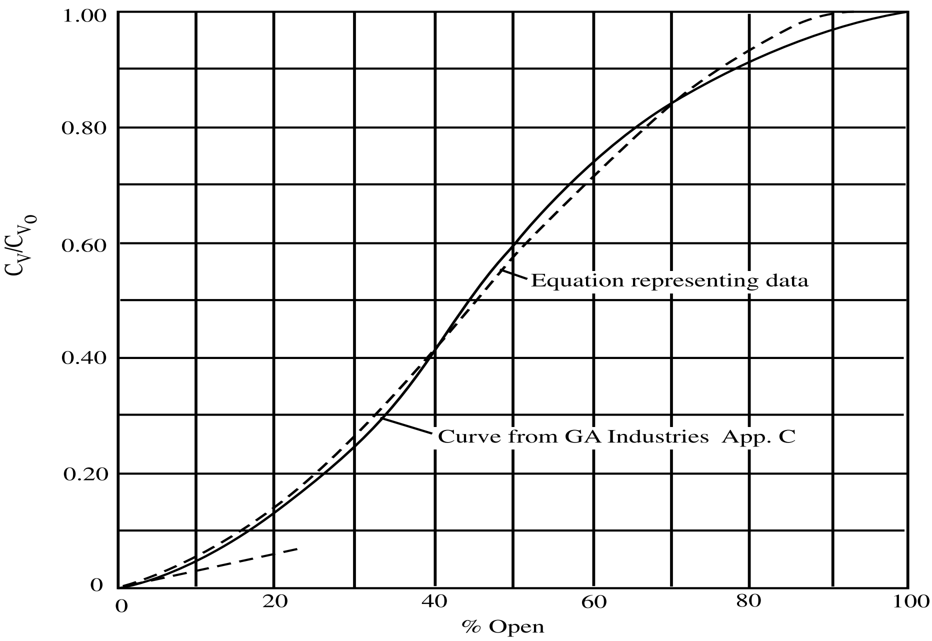
We return to the first two equations of the solution for part (a) and replace $K_V = 0$ with $K_V = 6.0$ there to obtain the steady velocity as $V_0 = 7.55$ ft/s and $f = 0.0201$.

Before we can apply the Runge-Kutta method, we must devise a way to determine the value of K_V as a function of time. The valve opening will be prescribed at any time by the closure schedule; in this case we assume a linear behavior. It only remains to find K_V for any given opening. Because the data for the GA Industries valve are given in terms of C_V, we will determine C_V at a given opening and convert to the corresponding K_V by using the formula from Appendix C.1. The data pairs of P = percent open vs. C_V/C_{V0} in the following table (C_{V0} is the fully open value) are read from Appendix Fig. C.1:

P	0	10	20	30	40	50	60	70	80	90	100
C_V/C_{V0}	0.00	0.055	0.13	0.26	0.42	0.60	0.74	0.83	0.92	0.97	1.00

From line two in this table we clearly see at the instant of valve closure that $C_V = 0.0$, with $K_V \to \infty$. We will avoid this problem by halting the numerical analysis two time increments before the valve closure is complete. If the maximum pressure head occurs before complete closure, little will be lost when the analysis is terminated then.

To develop an equation to fit the tabular data, we first replot the data. The fitting equation must capture the point of inflection that is seen in the plot. Hence we select a



cubic polynomial as the simplest function which will satisfy this requirement. The equation is

$$C_V = C_{V0}\left[aP^3 + bP^2 + cP + d\right]$$

where P is the measure of the valve position and a, b, c, d are the fitting coefficients.

The conditions we will impose on the polynomial to find the four coefficients are

C_V/C_{V0}	= 0	when	P = 0
C_V/C_{V0}	= 1.0	when	P = 100
Slope	= 0.003	when	P = 0
C_V/C_{V0}	= 0.42	when	P = 40

The third condition seems at first glance to be an odd requirement. However, a valve only generates enough head loss to decelerate the flow significantly when it is very close to complete closure. Hence, it is important to represent the empirical curve as accurately as possible near the closure point. One good way to assure that the shape of the curve near closure is accurate is to measure the slope of the plot at closure and then specify this slope.

Application of these four conditions results in the following equation, which is also plotted on the graph as a dashed curve:

$$C_V = C_{V0}\left[-1.96\times10^{-6}P^3 + 2.66\times10^{-4}P^2 + 3.00\times10^{-3}P\right]$$

The creation of this equation to represent the flow coefficient over the full range of valve motion suggests that Eq. 7.33 could be integrated to yield a closed form solution. However, the variables in the equation do not separate. Further, if the valve closure schedule is more complex than the motion of this example, the relation between C_V and time will become progressively more complex. Hence we will proceed with a numerical solution as the most broadly applicable approach.

