CHAPTER C11

SLURRY AND SLUDGE PIPING

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DEFINITION AND BACKGROUND

Slurry is a mixture of solids and liquid. A sludge denotes a mud or a concentrated slurry having a considerable amount of fine material that imparts high viscosity. Typical examples of slurries are the solid-liquid mixtures encountered in mineral processing plants and dredged material from waterways and dams. Most of the slurries are made up with water. However, industrial paints, rocket fuel, coal-oil mixture, and coal-methanol slurries are made up with liquids other than water.

River sediment in the form of slurry appears to have been handled since ancient times.[1] All ancient civilizations arose on river banks. Maintenance of waterways requires periodic dredging which results in a sand and silt water slurry. Today dredging represents the largest volume of solids handled in slurry form. Slurry transport is also used for dam construction.

Blatch[2] reported the first hydraulic test results for a sand-water slurry flowing through NPS 1 (DN 25) pipe. Gregory,[3] O'Brien and Folsom,[4] and Howard[5] reported results of tests of clay, sand, and gravel slurries. The flow of muds and sludges through pipes was first examined by Caldwell and Babbit.[6] The first large-scale experimental program on the flow of slurries through pipes was reported by Durand.[7] The correlations proposed by Durand and his coworkers serve as a basis for the present-day design methods.

Design of a slurry piping system involves

- Selection of pipe diameter
- Estimate of friction loss and pumping requirements
- Selection of pipe material, valves, and fittings
- Selection of pumps
- Selection of instruments and control system for safe and reliable operation

Pipelines transporting liquids such as oil and water can be operated at any velocity up to their design limits. In most slurry applications, a certain minimum velocity needs to be maintained, to keep solids from settling out in horizontal sections of the pipe. The velocity below which particles tend to settle out and form a deposit in the pipe is called the *deposition velocity*. The pipe diameter should be selected such that the velocity in the pipeline is maintained above the deposition velocity over the operating range of flow rates.

The operating flow rate range is determined by the expected range of solids throughput and slurry concentration. *Solids throughput* is defined as the weight of solids to be transported per unit time. It is normally expressed in tons per hour (tons/h). The slurry concentration is expressed as the weight of solids per unit weight of slurry, or volume of solids per unit volume of slurry.

The slurry concentration may be established by the requirements of the upstream or downstream processing facilities. This is normally the case with in-plant piping. In the case of long-distance pipelines, it becomes advantageous to adjust the slurry characteristics and concentration to reduce the cost of the pipeline system. An economic study is performed to select parameters acceptable to upstream and downstream plants while offering economies in pipeline construction and operation.

The deposition velocity and friction loss in a given size pipe at a given concentration depend upon the slurry flow behavior. The selection of pipe material, valves, fittings, and pumps depends upon the velocity of flow, abrasivity of the slurry, and pumping pressures which are in turn governed by the slurry flow behavior.

SLURRY FLOW BEHAVIOR

Flow of slurry in pipes depends upon the interaction between the solids and liquid as well as between the slurry and the pipe.

Depending upon the velocity of flow, pipe diameter, solids size distribution, fluid properties, and solids characteristics, four different flow conditions can be encountered in a horizontal or nearly horizontal pipeline.[8] These are homogeneous flow, heterogeneous flow, intermediate regime, and saltation regime.

Homogeneous Flow

Homogeneous flow implies that the solid particles are uniformly distributed across the pipeline cross section. Homogeneous flow, or a close approximation to it, is encountered in slurries of high concentrations and fine particle sizes. Slurries exhibiting homogeneous flow properties do not tend to settle and form a deposit under flowing conditions. Typical examples of homogeneous slurries are sewage sludge, coal-water fuel, clays, drilling mud, paper pulp, titania, fine limestone (cement kiln feed slurry), thorium oxide, and many other finely ground materials.

Heterogeneous Flow

In heterogeneous flow conditions, there is a pronounced concentration gradient across the pipeline cross section. Slurries at low concentration with rapidly settling (coarse particles) solids generally exhibit heterogeneous flow. Typical examples are sand and gravel slurries, coarse coal slurries, and coarse tailings slurries.

Intermediate Regime

This type of flow occurs when some of the particles are homogeneously distributed while others are heterogeneously distributed. Most industrial applications involve a wide range of particle sizes. Intermediate regime of flow is expected with transportation of tailings slurry from mineral processing plants and transportation of coal-water slurries.

Saltation Regime

The fluid turbulence may not be sufficient to keep fast settling particles in suspension. The particles travel by discontinuous jumps or roll along a sliding or stationary bed on the pipe bottom. This type of flow will occur with coarse sand and gravel slurries.

IN-PLANT SYSTEMS

In-plant systems generally involve horizontal, vertical, and inclined sections of pipe. The pipe lengths are generally short. A large number of bends, valves, and fittings may be present in such systems. The pressure losses due to bends, valves, and fittings may be a significant part of the total friction loss. Static head due to change in pipe elevation may be a significant part of the total pumping head requirements for in-plant systems. Typical examples of in-plant system are slurry preparation plants, mineral beneficiation plants, and municipal and industrial waste treatment plants.

In a mineral beneficiation plant, different types of slurries may be handled in the same plant. The slurry concentration as well as the particle size distribution of the slurry may change as the mineral passes through various grinding, separation, and settling stages. Large variations in slurry characteristics and flow rate may be encountered in the same section of pipe owing to changes in ore characteristics or plant operations. The pipes should be sized for these anticipated variations.

LONG-DISTANCE PIPELINES

A number of slurry pipelines have been built to transport solid particles. The materials transported include coal, limestone, kaolin clay, China clay, iron concentrate, copper and nickel concentrates, phosphate concentrates, gold ore, fly ash, sludges, and mineral tailings. Because of the relatively long length of these pipelines, pressure losses through bends and fittings are not a significant part of the total friction loss. Pumping requirements should include changes in pipeline elevation which could be substantial in long-distance pipelines traversing rugged terrain.

Because of the relatively large investment required for a long-distance pipeline, it is generally advantageous to adjust the characteristics of the slurry to suit the pipeline requirements. The slurry concentration, particle size distribution, and throughput are generally controlled within relatively narrow operating limits. The material is generally finely ground to obtain a pseudohomogeneous flow condition in the pipeline.

Slurry pipeline systems range from single-station low-pressure centrifugal pump installations to multistation high-pressure reciprocating pump systems. In all cases, the basic requirement for successful slurry pumping is to maintain pipeline flow above a minimum operating velocity. The minimum operating velocity is set at a desired margin of safety above the critical velocity. The critical velocity in turn is determined by the solids screen analysis, solids density, and concentration as well as the specific system characteristics—pipe diameter, slurry temperature, etc.

In a positive displacement system, the flow is controlled by varying the pump speed. This can be accomplished by the use of a fluid coupling, eddy current coupling, ac or dc variable-speed drive, or a hydraulic clutch system. Diesel-driven pumps have also been used in remote areas where electric power was not available. For optimum system efficiency, most pumps should be operated at their maximum design speed.

A short list representative of the long-distance slurry pipelines in use throughout the world is presented in Table C11.1.

TABLE C11.1 List of Selected Long-Distance Slurry Pipelines

Material	System, location	Throughput, mmtpy	Length, mi	Start of operation
Coal	Consolidation Coal, Ohio	1.2	105	1957
	Russia	4.0	7	1966
	France (Merlebach)	1.5	6	1954
	Black Mesa, Arizona	5.0	273	1970
	Japan	0.3	16	1965
Limestone	Trinidad	0.5	6	1959
	Rugby, England	1.5	57	1964
	Calaveras, California	1.4	17	1971
	Gladstone, Australia	2.0	15	1981
Iron concentrate	Savage River, Tasmania	2.3	53	1967
	Pena Colorada, Mexico	1.6	30	1974
	Las Truchas, Mexico	1.4	17	1975
	Sierra Grande, Argentina	1.9	20	1977
	Samarco, Brazil	12.0	247	1978
	Kudremukh, India	7.5	42	1980
	La Perla, Mexico	4.5	237	1983
Iron sand	Waipipi, New Zealand	1.0	4	1971
Copper concentrate	Bougainville, Papua, New Guinea	1.0	17	1972
	West Irian, Indonesia	0.3	69	1973
	Pinto Valley, Arizona	0.4	11	1974
	Kennecott Chino, New Mexico	0.7	7	1982
	KBI, Turkey	0.9	40	1973
	Kennecott, Utah	0.7	17	1987
Phosphate concentrate	Valep, Brazil	2.0	70	1979
	Chevron, Utah	1.8	94	1986
	Makon, India	0.1	7	1983

SLURRY CHARACTERISTICS

Slurries may be classified as settling suspensions and nonsettling suspensions. Settling suspensions require turbulence to maintain individual particles in motion or in suspension. With finely divided solids, homogeneous flow could also be achieved for settling suspensions in turbulent flow. Nonsettling suspensions, as the name implies, do not settle, even under the no-flow condition.

Mineral concentrates, tailings, and coal slurries require turbulence to maintain particles in suspension. Digested sludge and coal-water fuel slurries do not settle under static conditions.

The flow characteristics of a settling suspension are largely governed by the settling velocity of solids in it. The flow characteristics of a nonsettling suspension are governed by its rheological characteristics and densities. Most commercial slurries contain appreciable amounts of finely divided solids that change the rheological properties of the suspending fluid. For these slurries, both the settling characteristics of solids and rheological properties and the density of the slurry become important.

Slurry Density

The *density* of a slurry is given by

$$\rho_m = \frac{100}{C_w/\rho_s + (100 - C_w)/\rho_l} \tag{C11.1}$$

where ρ_l = density of suspending liquid, lb/ft^3 (kg/m^3).
ρ_m = density of mixture, lb/ft^3 (kg/m^3)
ρ_s = density of the solids, lb/ft^3 (kg/m^3)
C_w = solids concentration by weight in slurry, %

Measurements of slurry concentration, solids density, and liquid density are straightforward. The slurry concentration is obtained by evaporating a liquid component from a known weight of slurry and measuring the weight of dried solids.

The density of slurry in a pipe may be measured by using a nuclear density meter or by measuring head loss per unit length along each vertical leg of a test section arranged as an inverted U. The slurry density may also be measured by collecting a sample in a suitably designed specific-gravity bottle or by a Marcy balance. Note that these devices measure the specific gravity of slurry. The density of slurry is computed by multiplying its specific gravity by the density of water.