Program VALCLO1, to be found on the CD, generates a solution for this example problem for the specified conditions; in it the valve coefficient is modeled by the third-order polynomial. This data file will generate the solution:

```
EXAMPLE PROBLEM 7.3 RIGID COLUMN THEORY VALVE CLOSURE
&SPECS HR=60.,VZERO=7.57,D=12.,L=3000.,E=0.012,KE=0.5,TCLOSE=20.,
       DELT=1.0,A=-0.00000196,B=0.000266,C=0.00300,CVZERO=1750./
```

An increment $\Delta t = 0.10$ sec ought to produce accurate numerical results. To check this assumption, additional runs with $\Delta t =$ 0.50, 0.25, and 0.05 sec. were made, and $\Delta t =$ 0.10 sec does produce a solution that is independent of Δt. The solutions confirmed a maximum pressure head of 228 ft occurring about 18 sec after beginning the 20-sec valve closure. Some output from the computer analysis follows, using a print interval of 1.00 sec to conserve space:

```
              ***************
              * INPUT DATA  *
              ***************

          EXAMPLE PROBLEM 7.3 - RIGID COLUMN THEORY VALVE CLOSURE
                HR =    60.0 FT
             VZERO =    7.57 FT/S
                 D =   12.00 IN
                 L =  3000.0 FT
                 e =   0.012 IN
```

```
        KE =  0.50

    TCLOSE =   20.0 SEC
      DELT =  1.000 SEC

         A = - 0.196E-05
         B =   0.266E-03
         C =   0.300E-02
    CVZERO =  1750.0

        ***********
        * RESULTS *
        ***********

  TIME,SEC  V,FT/S  PRESSH,FT
  --------  ------  ---------
     0.00     7.57       5.4
     1.00     7.57       5.3
     2.00     7.56       5.4
     3.00     7.56       5.6
     4.00     7.55       6.1
     5.00     7.54       6.6
     6.00     7.52       7.5
     7.00     7.50       8.6
     8.00     7.46      10.2
     9.00     7.42      12.4
    10.00     7.35      15.6
    11.00     7.25      20.1
    12.00     7.11      26.8
    13.00     6.91      37.1
    14.00     6.60      53.3
    15.00     6.12      79.1
    16.00     5.37     118.9
    17.00     4.22     172.3
    18.00     2.62     215.0
    19.00     0.98     194.6
```

An alternate approach to the construction of the cubic-polynomial to represent the valve coefficient is to apply a spline fit to the coefficient data and otherwise proceed as before. Program VALCLO, also on the CD, implements this approach. The input and output data files for this alternative follow. The input file is

```
EXAMPLE PROBLEM 7.3 RIGID COLUMN THEORY VALVE CLOSURE
&SPECS HR=60.,VZERO=7.57,D=12.,L=3000.,E=0.012,KE=0.5,TCLOSE=20.,
       DELT=1.,CVZERO=1750./
0 0 10 0.055 20 0.13 30 0.26 40 0.42 50 0.6 60 0.74 70 0.83 80 0.92 90
0.97 100 1.0
```

The output file is

```
      **************
      * INPUT DATA  *
      **************

  EXAMPLE PROBLEM 7.3 - RIGID COLUMN THEORY VALVE CLOSURE

        HR =   60.0 FT
     VZERO =   7.57 FT/S
         D =  12.00 IN
```

```
         L = 3000.0 FT
         e =  0.012 IN
        KE =  0.50

    TCLOSE =   20.0 SEC
      DELT =  1.000 SEC

    CVZERO =  1750.0

        ***********
        * RESULTS *
        ***********

  TIME,SEC  V,FT/S  PRESSH,FT
  --------  ------  ---------
       .00    7.57        5.4
      1.00    7.57        5.5
      2.00    7.56        5.7
      3.00    7.55        5.9
      4.00    7.54        6.3
      5.00    7.53        6.9
      6.00    7.51        7.7
      7.00    7.49        8.5
      8.00    7.46        9.5
      9.00    7.42       11.2
     10.00    7.37       14.1
     11.00    7.28       19.0
     12.00    7.14       27.1
     13.00    6.92       39.6
     14.00    6.57       59.8
     15.00    5.99       93.8
     16.00    5.06      141.6
     17.00    3.77      175.0
     18.00    2.40      177.4
     19.00    1.14      169.8
```

The differences in pressures between the two solutions are clearly a consequence of the differences in the numerical representation of the valve coefficient toward the end of the closure schedule. As the valve approaches its closed position, the loss coefficient K_V, which varies inversely with C_V, increases very rapidly. The functional relationship chosen to represent C_V can have a significant effect on the value of K_V and thus on the pressure. Since the maximum value of the pressure is usually an important part of an analysis, it is imperative to try to define C_V as accurately as possible near closure. Valve closure data from manufacturers is generally provided in a table or graph, which works very well for steady-state analyses. But the precise nature of the change in C_V near closure, which is needed for an accurate analysis of unsteady flow behavior, is rarely available. As a consequence, practitioners must use their judgment and experience in modeling the valve closure and appraising in a conservative way the results of the analysis.

7.3.3. WATER HAMMER*

When velocities in a pipe system change so rapidly that the elastic properties of the pipe and liquid must be considered in an analysis, we have a hydraulic transient commonly known as water hammer. While this type of analysis is more complex than a rigid-column

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* Material in this section on pp. 295-299 is adapted from Elementary Fluid Mechanics, by R. L. Street, G. Z. Watters, and J. K. Vennard, Ed. 7, Copyright 1996 by John Wiley & Sons, Inc. Reprinted by permission.

analysis, it more accurately represents the actual behavior of the flow. Before we embark on an extensive investigation of this type of phenomenon in the next chapter, we will look at a simple water hammer situation. The problem will help us to see what happens in a pipe when velocities change rapidly, and it will introduce fundamental concepts that are important in understanding the phenomenon.

With the aid of the simple pipeline and valve that is attached to the reservoir in Fig. 7.6, we can now observe how water hammer waves evolve in time, according to our simplified equation set. We assume that steady flow occurs in the pipe at velocity V. The piezometric head everywhere in the pipe is H in the absence of friction. If the valve setting is changed in any way, a transient will be caused in the pipe, both upstream and downstream of the valve; we will concentrate only on the pipe section that is upstream.

Now assume we can completely close the valve rapidly, indeed instantaneously. At the valve the water velocity is suddenly forced to zero. As a consequence the head at the valve abruptly increases by an amount $\Delta H = aV/g$, as Chapter 8 will show. The amount of this increase is just sufficient to reduce the momentum of the moving water to zero.

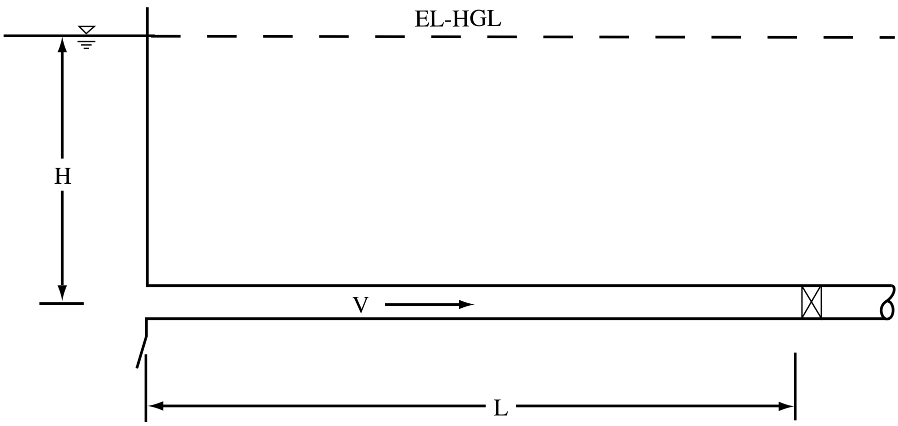


Figure 7.6 Steady flow from a reservoir in the absence of friction.

The increased head immediately creates two other changes at the valve; the pressure increase slightly enlarges the pipe and also increases the density of the fluid. The amount of the stretching of the pipe depends on the diameter and thickness of the pipe and on the compressibility of the pipe material and the liquid, but it normally changes by less than one-half percent. In Fig. 7.7 the amount of the deformation is exaggerated.

The rise in pressure head causes a sharp-fronted pressure wave to propagate upstream at speed a, the magnitude of which is a function of properties of the conduit and the fluid. This wave speed remains constant until the conduit properties or the fluid properties change. The wave front reaches the reservoir L/a seconds after valve closure. At that instant the velocity is zero throughout the pipe, the pressure head is everywhere $H + \Delta H$, the pipe is enlarged and the fluid is compressed.

Under these conditions the fluid in the pipe near the reservoir connection is locally not in equilibrium since the reservoir pressure head is only H. Hence fluid begins to flow toward the region of lower head (the reservoir) as the distended pipeline forces flow in that direction. In the absence of friction this left-ward velocity is equal in magnitude to the original steady velocity as it is driven by the same head increment ΔH; and the source of the liquid for this flow is the compressed liquid that is stored in the enlarged pipe cross section under the increased pressure head.