Example C11.1. Slurry concentration is determined by drying to constant weight a sample of slurry in an oven maintained at 220°F (104°C). Determine the slurry concentration based on the following data:

Weight of empty dry container	0.1 lb (0.0454 kg)
Weight of container plus slurry	0.32 lb (0.1454 kg)
Weight of container plus dry solids	0.21 lb (0.0954 kg)

Solution

$$\text{Weight of dry solids} = 0.21 - 0.1$$
$$= 0.11\text{ lb } (0.05\text{ kg})$$
$$\text{Weight of slurry} = 0.32 - 01$$
$$= 0.22\text{ lb } (0.01\text{ kg})$$
$$\text{Weight percent solids } C_w = \text{weight of solids} \times \frac{100}{\text{weight of slurry}}$$
$$= 50\%$$

Example C11.2. Determine the density of the slurry considered in Example C11.1 if the solids and liquid specific gravities are 3.0 and 1.0, respectively.

Solution

$$\text{Density of solids } \rho_s = 3.0 \times 62.4\text{ lb/ft}^3 (3000\text{ kg/m}^3)$$
$$\text{Density of liquid } \rho_l = 1.0 \times 62.4\text{ lb/ft}^3 (1000\text{ kg/m}^3)$$
$$\text{Solids concentration } C_w = 50\%$$

Substituting into Eq. (C11.1), we get

$$\text{Density of slurry } \rho_m = 93.6\text{ lb/ft}^3 (1500\text{ kg/m}^3)$$

Example C11.3. A nuclear density meter gives a slurry specific gravity of 1.167 for a coal-water slurry. If the specific gravity of coal is 1.4, find the weight percent coal in slurry.

Solution

$$\text{Density of coal slurry } \rho_m = 1.167 \times 62.4\text{ lb/ft}^3$$
$$= 72.82\text{ lb/ft}^3 (1167\text{ kg/m}^3)$$
$$\text{Density of water } \rho_l = 62.4\text{ lb/ft}^3 (1000\text{ kg/m}^3)$$
$$\text{Density of coal } \rho_s = 1.4 \times 62.4\text{ lb/ft}^3$$
$$= 87.36\text{ lb/ft}^3 (1400\text{ kg/m}^3)$$

Rearranging Eq. (C11.1), we get

$$C_w = 100\frac{\rho_s(\rho_m - \rho_l)}{\rho_m(\rho_s - \rho_l)}$$
$$= 50\%$$

Slurry Rheology

In the presence of subsieve particles (those smaller than 35 μm) and at relatively high concentrations, the slurry flow properties are governed by its rheology. Slurries that do not contain particles smaller than 35 μm or that are at low concentrations exhibit heterogeneous flow behavior. Heterogeneous flow properties are not governed by slurry rheology.

Rheology is the relationship between the shear stress and the corresponding rate of shear in a slurry under laminar-flow conditions. The friction loss in a pipeline depends upon the rheology of the slurry in homogeneous and intermediate flow regimes. In the case of pure liquids, the shear stress is directly proportional to the rate of shear in laminar flow. The proportionality constant is called the *viscosity* of the liquid. This type of flow behavior is called *newtonian.* Liquids containing long-chain polymers and finely ground solids exhibit a nonlinear relationship between shear stress and the rate of shear under laminar-flow conditions. Such slurries are said to exhibit *nonnewtonian* flow properties. Depending upon the size distribution of solids, slurry concentration, and interaction between solids and liquid, the slurry may have newtonian or nonnewtonian flow properties.

Slurries containing nonflocculated particles generally exhibit newtonian flow behavior. Nonnewtonian flow behavior is generally encountered with flocculated suspensions.

Some slurries require a certain minimum stress before flow starts. For example, fresh concrete does not flow over a chute until a certain slope is exceeded. The slurry is said to possess a yield stress which must be exceeded to initiate flow.

The rheology of a newtonian fluid is expressed by its viscosity, which is the ratio of shear stress to the corresponding rate of shear. Two or more parameters are needed to describe the rheological properties of a nonnewtonian liquid. Bingham plastic, pseudoplastic, and yield pseudoplastic models are generally used to describe the flow behavior of slurries. The relationships between the shear stress and shear rate for these rheological models are as follows:

Newtonian:

$$\tau = \mu\gamma \tag{C11.2}$$

Bingham plastic:

$$\tau = \tau_y + \eta\gamma \tag{C11.3}$$

pseudoplastic:

$$\tau = K\gamma^n \tag{C11.4}$$

yield pseudoplastic:

$$\tau = \tau_y + K\gamma^n \tag{C11.5}$$

where τ = shear stress, lbf/ft^2 (Pa)
τ_y = yield stress, lbf/ft^2 (Pa)
γ = rate of shear (velocity gradient), 1/s
μ = newtonian viscosity, lbf·s/ft^2 (Pa·s)
n = flow behavior index
K = consistency index, lbf·s^n/ft^2 (Pa·s^n)

Example C11.4. The following rheology test results were obtained for a sample of a mineral tailings slurry containing 50 percent solids by weight.

Rate of shear, 1/s	Shear stress τ, lbf/ft^2 (Pa)	$\tau - \tau_y$, lbf/ft^2 (Pa)
0	0.1250 (6.000)	0.0000 (0.000)
0.1	0.1256 (6.029)	0.0006 (0.029)
1.0	0.1280 (6.144)	0.0030 (0.144)
5.0	0.1343 (6.445)	0.0093 (0.445)
10	0.1400 (6.718)	0.0150 (0.718)
20	0.1494 (7.168)	0.0244 (1.168)
40	0.1647 (7.900)	0.0397 (1.900)
80	0.1895 (9.088)	0.0645 (3.088)
100	0.2004 (9.610)	0.0754 (3.610)
150	0.2250 (10.79)	0.1000 (4.79)
200	0.2474 (11.86)	0.1224 (5.86)
300	0.2876 (13.79)	0.1626 (7.79)
400	0.3240 (15.53)	0.1990 (9.53)
500	0.3575 (17.13)	0.2325 (11.13)
600	0.3890 (18.64)	0.2640 (12.64)
700	0.4190 (20.08)	0.2940 (14.08)
800	0.4480 (21.47)	0.3230 (15.47)

Solution. The shear stress at zero shear rate is 0.125 lbf/ft^2. The slurry yield stress is therefore 0.125 lbf/ft^2 (6 Pa). Find the difference between the observed shear stress τ and the yield stress τ_y at each shear rate shown in the third column of the above table.

A plot of $\tau - \tau_y$ versus shear rate on an arithmetic scale (Fig. C11.1) shows a nonlinear relationship. A similar plot on log-log scale (Fig. C11.2) shows a linear relationship (note that the data for zero shear rate is excluded). The value of τ at a shear rate of 1 gives the value of K in $lbf{\cdot}s^n/ft^2$. ($Pa{\cdot}s^n$). The slope of the line gives the value of the flow behavior index n. The results from the graph are

$$K = 0.003\,\mathrm{lbf \cdot s^{\mathit{n}}/ft^2}\,(0.144\,\mathrm{Pa \cdot s^{\mathit{n}}})$$

$$n = 0.7$$

Estimate of Slurry Rheology

Correlations between slurry concentration and rheology of the slurry for newtonian and Bingham plastic slurries have been proposed by various investigators. These relationships may be used for preliminary estimates when rheology test results are not available.

The viscosity of a slurry depends upon the volume fraction of solids in slurry. The volume fraction of solids is determined by using

$$C_v = C_w \frac{\rho_m}{100\rho_s} \tag{C11.6}$$

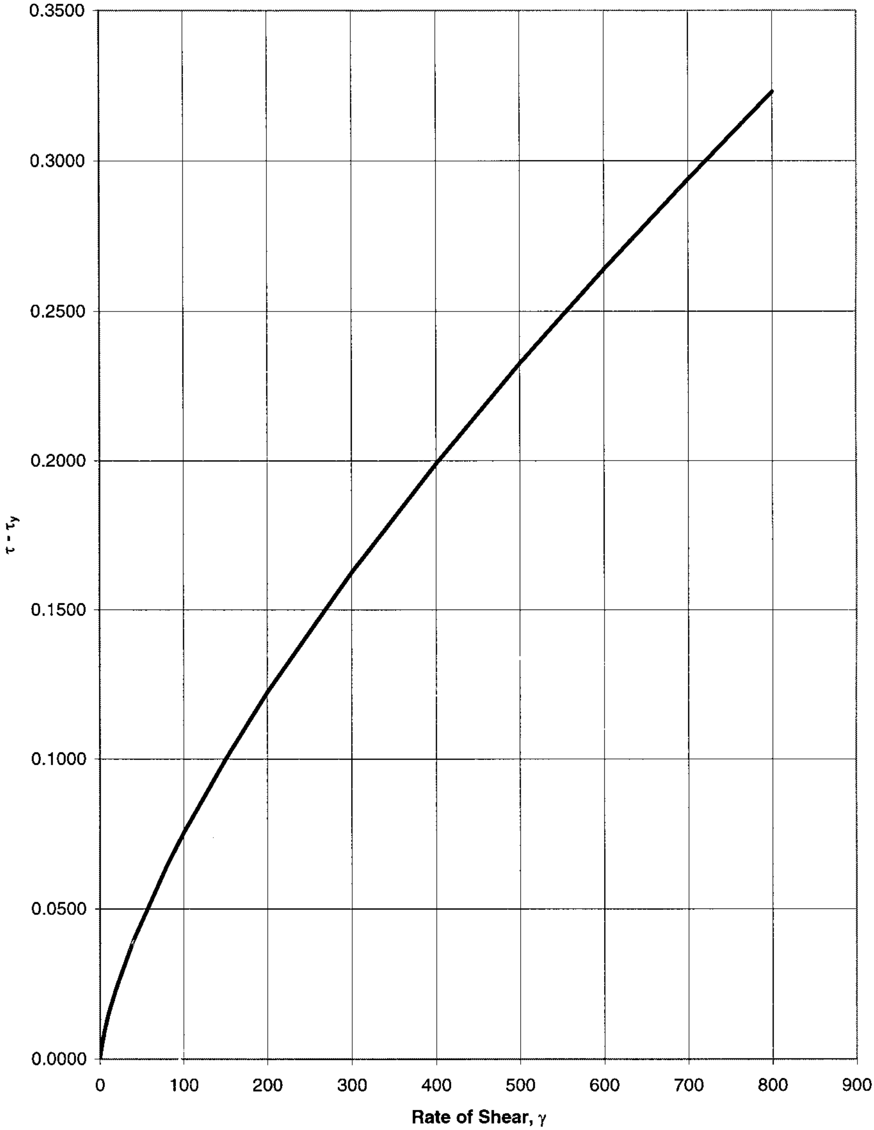

FIGURE C11.1 Plot of $\tau - \tau_y$ versus shear rate on an arithmetic scale.