The process continues to evolve with time. At time $2L/a$ after the beginning, the pressure throughout the pipe has returned to its original value, but with the velocity reversed from its original direction. At this instant the store of compressed liquid is

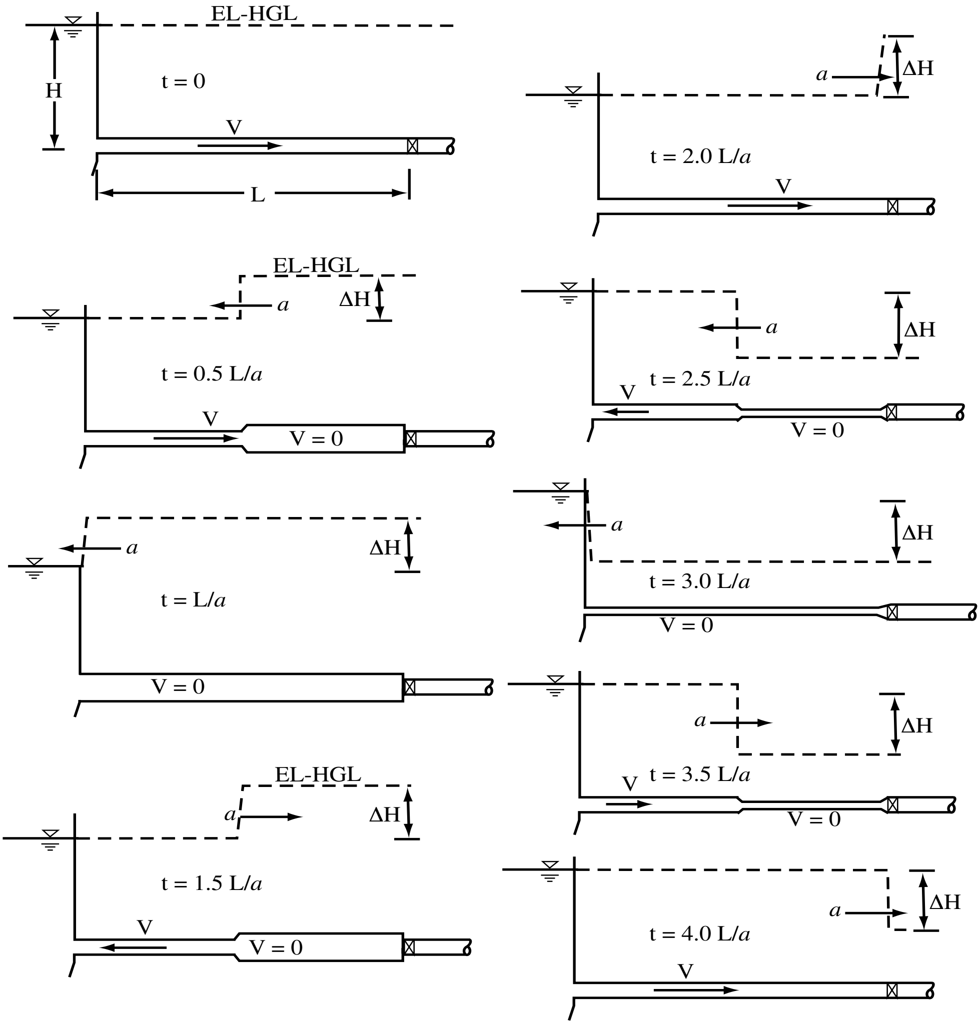


Figure 7.7 Evolution of a transient pressure wave in the pipe in Fig. 7.6.

exhausted, and the pressure wave appears to undergo a reflection. That is, the pressure head drops an amount ΔH below the original steady head, and this pressure drop and the closed valve cause the velocity behind the wave front to return to zero. Behind this negative wave the pipe cross section shrinks and the liquid expands.

By time $3L/a$ this negative wave has reached the reservoir, and the velocity is everywhere zero. However, the pressure head at the reservoir is again not in equilibrium with the reservoir head, so fluid is drawn from the reservoir into the pipe at velocity V. Behind the new, advancing wave the head is in equilibrium with the reservoir head.

At time $4L/a$ the wave has reached the valve; at this instant all variables have returned to the original steady state that existed before the valve was closed. This time interval that has just been described is one full cycle in a hydraulic transient that would, in the absence of friction, continue without abating. The simple fact that this sequence of events is unending, unless friction is present, points out the necessity of retaining the otherwise