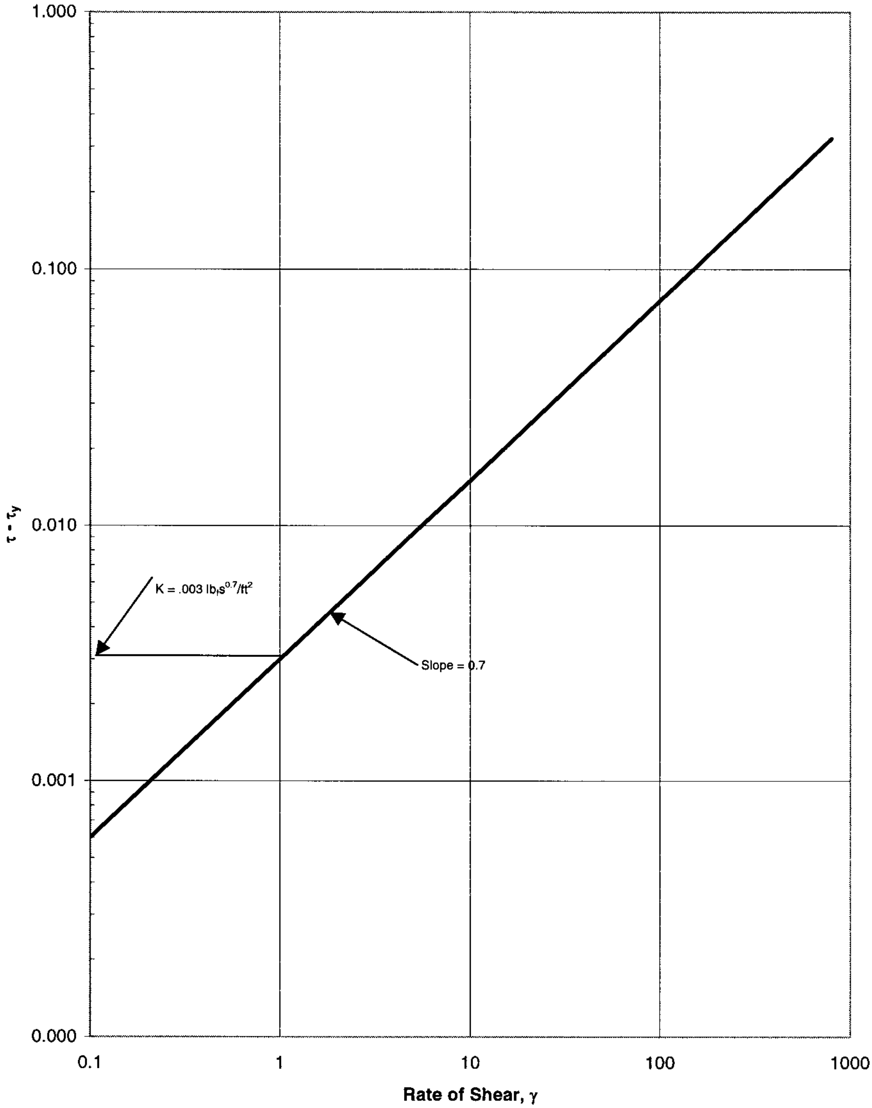

FIGURE C11.2 Plot of $\tau - \tau_y$ versus shear rate on a logarithmic scale.

where C_v is the volume fraction of solids in slurry. The viscosity of slurries exhibiting newtonian behavior can be estimated by the correlation proposed by Thomas[9]

$$\frac{\eta_m}{\eta\gamma} = 1 + 2.5C_v + 10.05C_v^2 + 0.00273e^{16.6C_v} \qquad \text{(C11.7)}$$

where η_m = slurry viscosity
η_o = suspending fluid viscosity

Chong et al.[10] have proposed the following equation for concentrated suspensions of spherical particles:

$$\frac{\eta_m}{\eta_o} = 1 + 0.75 \frac{C_v/C_{voo}}{1 - C_v/C_{voo}} \qquad \text{(C11.8)}$$

where C_{voo} is the maximum packing concentration of the solids in slurry. Equation (C11.8) should be used for values of C_v greater than 0.4.

Gay et al.[11] have proposed the following correlations for estimating the Bingham plastic viscosity and yield stress, based on their experimental data:

$$\eta_m = \mu \exp\left\{\left[2.5 + \left(\frac{C_v}{C_{voo} - C_v}\right)^{0.48}\right]\frac{C_v}{C_{voo}}\right\} \qquad \text{(C11.9)}$$

$$\tau_y = 200\left(\frac{d}{C_{voo} - C_v}\right)\left(\frac{C_{voo}}{1 - C_{voo}}\right)^2 \frac{1}{\zeta^{1.5}\sigma_g^2} \qquad \text{(C11.10)}$$

where μ = viscosity of suspending medium, lbf·s/ft^2 (Pa·s)
η_m = Bingham plastic viscosity of slurry, lbf·s/ft^2 (Pa·s)
τ_y = yield stress of slurry, lbf/ft^2 (Pa)
d = geometric mean particle diameter, ft (m)
ξ = particle shape factor, defined as ratio of surface area of sphere of equivalent volume to surface area of particle
σ_g = geometric standard deviation of particle diameter

SLURRY HYDRAULICS

Homogeneous Flow

The friction loss for a homogeneous slurry depends upon the rheological characteristics of the slurry. The flow through a pipeline can be laminar or turbulent depending upon the velocity of flow. For a nonsettling suspension such as sewage sludge or a highly concentrated coal-water fuel slurry, laminar flow may be encountered. Turbulent flow should be maintained when the suspension exhibits a settling tendency. It is, therefore, necessary to estimate the velocity at which transition from laminar to turbulent flow occurs.

Transition Velocity

The *transition velocity* is defined as the velocity below which laminar flow is encountered. For a newtonian slurry, the transition velocity corresponds to a Reynolds number of 2000. The Reynolds number should be based on the viscosity of the slurry.

For slurries exhibiting Bingham plastic rheology, the transition velocity is governed by the Reynolds number as well as the Hedstrom number. The Reynolds number should be defined using the plastic viscosity. The critical Reynolds number corresponding to the transition velocity can be estimated from a knowledge of the

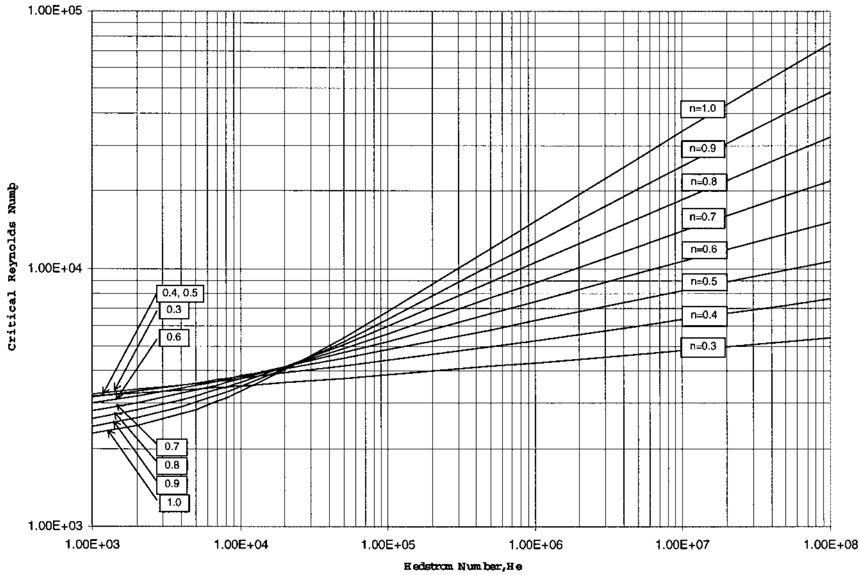

FIGURE C11.3 Laminar-turbulent transition Reynolds number $Re_c = 8\rho D^n V^{2-n}[n/(2 + 6n)]^n(Kg_c)$ as a function of Hedstrom number $He = [D^2\rho\tau_y(t_y/K)^{2/n-2}]/(K^2 g_c)$ for Bingham plastic slurries, where D = pipe ID (ft), V = velocity (ft/s), ρ = density (lb/ft^3), K = consistency (lbf · s^n/ft^2), τ_y = yield stress (lbf · /ft^2), g_c = 32.2 lb · s/(lbf · ft^2).

physical properties of the slurry and the pipe system from Fig. C11.3, proposed by Hanks and Pratt.[12] In this figure the Reynolds number Re and the Hedstrom number He are defined as follows:

$$Re = \frac{vd\rho}{\eta} \tag{C11.11}$$

$$He = \frac{D^2\rho\tau_y}{\eta^2 g_c} \tag{C11.12}$$

where D = pipe inside diameter, ft (m)
V = average flow velocity, ft/s (m/s)
g_c = dimension conversion factor = 32.2 lbm·ft/(lbf·s^2) (1 for SI units)
η = Bingham plastic viscosity, lbf·s/ft^2 (Pa·s)
ρ = fluid density, lb/ft^3 (kg/m^3)
τ_y = yield stress, lbf/ft^2 (Pa)

For slurries exhibiting pseudoplastic rheology, the transition velocity is governed by the flow behavior index of the slurry. Figure C11.4 shows the variation of the transition critical Reynolds number with the flow behavior index n. Note that the Reynolds number for a pseudoplastic slurry is given by

$$Re_p = 8\rho D^n V^{2-n}\left(\frac{n}{2+6n}\right)^n\left(\frac{1}{Kg_c}\right) \tag{C11.13}$$

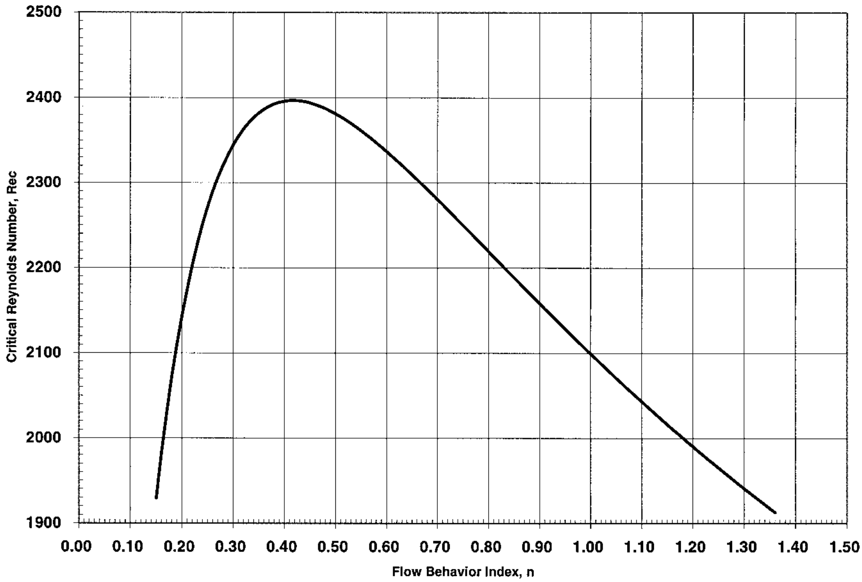

FIGURE C11.4 Laminar-turbulent transition Reynolds number (Re_c) as a function of flow behavior index, n for pseudoplastic slurries, where D = pipe ID (ft), V = velocity (ft/s), ρ = density (lb/ft^3), K = consistency (lbf·s^{n}ft^2), g_c = 32.2 lb·s/(lbf·ft^2).