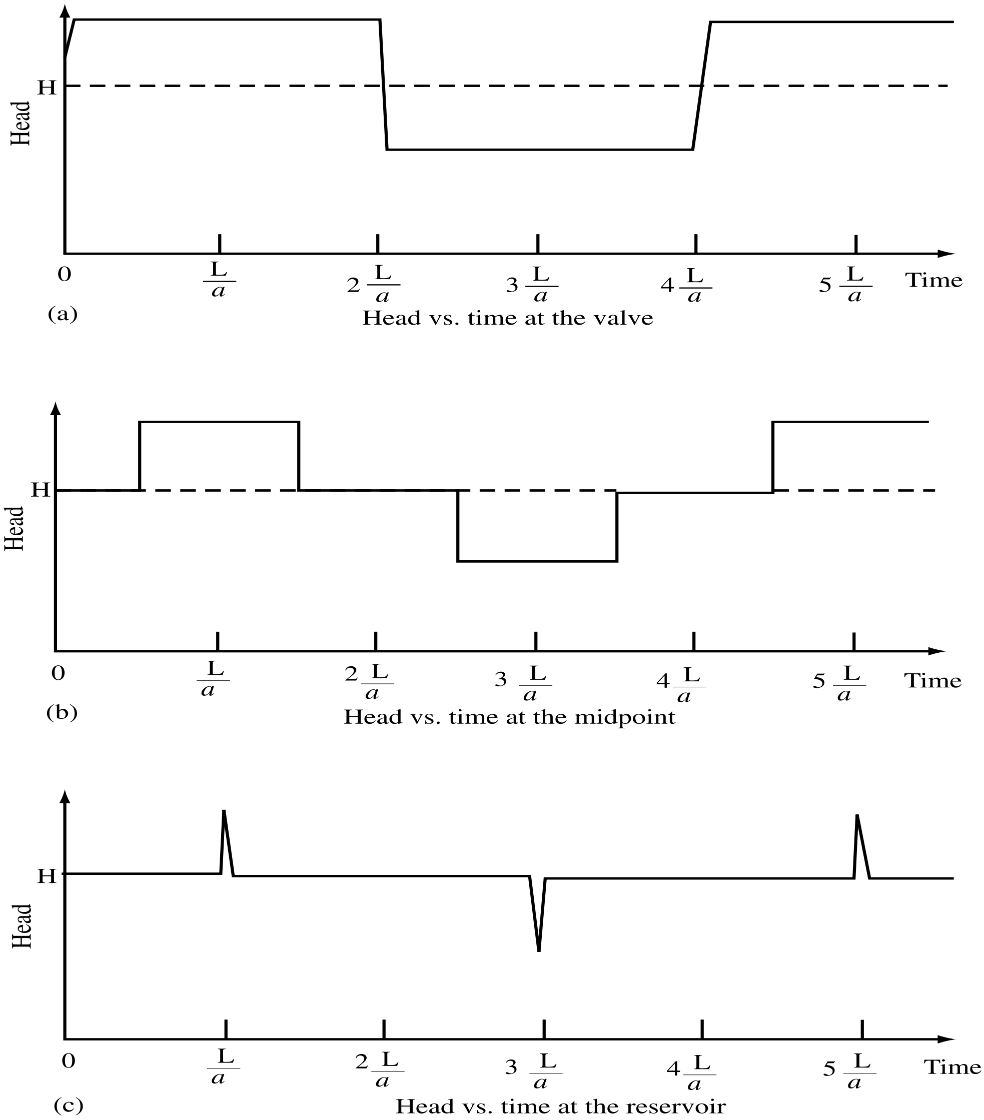


Figure 7.8 Head vs. time at three locations.

small friction term if we are to achieve realistic, practical results in our simulations of such events.

If the variation of piezometric head is plotted as a function of time for selected locations along the pipe, as is done in Fig. 7.8, we can use these results to infer several additional fundamental features. For example, these plots make clear that it is not time alone, but time in units of L/a, that describes the head variation at a point in the most meaningful way. In Fig. 7.8a we see that the head takes on only the values $H + \Delta H$ and $H - \Delta H$

with abrupt transitions, but intermediate points such as the midpoint of the line (see Fig. 7.8b) also have intervals when the head is H itself.

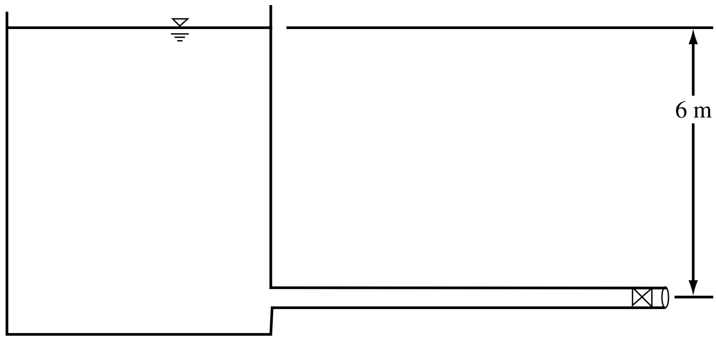
From Figs. 7.8b and 7.8c we see that the disturbance is not initially noticed everywhere; instead the fact that the valve is suddenly closed travels at the finite wave propagation speed a to the other locations and arrives at the midpoint in the pipe in half the time that is needed to traverse the entire pipe. The sudden increase in head at the valve, shown in Fig. 7.8a, remains in place at the valve until one round-trip wave travel time has elapsed, and only then is the returning wave from the reservoir able to reduce the head there. In fact the head increment ΔH need not be created instantaneously for the full incremental head to be present at the valve; it is only necessary for a set of incremental increases in head, which sum to ΔH, to be developed at the valve in a time interval that is less than this round-trip travel time $2L/a$ for the full increment in head ΔH to be present at the valve for a while. We shall later see that, owing to the manner in which a valve decreases the discharge in a pipeline by creating large head losses, it may be necessary to close a valve in a time that is much greater that $2L/a$ if we are to avoid the creation of large transient pressures.

7.4 PROBLEMS

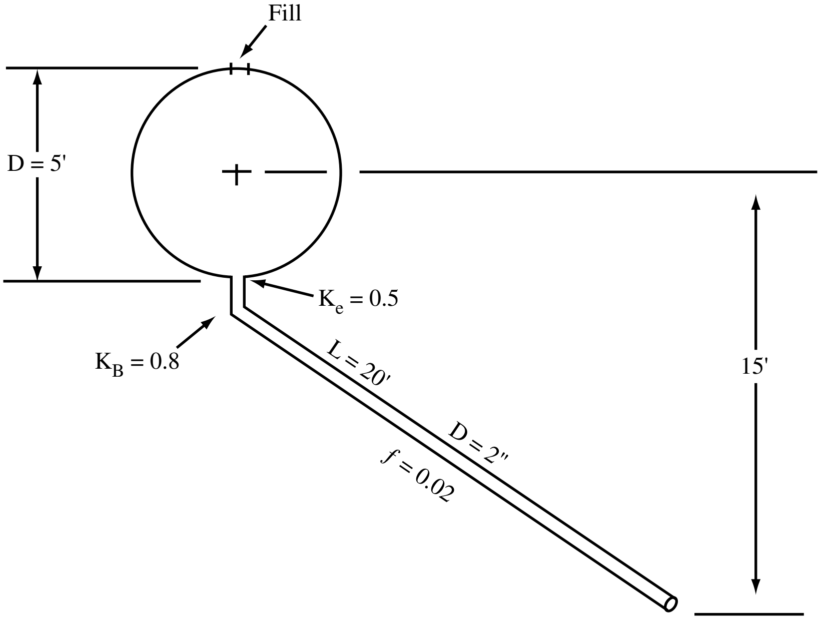
7.1 The water storage tank shown below is square with a side length of 4 m. Initially it is filled to a depth of 6 m. The exit pipe is 20 m long; its diameter is 5 cm, and the pipe entrance is sharp-edged. Assume quasi-steady flow.

(a) If the friction factor is 0.02, how much time is required for the water surface to fall 2.0 m? Plot the water surface elevation as a function of time during this fall.

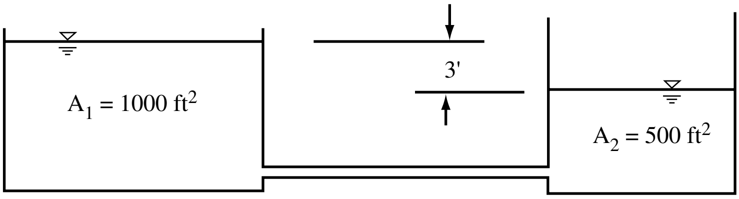
(b) Repeat part (a), but now assume that the pipe is smooth, new PCV pipe. How much do the results change?



7.2 A cylindrical tank 10 ft. long with an outlet pipe is shown below. The tank is filled with water through an opening at the top. Find the time Δt, to within approximately 15 sec, to empty the tank completely. Also plot the water surface elevation vs. time to a reasonable scale.

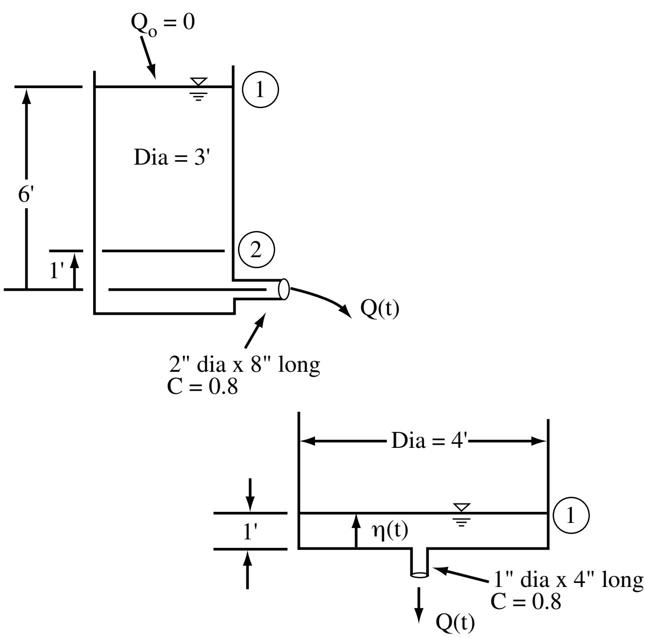


7.3 The two tanks shown atop the next page in cross section are connected by 50 feet of 18-inch-diameter cast iron pipe with a sharp-edged entrance and exit. The initial difference in water surface elevation is 3 ft. Assuming quasi-steady flow and a friction factor of 0.017, find the time for the water surfaces to reach equilibrium.