where K = consistency index, lbf·s^n/ft^2 (Pa·s^n)
Re_p = Reynolds number for pseudoplastic slurry
n = flow behavior index

For a yield pseudoplastic slurry, the generalized Reynolds number corresponding to the transition critical velocity can be estimated by using the following equations, proposed by Hanks and Ricks.[13]

$$Re_{cp} = \frac{6464n}{(1+3n)^n}(2+n)^{(2+n)/(1+n)} \frac{\left[\frac{(1-x)^2}{1+3n} + \frac{2x(1-x)}{1+2n} + \frac{x^2}{1+n}\right]^{2-n}}{(1-x)^n} \tag{C11.14}$$

The value of x, which is the ratio of yield stress to wall shear stress, at the critical Reynolds number is obtained from the following equation:

$$He_{yp} = \frac{3232}{n}(2+n)^{(2+n)/(1+n)}\left[\frac{x}{(1-x)^{1+n}}\right]^{(2-n)/n}\left(\frac{1}{1-x}\right)^n \tag{C11.15}$$

The Reynolds number Re_{cp} in Eq. (C11.14) is the same as that for a pseudoplastic in Eq. (C11.13). The Hedstrom number He_{yp} is defined as follows:

$$He_{yp} = \frac{D^2 \rho \tau_y}{K^2 g_c}\left(\frac{\tau_y}{K}\right)^{2/n-2} \tag{C11.16}$$

For given slurry characteristics and pipe diameter, the Hedstrom number is computed by using Eq. (C11.16). This value of the Hedstrom number is used in Eq. (C11.15) to compute the value of x. This value is then used in Eq. (C11.14) to compute the critical Reynolds number. The corresponding transition velocity is calculated by using Eq. (C11.13).

Figure C11.3 also shows the variation in critical Reynolds number with Hedstrom number for the yield pseudoplastic slurries.

Example C11.5. Estimate the laminar-turbulent transition critical velocity for the slurry considered in Example C11.4 if the pipe inside diameter is 12 in and the solids specific gravity is 3.0.

Solution. Given data:

$$\text{Pipe ID, } D = 12\,\text{in}(0.3048\,\text{m})$$
$$= 1\,\text{ft}$$
$$\text{Yield stress of slurry} = 0.125\,\text{lb/ft}^2\ (6\,\text{Pa})$$
$$\text{Slurry consistency } k = 0.003\,\text{lbf}\cdot\text{s}^n/\text{ft}^2\ (0.144\,\text{Pa}\cdot\text{s}^n)$$
$$\text{Solids specific gravity} = 3.0$$
$$\text{Solids concentration } C_w = 50$$

Computed results:

$$\text{Solids density } \rho_o = 3 \times 62.4\,\text{lbm/ft}^2\ (3000\,\text{kg/m}^3)$$
$$\text{Liquid density } \rho_1 = 62.4\,\text{lbm/ft}^3 (1000\,\text{kg/m}^3)$$

Substituting in Eq. (C11.1), we get

$$\text{Slurry density } \rho_m = 93.6\,\text{lbm/ft}^3\ (1500\,\text{kg/m}^2)$$

Compute the Hedstrom number He_{yp}, using Eq. (C11.16):

$$\text{He}_{yp} = 9.9 \times 10^5$$

From Fig. C11.3 find the critical Reynolds number corresponding to $n = 0.7$ and the computed value of the Hedstrom number.

$$\text{Re}_{yp} = 8850$$

Next compute the laminar-turbulent transition velocity, using Eq. (C11.13):

$$v = 3.58\,\text{ft/s}(1.09\,\text{m/s})$$

Friction Loss in Laminar Flow. For a homogeneous flow condition, the friction loss in a pipeline is estimated by using the following equation:

$$h = 4f\left(\frac{L}{D}\right)\frac{V^2}{2_g} \qquad \text{(C11.17)}$$

where D = pipe inside diameter, ft (m)
L = pipe length, ft (m)
V = velocity of flow, ft/s (m/s)
f = friction factor
h = friction loss, ft (m) of slurry head
g = acceleration due to gravity, ft/s^2 (m/s^2)

The friction factor for a newtonian slurry is given by the following:

$$f = \frac{16}{\text{Re}} \tag{C11.18}$$

$$\text{Re} = \frac{VD\rho}{\eta_m g_c} \tag{C11.19}$$

where ρ = slurry density, lbm/ft^3 (kg/m^3)
η_m = slurry viscosity lbf·s/ft^2 (Pa·s)
g_c = 32.2 lbm·ft/(lbf·s^2 (1 for SI units)

The laminar-flow friction factor for Bingham plastic slurries is given by

$$\frac{f}{16} = \frac{1}{\text{Re}} + \frac{\text{He}}{6\text{Re}^2} - \frac{\text{He}^4}{3f^3\text{Re}^8} \tag{C11.20}$$

The Reynolds number Re and the Hedstrom number He are given by Eqs. (C11.11) and (C11.12), respectively.

The laminar-flow friction factor for a pseudoplastic slurry is given by Dodge and Metzner[14] as

$$f = \frac{16}{\text{Re}_p} \tag{C11.21}$$

where Re_p is the generalized Reynolds number given by Eq. (C11.13). The laminar-flow friction factor for a slurry exhibiting yield pseudoplastic flow behavior is given by

$$f = \frac{16}{\text{Re}_p \Psi} \tag{C11.22}$$

where the Reynolds number Re_p is given by Eq. (C11.13).

$$\Psi = (1+3n)^n (1-x)^{1+n} \left[\frac{(1-x)^2}{1+3n} + \frac{2x(1-x)}{1+2n} + \frac{x^2}{1+n} \right]^n \tag{C11.23}$$

The value of the ratio x of the yield stress to wall shear stress is obtained from

$$\text{Re}_p = 2\,\text{He}_{yp} \left(\frac{n}{1+3n} \right)^2 \left(\frac{\Psi}{x} \right)^{(2-n)/n} \tag{C11.24}$$

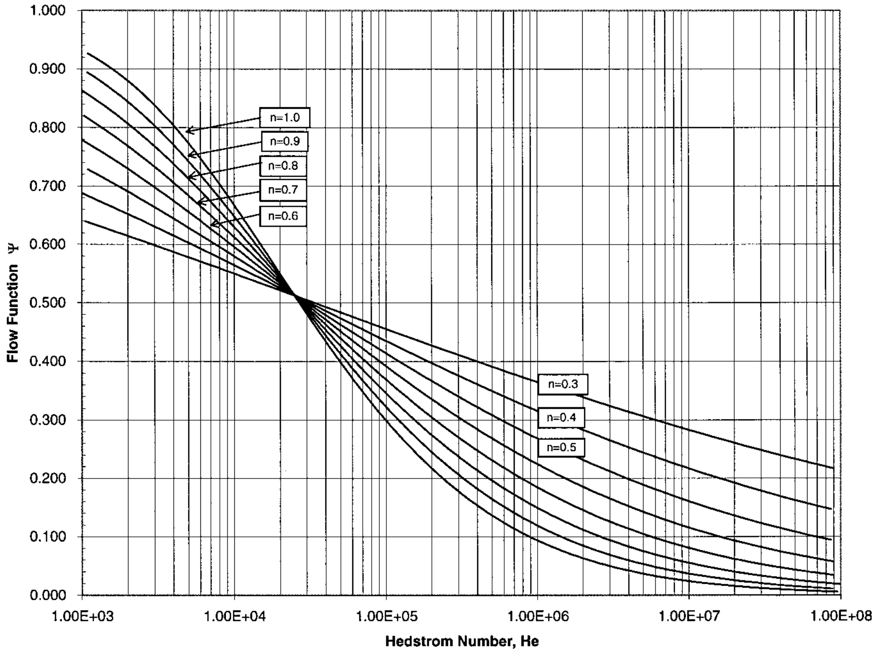

FIGURE C11.5 Laminar flow function Y as a function of Hedstrom number He $= [D^2\ \rho\tau_y(t_y/K)^{2/n-2}]/(K^2 g_c)$, for yield pseudoplastic slurries, where D = pipe ID (ft), V = velocity (ft/s), ρ = density (lb/ft^3), K = consistency (lbf·s^n/ft^2), τ_y = yield stress (lbf/ft^2), g_c = 32.2 lb·s/(lbf·ft^2).

Figure C11.5 shows a plot of ψ as a function of the Hedstrom number for various values of the flow behavior index n.

Stainsby and Chilton[15] have proposed a hybrid model incorporating yield pseudoplastic behavior at low shear rates and Bingham plastic behavior at high shear rates. They claim that their model gives better correlations with pipeline friction loss data than those of other models.

Example C11.6. Estimate the friction loss in a 12-in-ID (304.8-mm-ID) pipe at a velocity of 3 ft/s (0.914 m/s) for the slurry considered in Example C11.5.

Given data:

$$\text{Pipe ID } D = 1\text{ ft } (0.3048\text{ m})$$

$$\text{Slurry density} = 93.6\text{ lbm/ft}^3\ (1500\text{ kg/m})^3$$

$$\text{Velocity } v = 3\text{ ft/s } (0.914\text{ m/s})$$

$$K \text{ (from Example C11.5)} = 0.003\text{ lbf}\cdot\text{s}^n/\text{ft}^2\ (0.144\text{ Pa}\cdot\text{s}^n)$$

$$\tau_y \text{ (from Example C11.5)} = 0.125\text{ lbf/ft}^2\ (6\text{ Pa})$$

$$n \text{ (from Example C11.5)} = 0.7$$

$$g = 32.2\text{ ft/s}^2\ (9.8\text{ m/s})^2$$

$$\text{He}_{yp} \text{ (from Example C11.5)} = 9.9\times 10^5$$

Using Eq. (C11.13), find $Re_p = 7023$. The Reynolds number is less than the critical Reynolds number. The flow is laminar.