7.4 You are asked to assist in the design of a tank for a client; when this tank is drained (through a bottom orifice of area a and discharge coefficient C), the fall of the water surface is to be linearly proportional to the elapsed time. (That is, if the surface drops one foot in 30 minutes, then it drops another one foot in another 30 minutes.) What should be the shape of this tank?

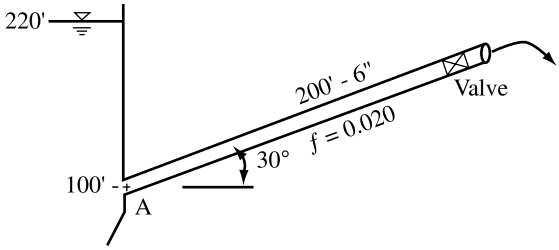
7.5 A two-tank cascade is shown below. Both tanks initially have their outlets closed, and no flow occurs; each water surface elevation is at level 1. At $t = 0^+$ both outlets are opened. When the water level in the upper tank has fallen 5 ft to level 2, the upper outlet is closed. Determine, and plot to a reasonable scale, the water surface elevation $\eta(t)$ so long as η is 1 ft or more.



7.6 Repeat the computations in Example Problem 7.1, assuming now that $R_1 = 15$ ft and $R_2 = 10$ ft.

7.7 A horizontal pipe that is 12,000 ft long and has a 3 ft diameter leaves a reservoir under a head of 125 ft and ends with a valve. Assume a Darcy-Weisbach friction factor of 0.022. If the valve is completely opened and causes no loss, what is the steady state velocity? After sudden opening of the valve, what time interval is required for the fluid velocity to attain (a) 50 %, (b) 99 %, of the steady state velocity?

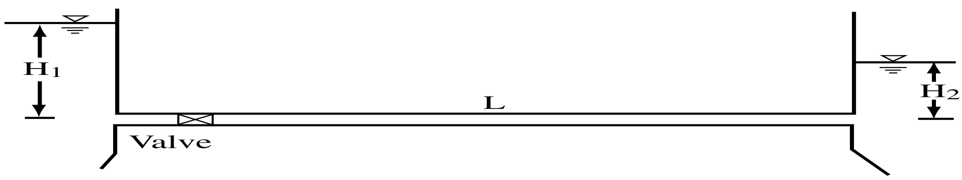
7.8* The pipe shown is initially full of water with the valve completely closed. Compute the time for the velocity to reach 99% of its steady-flow value after the valve is opened suddenly. When the valve is fully open, its head loss is negligible. Neglect the entrance loss.



7.9 For the physical system shown with Problem 7.8, assume an equivalent sand grain roughness for the pipe of e = 0.006 in., allow f to vary and repeat the computation requested in that problem.

7.10 For the same conditions, solve Problem 7.8 with the valve located at section A.

7.11 The pressure head in this horizontal pipeline is H_2 before the valve is opened.



(a) If the valve is opened suddenly, find an equation for the time that is needed for the velocity to reach 99% of its final value. Neglect the head loss across the valve and the entrance loss.

(b) In shutting off flow in the pipeline, the valve is operated so that the fluid velocity decreases linearly with time. The steady-state velocity is 9.92 ft/s, and the time of closure is 100 sec. Find the minimum pressure head in the system, where it occurs, and the time it occurs. Use the following values for this calculation:

L = 3220 ft, H_2 = 100 ft, H_1 = 200 ft, D = 1.0 ft, f = 0.020.

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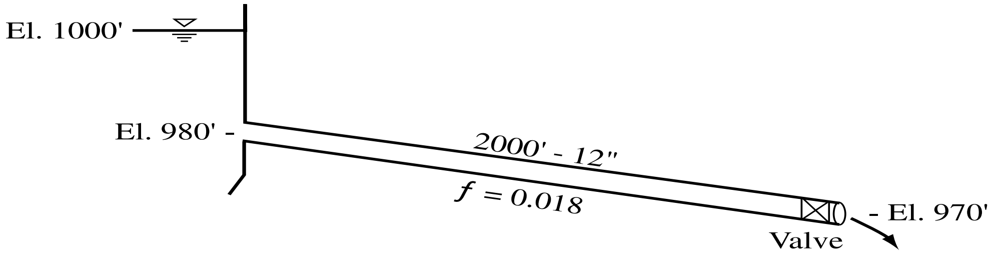
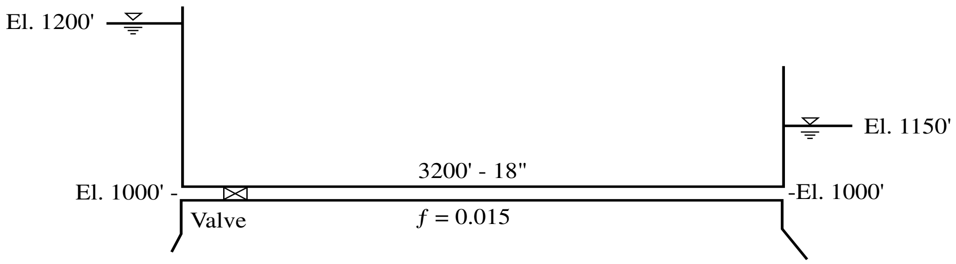
* Material in Problems 7.8 and 7.10-7.16 is adapted from Elementary Fluid Mechanics, by R. L. Street, G. Z. Watters, and J. K. Vennard, Ed. 7, Copyright 1996 by John Wiley & Sons, Inc. Reprinted by permission.