From Fig. C11.5, the value of ψ is found to be 0.184. The friction factor

$$f = \frac{16}{\Psi} Re_{yp}$$
$$= 0.0124$$

$$\text{Friction loss per unit length} = \frac{4fv^2}{2gD}$$
$$= 6.92 \times 10^{-3} \text{ ft/ft (m/m)}$$

Friction Loss in Turbulent Flow. The friction factor for newtonian liquids in a turbulent-flow regime is given by the Colebrook equation:

$$\frac{1}{\sqrt{f}} = 4 \log \frac{D}{2\varepsilon} + 3.48 - 4 \log \left(1 + \frac{9.35D}{2\varepsilon \mathrm{Re}\sqrt{f}}\right) \qquad \text{(C11.25)}$$

where ε = the roughness of the pipe, ft (m). Figure C11.6 shows the friction factor as a function of Reynolds number with the relative roughness ε/D as a parameter.

Hanks and Dadia[16] have developed a friction factor–Reynolds number relationship for Bingham plastic liquids. This relationship was later modified by Hanks[17]

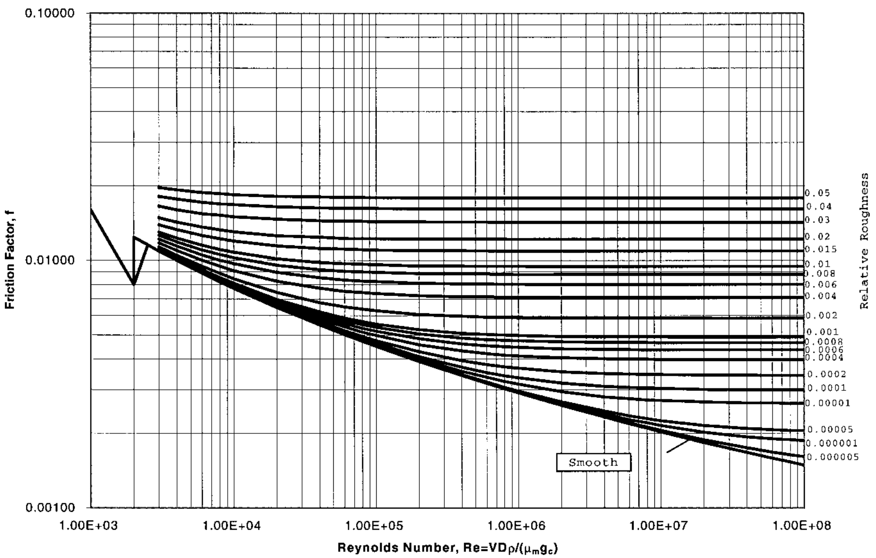

FIGURE C11.6 Friction factor f as a function of Reynolds number for newtonian slurries, where D = pipe ID (ft), V = velocity (ft/s), r = density (lb/ft^3), μ = viscosity (lbf·s/ft^2), g_c = 32.2 lb·s/(lbf·ft^2).

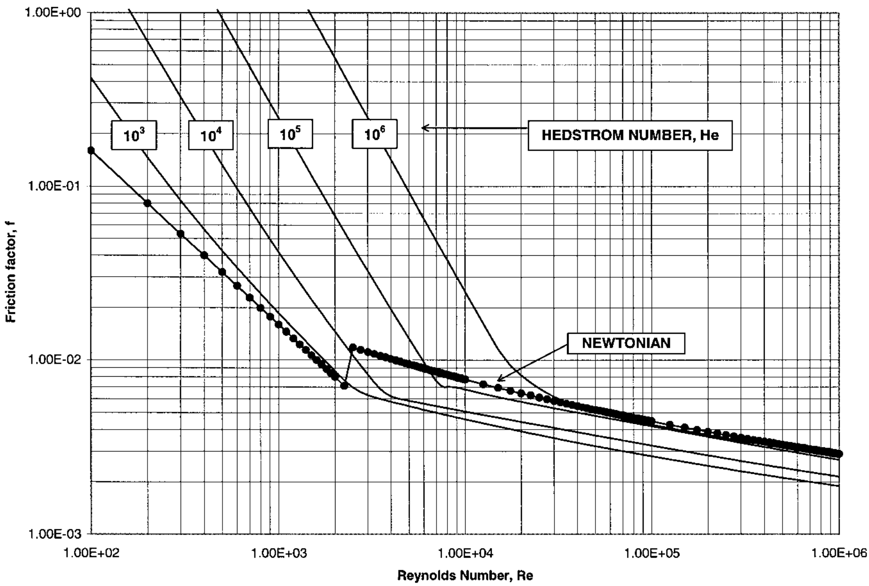

FIGURE C11.7 Friction factor f as a function of Reynolds number for Bingham plastic slurries, $\mathrm{Re} = \rho DV/(\eta g_{cy})$, $\mathrm{He} = D^2 \rho\tau_y/(\eta g_c)$, where D = pipe ID (ft), V = velocity (ft/s), ρ = density (lb/ft^3), η = viscosity (lbf·s/ft^2), τ_y = yield stress (lbf/ft^2), g_c = 32.2 lb·s/(lbf·ft^2).

based on further analysis of their data. Figure C11.7 shows the friction factor–Reynolds number relationship for Bingham plastics. Note that for Hedstrom numbers greater than 500,000, there is a long transition region between the laminar-turbulant transition critical Reynolds number and the Reynolds number at which the friction factor curve for Bingham plastics intersects the newtonian curve. The friction factor for Bingham plastic in this transition region is significantly greater than the newtonian friction factor.

Dodge and Metzner[18] carried out a semitheoretical analysis of the turbulent flow of pseudoplastic liquids in smooth pipe. They proposed the following equation for the friction factor in turbulant flow:

$$\frac{1}{\sqrt{f}} = \frac{4}{n^{0.75}} \log\left(\mathrm{Re}_p f^{1-n/2}\right) - \frac{0.4}{n^{1.2}} \qquad \text{(C11.26)}$$

Figure C11.8 shows a plot of friction factor as a function of Reynolds number for various values of n based on Eq. (C11.26).

Hanks and Ricks[19] have used the concept of mixing length in developing a semi-theoretical relationship between friction factor and Reynolds number for turbulent flow of pseudoplastics. This method was extended by Hanks[20] for estimating turbulent-flow friction factors for yield pseudoplastic fluids. Interested readers should refer to the referenced article for further details.

Figure C11.9 presents the friction factor–Reynolds number relationship with Hedstrom number as a parameter for selected values of flow behavior index n.

The following friction factor–Reynolds number relationship, developed by Tor-

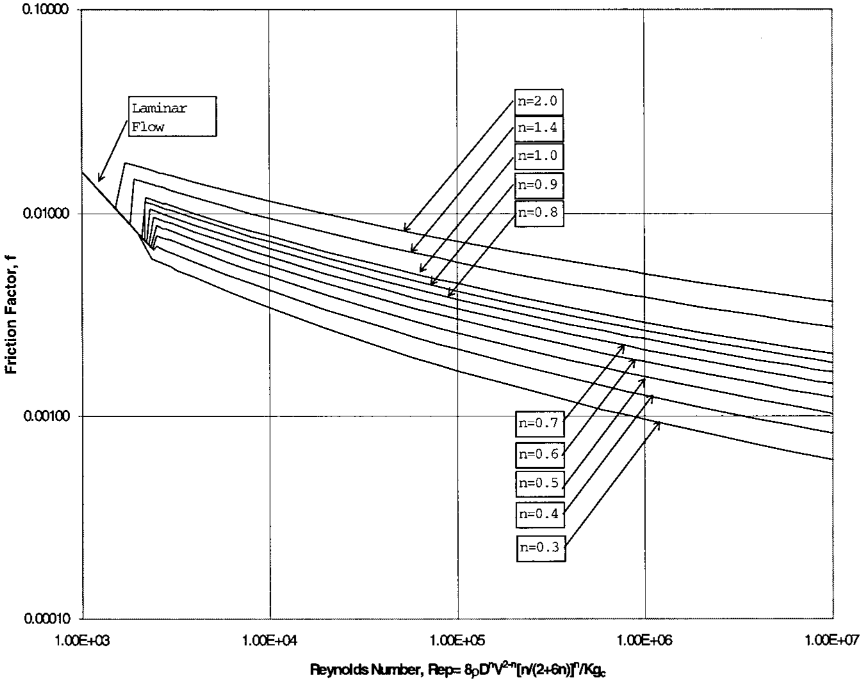

FIGURE C11.8 Friction factor f as a function of Reynolds number for pseudoplastic slurries. $Re_p = 8\rho D^n V^{2-n}(n/(2 + 6n))^n/(Kg_c)$ where D = pipe ID (ft), V = velocity (ft/s), ρ = density (lb/ft^3), K = consistency (lbf·s^n/ft^2), g_c = 32.2 (lb·s/(lbf·ft^2).

rance,[21] is applicable to turbulent flow of newtonian, pseudoplastic, and Bingham plastic as well as yield pseudoplastic fluids in smooth pipes.

$$\frac{1}{\sqrt{f}} = \frac{2.69}{n} - 2.95 + \frac{4.53}{n}\log(1-x) - \frac{4.53}{n}\log\left(\text{Re}_r\sqrt{f^{2-n}}\right) + \frac{0.68}{n} \tag{C11.27}$$

where

$$\text{Re}_T = \frac{8\rho D^n V^{2-n}}{K(8^n)} \tag{C11.28}$$

Torrance has also extended his analysis to rough wall fully turbulent friction factor for nonnewtonian fluids and obtained the following relationship:

$$\frac{1}{\sqrt{f}} = 4.07\log\frac{D}{2\varepsilon} + 6.0 - \frac{2.65}{n} \tag{C11.29}$$

Wilson and Thomas[22] and Thomas and Wilson[23] have used a mixing-length approach for the yield pseudoplastic fluids. Their model includes corrections to change

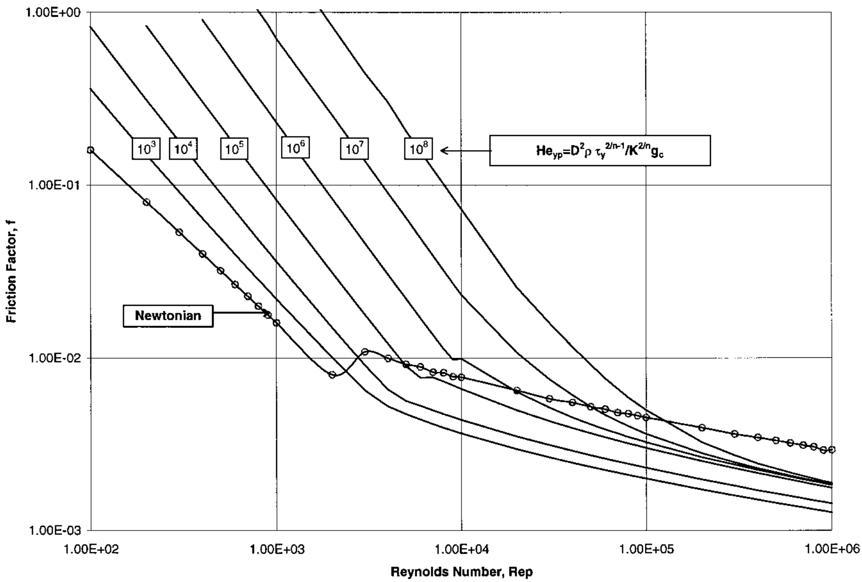

FIGURE C11.9a Friction factor f as a function of Reynolds number for yield pseudoplastic slurries for $n = 0.7$, $\mathrm{He} = [D^2\rho\tau_y(t_y/K)^{2/n-2}]/(K^2g_c)$, $\mathrm{Re} = 8\rho D^nV^{2-n}[n/(2+6n)]^n/(Kg_c)$, where D = pipe ID (ft), V = velocity (ft/s), ρ = density (lb/ft^3), K = consistency (lbf·s^n/ft^2), τ_y = yield stress (lbf/ft^2), $gc = 32.2$ lb·s/(lbf·ft^2).

the thickness of the viscous sublayer according to the rheology and a term to account for the central unsheared core. Their method has been shown to give fairly good agreement with experimental data.

Stainsby and Chilton[24] have given a turbulent-flow friction factor correlation based on their model.

Example C11.7. Estimate the friction loss in a 12-in-ID (0.3048-m-ID) pipe at a velocity of 5 ft/s (1.52 m/s) for the slurry considered in Example C11.5. Since the velocity of flow is greater than the transition velocity (see Example C11.5), the flow will be turbulent. Find the Reynolds number corresponding to the given velocity, using the slurry rheological data given in Example C11.5.

$$\mathrm{Re}_p = 13{,}644$$

The Hedstrom number from Example C11.5 is 9.9×10^5. Using Fig. C11.9*c*, find the friction factor $f = 0.008$. Substituting the values of D, V, f, and g in Eq. (C11.17), find the head loss per unit length of pipe:

$$\text{Head loss} = 0.0124 \text{ ft (m) slurry/ft (m) pipe}$$

Heterogeneous Flow

Fluid turbulence is needed to maintain particles in suspension or motion in a horizontal pipe carrying setting suspension. At low velocities, particles settle and form a stationary bed on the bottom of a horizontal pipe.

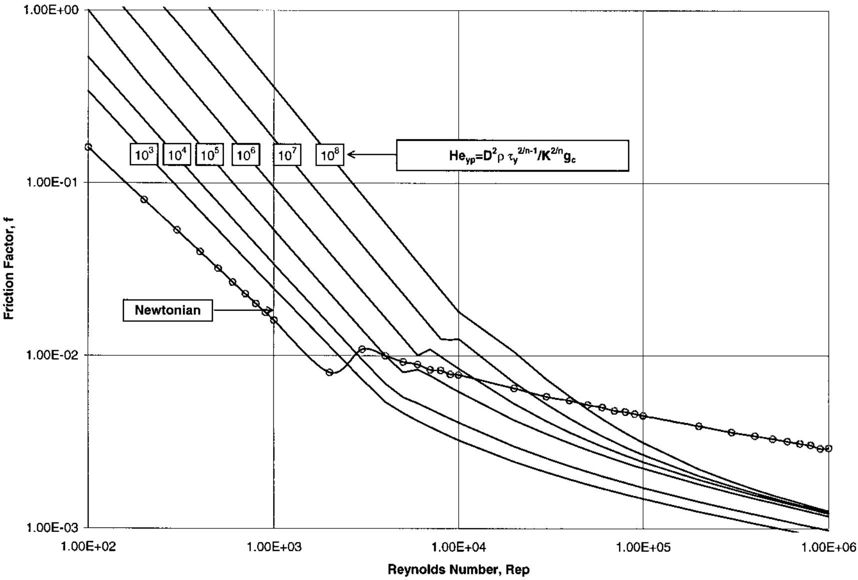

FIGURE C11.9b Friction factor f as a function of Reynolds number for yield pseudoplastic slurries for $n = 0.5$, $\text{He} = [D^2\rho\tau_y(t_y/K^{2/n-2}]/(K^2g_c)$, $\text{Re} = 8\rho D^nV^{2-n}[n/(2+6n)]^n/(Kg_c)$, where D = pipe ID (ft), V = velocity (ft/s), ρ = density (lb/ft^3), K = consistency (lbf · s^n/ft^2), τ_y = yield stress (lbf/ft^2), $g_c = 32.2$ (lb · s/(lbf · ft^2).

Deposition Velocity. The velocity below which bed deposits form is called the deposition velocity. Operating the pipeline at or below the deposition velocity for prolonged time could result in a pipeline blockage. The minimum operating velocity in a slurry pipeline should be kept greater than the deposition velocity, to prevent pipeline blockages.

A number of empirical correlations have been proposed for estimating the deposition velocity. For uniform-size particles, the Durand[25] correlation, given as follows, is widely used:

$$V_D = F_L\sqrt{2gD(s-1)} \tag{C11.30}$$

The value of F_L in Durand's correlation can be obtained from Fig. C11.10. In Eq. (C11. 30).

D = pipe ID, ft (m)

V_D = deposition velocity, ft/s (m/s)

g = acceleration due to gravity, ft/s^2 (m/s^2)

s = solids specific gravity

Zandi and Govatos,[26] Wasp et al.[27] and Graf et al.[28] have proposed minor modifications of Durand's correlation based on additional data.

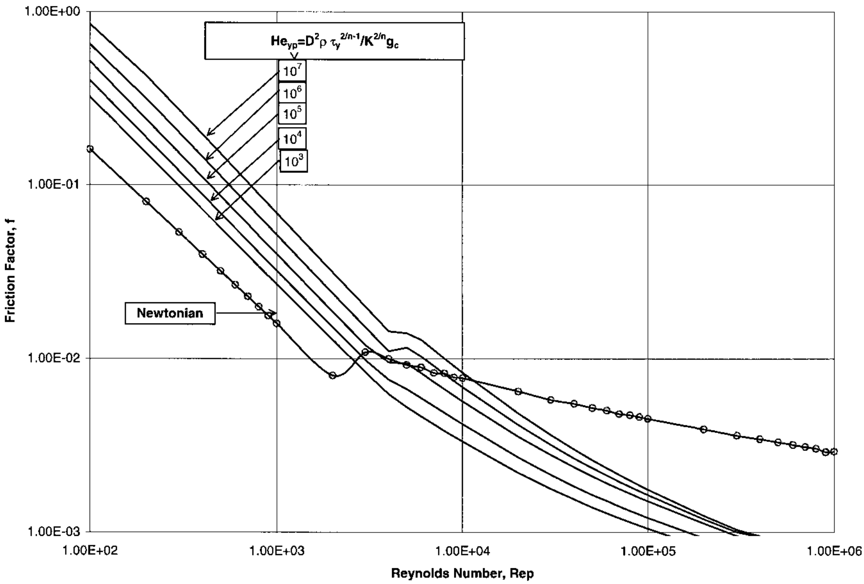

FIGURE C11.9c Friction factor f as a function of Reynolds number for yield pseudoplastic slurries for $n = 0.3$, $\mathrm{He} = [D^2\rho\tau_y(t_y/K)^{2/n-2}]/(K^2g_c$, $\mathrm{Re} = 8\rho D^nV^{2-n}[n/(2+6n)]^n/(Kg_c)$, where D = pipe ID (ft), V = velocity (ft/s), ρ = density (lb/ft^3), K = consistency (lbf · s^n/ft^2), τ_y = yield stress (lbf/ft^2), $g_c = 32.2$ (lb · s/(lbf · ft^2).

Most industrial applications involve nonuniform-size particles. Pilot plant test results or prior experience with similar material is generally used for estimating deposition velocity for nonuniform-size particles.

Oroskar and Turian[29] have developed a semiempirical correlation which can be used for nonuniform-size particles. Their correlation is as follows:

$$V_D = \left\{5C_V(1-C_V)^{2m-1}\left(\frac{D}{d}\right)\left[\frac{D\sqrt{gd(s-1)}}{g_c}\right]^{1/8}\left(\frac{1}{Z}\right)\right\}^{8/15}\sqrt{gd(s-1)} \tag{C11.31}$$

where V_D = deposition velocity, ft/s (m/s)
z = function of V/V_D, as shown in Fig. C11.11
d = mean diameter of particles, ft (m)
w = settling velocity of solid particle in slurry, ft/s (m/s)
m = hindered settling velocity exponent as a function of particle Reynolds number, shown in Fig. C11.12
w_o = settling velocity of solid particle in clear water of infinite extent, ft/s (m/s)
μ = viscosity of water, lbf·s/ft^2 (Pa·s)

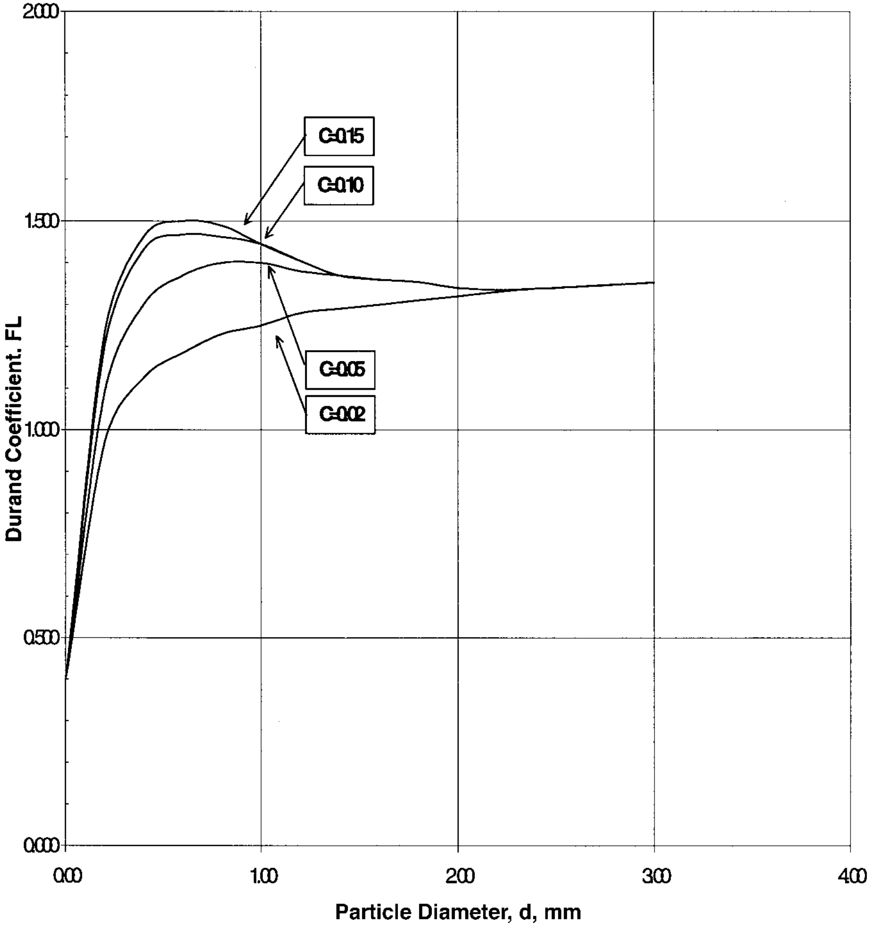

FIGURE C11.10 Durand coefficient F_L as a function of particle size where $V_D = F_L \sqrt{2gD(s-1)}$, where V_D = deposition velocity (ft/s), D = pipe ID (ft), g = 32.2 ft/s^2, s = solids specific gravity.