7.12 The globe valve in the pipeline shown is opened instantaneously. If the loss coefficient for the wide-open valve is 3.0, how many seconds are required for the velocity to reach 99% of its final value? Neglect the entrance loss.



7.13 The globe valve in Problem 7.12 is opened instantaneously to establish flow in the pipeline. If the valve's fully-open loss coefficient is 6.3, what is the elapsed time for the velocity to reach 99.9% of its final value? Neglect the entrance loss.

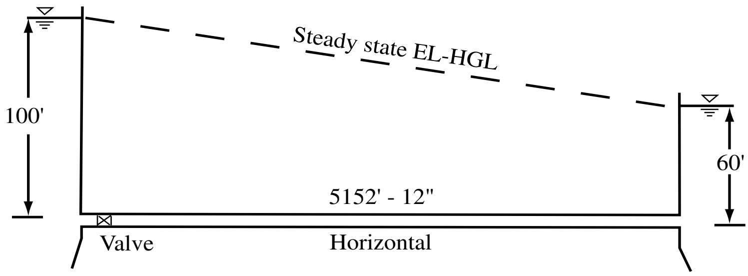
7.14 When the valve in the pipeline shown below is fully open, the steady-state velocity is 9.88 ft/s. Under these conditions the local losses and the valve loss are negligible. The valve is closed in a manner which causes the velocity to decrease linearly with time to 5 ft/s in 10 sec. Find the maximum and minimum pressure heads in the system and where and when they occur. Finally, determine the loss coefficient for the partially-open valve after valve movement has ceased and a steady flow has been re-established.



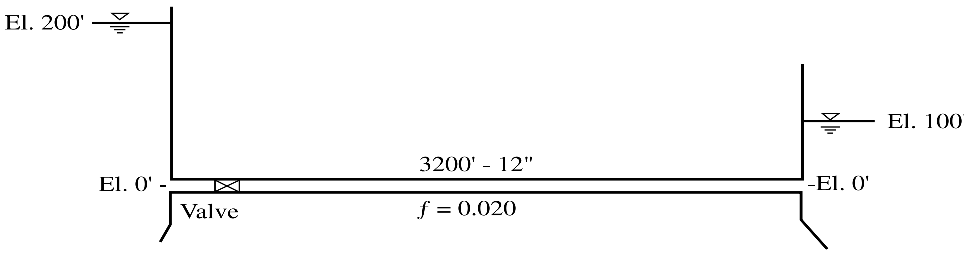
7.15 The valve in the pipeline shown below is closed in such a manner that the velocity follows the quadratic relation

$$V = V_0\left(1 - \frac{t}{T}\right)^2$$

in which the initial velocity is $V_0 = 5$ ft/s, and the time of valve closure is $T = 30$ sec. Find the minimum pressure head in the system and when and where it occurs. Neglect local losses.



7.16 At time zero the valve in the pipeline below is in the closed position. It is proposed to open the valve in such a manner that the velocity will increase linearly with time to its steady-state value of 10 ft/s in 100 sec. Find the maximum and minimum pressure heads occurring in the pipeline for the proposed program of valve movement. Can this operating program work? Explain.



7.17 In the pipeline of Problem 7.8 the valve at the downstream end, which is a GA Industries 6-inch globe valve (see App. C), will be closed linearly in time over 10 sec. Using a time increment of 0.05 sec, compute the maximum pressure which will occur at the valve. Refer to Example Problem 7.3 for numerical procedures and computer programs.

7.18 The valve in the pipeline in Problem 7.12 is a 12-inch Pratt butterfly valve (see App. C). If the valve is closed in 30 sec at an angular rate that is linear in time, calculate the maximum pressure head at the valve. The entrance loss coefficient is 0.50. Use a time increment of 0.10 sec, and refer to Example Problem 7.3 regarding numerical procedures and computer programs.

7.19 Solve Problem 7.18 if the valve in the pipeline is a Pratt 12-inch ball valve (see App. C).