The settling velocity w_o of a single particle in a fluid is given as

$$w_o^2 = \frac{4gd(\rho_s - \rho_l)}{3C_D\rho_l} \tag{C11.32}$$

where C_D = drag coefficient
d = particle diameter, ft (m)
g = acceleration due to gravity, 32.2 ft/s^2 (9.81 m/s^2)
ρ_s = density of solid particle, lbm/ft^3 (kg/m^3)
ρ_l = density of liquid, lbm/ft^3 (kg/m^3)
w_o = settling velocity, ft/s (m/s)

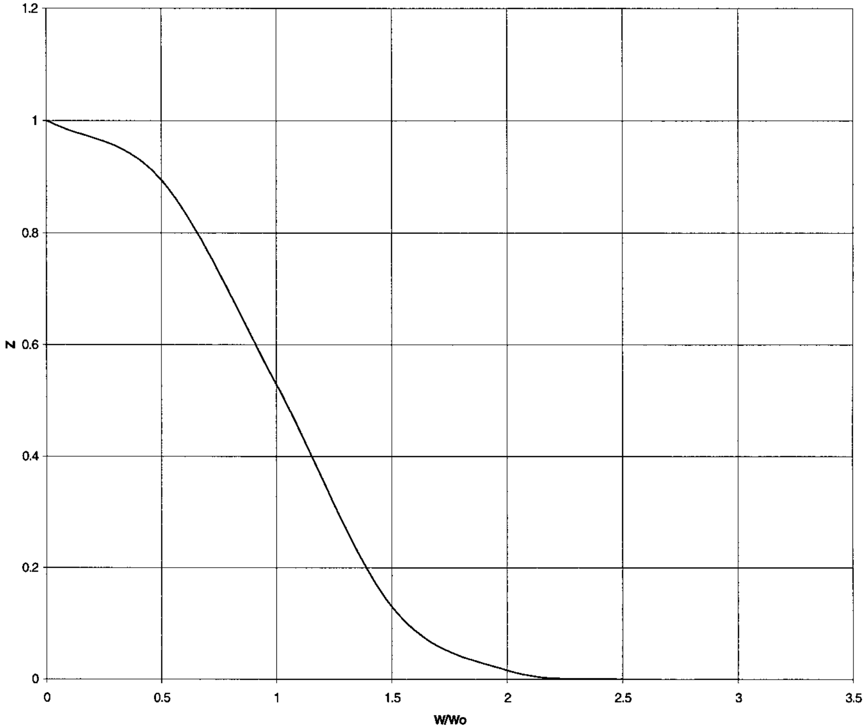

FIGURE C11.11 Function z as a function of w/w_o for use in Eq. (C11.31), where w = hindered settling velocity of solid particle in slurry (ft/s), and w_o = settling velocity of single particle in suspending liquid (ft/s).

The drag coefficient of spherical particles depends upon the particle Reynolds number Re_w.

For $\mathrm{Re}_w < 0.1$, the drag coefficient, C_D is given by

$$C_D = 24/\mathrm{Re}_w \tag{C11.33}$$

where

$$\mathrm{Re}_W = \frac{d\rho_l w_o}{\mu g_c} \tag{C11.34}$$

and μ = viscosity of liquid, lbf·s/ft^2 (Pa·s). The value of C_D equals 0.4 for a particle Reynolds number greater than 1000. Figure C11.13 shows variation in the drag coefficient with particle Reynolds number for spherical particles.

Use of $C_D\ \mathrm{Re}_w^2$ instead of C_D enables determination of the particle Reynolds number from a plot of $C_D\ \mathrm{Re}_w^2$ versus Re_w, shown in Fig. C11.13.

$$C_D \mathrm{Re}_w^2 = \frac{4gd^3(\rho_s - \rho_l)}{3\mu^2 g_c^2} \tag{C11.35}$$

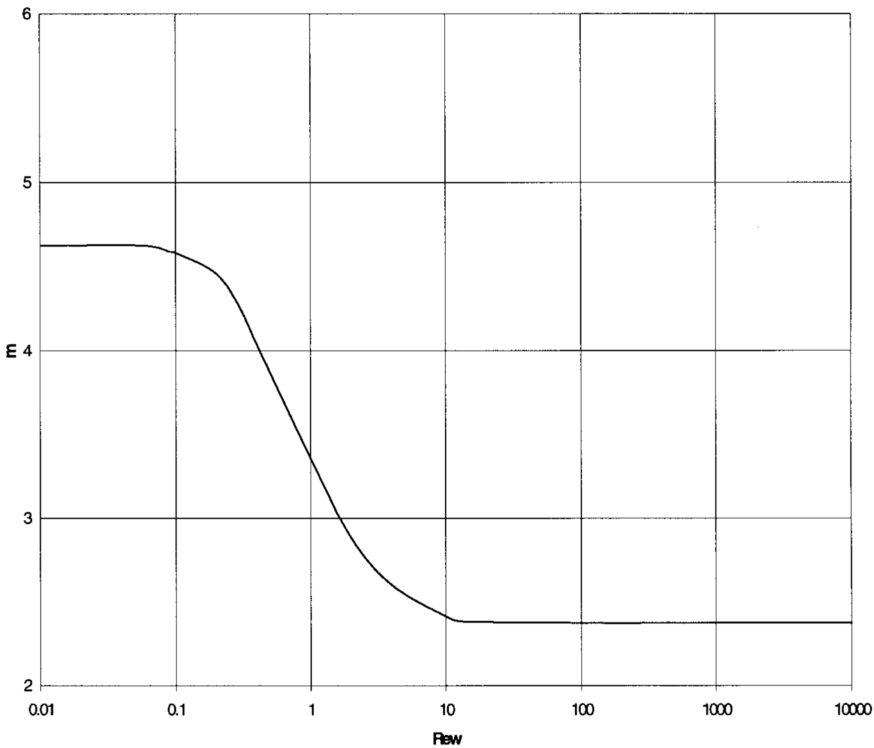

FIGURE C11.12 Hindered settling velocity exponent m as a function of particle Reynolds number $Re_w = d\rho_l w_o/(\mu g_c)$, where d = particle diameter (ft), w_o = particle settling velocity (ft/s), ρ_l = liquid density (lb/ft^3), μ = viscosity of fluid (lb·s/ft), g_c = 32.2 lb·s/(lbf·ft).

The drag coefficient is larger for a nonspherical particle than for a spherical particle of the same diameter. Experimental data on settling velocity as a function of particle diameter may be used to establish the relationship between C_D and Re_w.

The settling velocity of a solid particle in slurry (hindered settling velocity) is given by

$$w = w_o(1 - C_v)^m \tag{C11.36}$$

where the exponent m varies with the particle Reynolds number, as shown in Fig. C11.12.

Wilson[30,31] has developed a semitheoretical analysis for heterogeneous slurries. He has presented nomographs for estimating the deposition velocity for particles larger than 0.15 mm in diameter.

Example C11.8. Estimate the deposition velocity for a sand-water slurry in a 12-in (0.3048-m) ID pipeline. The sand particle diameter is 0.2 mm, and its specific gravity is 2.65. The slurry concentration is 31.9 percent solids.

Solution. Using Eq. (C11.1), we find the slurry density, ρ_m = 77.9 1b/ft^3 (1248 kg/m^3). Using Eq. (C11.6), we find the volume fraction of solids C_v = 0.15. From

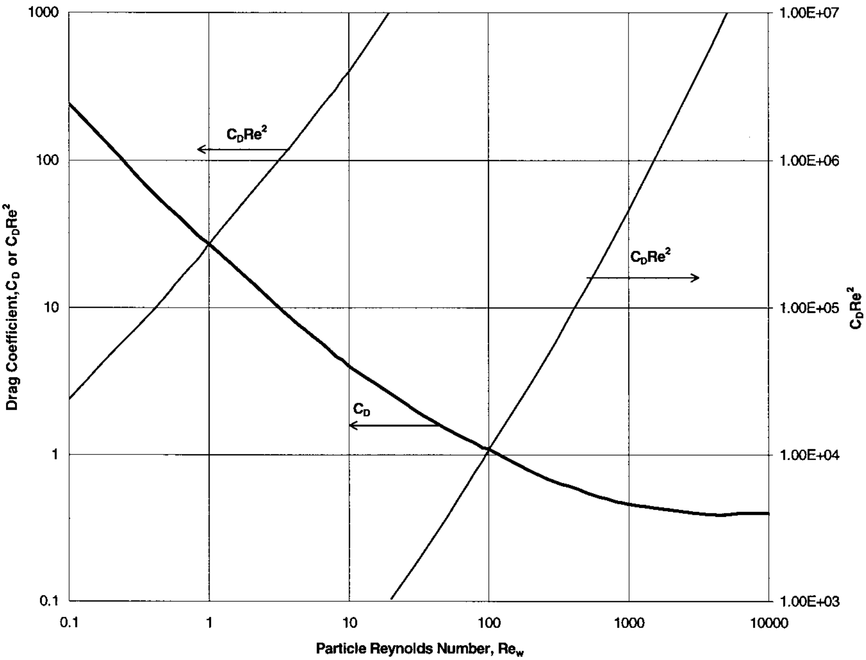

FIGURE C11.13 Drag coefficient C_D of spherical particles settling in liquid. $C_D = 4gd(\rho_s - \rho_l)/[3W_o^2\rho_l]$, where d = particle diameter (ft), w_o = settling velocity (ft/s), g = acceleration due to gravity (32.2 ft/s^2), ρ_s = particle density (lb/ft^3), ρ_l = liquid density (lb/ft^3).

Fig. C11.10, we find the value of F_L equal to 1.3 based on a particle diameter equal to 0.2 mm and C_v equal to 0.15. In Eq. (C11.30) we substitute the following values to compute the deposition velocity V_D:

$$F_L = 1.3 \qquad D = 1\,\text{ft}\,(0.3048\,\text{m}) \qquad g = 32.2\,\text{ft/s}^2\,(9.8\,\text{m/s}^2) \qquad S = 2.65$$

$$V_D = 13.4\,\text{ft/s}\,(4.08\,\text{m/s})$$

Example C11.9. Estimate the deposition velocity for a coal-water slurry having a mean particle diameter equal to 0.01 in (0.0254 mm) in a 12-in (0.3038-m) pipe. The volume fraction of coal in the slurry is 0.4, and the specific gravity of coal is 1.4. The viscosity of water is 2.1×10^{-5} lbf·s/ft^2 (1.0 m·Pa·s).

We are given

$$d = 0.01\,\text{in}\,(0.254\,\text{mm})$$

$$= 8.33 \times 10^{-4}\,\text{ft}\,(2.54 \times 10^{-4}\,\text{m})$$

$$\rho = 62.4\,\text{lb/ft}^3\,(1000\,\text{kg/m}^3)$$

$$\rho_s = 87.36\,\text{lb/ft}^3\,(1400\,\text{kg/m}^3)$$

$$\mu = 2.1 \times 10^{-5}\,\text{lbf}\cdot\text{s/ft}^2\,(0.001\,\text{Pa})$$

$$g_c = 32.2\,\text{lbm}\cdot\text{ft/(lbf}\cdot\text{s}^2)\,(1\text{ in SI units})$$

Thus

$$C_D \mathrm{Re}_w^2 = 84.5$$

From Fig. C11.13, Re_w = 2.7. Using Eq. (C11.32), we find w_o equal to 0.035 ft/s (0.0107 m/s). Next we find the hindered settling velocity exponent m from Fig. C11.12: m = 2.8.

Now we compute the hindered settling velocity, using Eq. (C11.36):

$$w = 0.0084\,\mathrm{ft/s}\,(0.0026\,\mathrm{m/s})$$

The ratio w/w_o = 0.24. From Fig. C11.11, we find z equal to 0.98. Next we compute the deposition velocity, using Eq. (C11.31).

$$V_D = 3.5\,\mathrm{ft/s}\,(1.07\,\mathrm{m/s})$$

Friction Loss for Heterogeneous Flow-Horizontal Pipes. The formula proposed by Durand,[32] Eq. (C11.37), is widely used for graded solids. The formula is based on sand-and-gravel slurries with particle sizes ranging from 0.2 to 25 mm, pipe diameters from 38 to 580 mm, and solids concentrations up to 60 percent by volume.

$$\frac{i - i_w}{C_v i_w} = 81 \left[\frac{V^2 \sqrt{C_D}}{(s-1) g D} \right]^{-1.5} \tag{C11.37}$$

where i = friction loss for slurry, ft (m) water per ft (m)
i_w = friction loss for water, ft (m) water per ft (m) at same velocity
C_D = drag coefficient of suspended solid particle settling in fluid of infinite extent
S = specific gravity of solid particles

Zandi and Govatos[33] concluded from an examination of 2549 data points that Durand's formula predicted the observed head losses fairly well once the saltation data were separated from the heterogeneous flow data. Turian and Yuan[34] have divided the available data into heterogeneous flow, saltation flow, and flow with stationary bed and have developed correlations applicable to each individual type of flow. Their correlations fit the available data better than Durand's correlation.

In Eq. (C11.37), the difference $i - i_w$ represents an increase in pressure drop due to the presence of solids in the slurry. The effect of particle size on slurry pressure drop is accounted for by the inclusion of the drag coefficient C_D.

In most industrial applications, the particle size is not uniform. A mean value of C_D is developed in Eq. (C11.37) to account for actual size variations. Equation (C11.37) can be written as follows:

$$\frac{i - i_w}{i_w} = 81 \left[\frac{V^2}{(s-1) g D} \right]^{-1.5} C_v C_D^{-0.75} \tag{C11.38}$$

$$\frac{i - i_w}{i_w} = 81 \left[\frac{V^2}{(s-1) g D} \right]^{-1.5} \sum_{i=1}^{N} C_{vi} C_{Di}^{-0.75} \tag{C11.39}$$

where C_{vi} = volume fraction of solids having size d_i
d_I = particle size of ith fraction
C_{Di} = drag coefficient of particle having size d_i
N = total number of size fractions into which given particle size distribution is divided

TABLE C11.2 Tyler Screen Sizes

Sieve designation		Sieve opening		Nominal wire diameter		Tyler equivalent designation
Standard	Alternate	mm	in (approx.)	mm	in (approx.)	
107.6 mm	4.24 in	107.6	4.24	6.40	0.2520	
101.6 mm	4 in	101.6	4.00	6.30	0.2480	
90.5 mm	3.5 in	90.5	3.50	6.08	0.2394	
76.1 mm	3 in	76.1	3.00	5.80	0.2283	
64.0 mm	2.5 in	64.0	2.50	5.50	0.2165	
53.8 mm	2.12 in	53.8	2.12	5.15	0.2028	
50.8 mm	2.00 in	50.8	2.00	5.05	0.1988	
45.3 mm	1¾ in	45.3	1.75	4.85	0.1909	
38.1 mm	1½ in	38.1	1.50	4.59	0.1807	
32.0 mm	1¼ in	32.0	1.25	4.23	0.1665	
26.9 mm	1.06 in	26.9	1.06	3.90	0.1535	1.050 in
25.4 mm	1.00 in	25.4	1.00	3.80	0.1496	
22.6 mm	⅞ in	22.6	0.875	3.50	0.1378	0.883 in
19.0 mm	¾ in	19.0	0.750	3.30	0.1299	0.742 in
16.0 mm	⅝ in	16.0	0.625	3.00	0.1181	0.624 in
13.5 mm	0.530 in	13.5	0.530	2.75	0.1083	0.525 in
12.7 mm	½ in	12.7	0.500	2.67	0.1051	
11.2 mm	7⁄16 in	11.2	0.438	2.45	0.0965	0.441 in
9.51 mm	⅜ in	9.51	0.375	2.27	0.0894	0.371 in
8.00 mm	5⁄16 in	8.00	0.312	2.07	0.0815	2½ mesh
6.73 mm	0.265 in	6.73	0.265	1.87	0.0736	3
6.35 mm	¼ in	6.35	0.250	1.82	0.0717	3½
5.66 mm	No. 3½ in	5.66	0.223	1.68	0.0661	4
4.76 mm	No. 4	4.76	0.187	1.54	0.0606	5
4.00 mm	No. 5	4.00	0.157	1.37	0.0539	6
3.36 mm	No. 6	3.36	0.132	1.23	0.0484	7
2.83 mm	No. 7	2.83	0.111	1.10	0.0430	8
2.38 mm	No. 8	2.38	0.0937	1.00	0.0394	9
2.00 mm	No. 10	2.00	0.0787	0.900	0.0354	10
1.68 mm	No. 12	1.68	0.0661	0.810	0.0319	12
1.41 mm	No. 14	1.41	0.0555	0.725	0.0285	14
1.19 mm	No. 16	1.19	0.0469	0.650	0.0256	16
1.00 mm	No. 18	1.00	0.0394	0.580	0.0228	20
841 μm	No. 20	0.841	0.0331	0.510	0.0201	24
707 μm	No. 25	0.707	0.0278	0.450	0.0177	28
595 μm	No. 30	0.595	0.0234	0.390	0.0154	32
500 μm	No. 25	0.500	0.0197	0.340	0.0134	35
420 μm	No. 40	0.420	0.0165	0.290	0.0114	42
354 μm	No. 45	0.354	0.0139	0.247	0.0097	48
297 μm	No. 50	0.297	0.0117	0.215	0.0085	60

TABLE C11.2 Tyler Screen Sizes (*Continued*)

Sieve designation		Sieve opening		Nominal wire diameter		Tyler equivalent designation
Standard	Alternate	mm	in (approx.)	mm	in (approx.)	
250 μm	No. 60	0.250	0.0098	0.180	0.0071	65
210 μm	No. 70	0.210	0.0083	0.152	0.0060	80
177 μm	No. 80	0.177	0.0070	0.131	0.0052	100
149 μm	No. 100	0.149	0.0059	0.110	0.0043	115
125 μm	No. 120	0.125	0.0049	0.091	0.0036	150
105 μm	No. 140	0.105	0.0041	0.076	0.0030	170
88 μm	No. 170	0.088	0.0035	0.064	0.0025	200
74 μm	No. 200	0.074	0.0029	0.053	0.0021	250
63 μm	No. 230	0.063	0.0025	0.044	0.0017	270
53 μm	No. 270	0.053	0.0021	0.037	0.0015	325
44 μm	No. 325	0.044	0.0017	0.030	0.0012	400
37 μm	No. 400	0.037	0.0015	0.025	0.0010	

Note that the particle size distribution is determined by screen analysis. Tyler mesh screens are widely used. The diameter of openings for Tyler screens is given in Table C11.2.

Example C11.10. Estimate the friction loss for a coal-water slurry in a 12-in-ID (0.3048-m-ID) pipe at a velocity of 8 ft/s (2.44 m/s) based on the following data:

Coal specific gravity	1.4
Volume fraction of coal in slurry	0.2
Pipe roughness	0.002 in (0.05 mm)
Viscosity of water	2.1×10^{-5} lbf·s/ft² (1 m·Pa·s)

The particle size distribution is as follows:

Diameter, in (mm)	Weight %
0.24 (6.1)	10
0.12 (3.05)	40
0.06 (1.52)	40
0.03 (0.76)	10

Solution. Find the drag coefficient of particles of individual size fraction.

Particle diameter d_i, ft (m)	$C_D \text{Re}_{w^2} i$	Re i	C_{Di}
0.02 (0.0061)	1.17×10^6	1710	0.4
0.01 (0.00305)	1.46×10^5	520	0.54
0.005 (0.00152)	1.83×10^4	145	0.87
0.0025 (0.00076)	2.29×10^3	36	1.76

Now,

$$\Sigma C_{vi} C_{Di}^{-0.75} = 0.269$$

Next we find the friction loss i_w for water. The Reynolds number for water is 2.38×10^7. And relative roughness of pipe = 0.0002/12 = 0.000167. From Fig. C11.6, f = 0.00325. Next we find

$$i_w = 4f \frac{V'^2}{2gD}$$
$$= 0.0129 \text{ ft/ft}$$

Using Eq. (C11.39), we get

$$\frac{i - i_w}{i_w} = 1.97$$

Thus,

$$\text{Friction loss for slurry } i = (1 + 1.97)i_w$$
$$= 0.0383 \text{ ft water/ft}$$

Friction Loss for Saltation Flow-Horizontal Pipes. At low velocities or with particles having large settling velocities, flow with a moving bed or saltating particles may arise.

The saltation flow regime is encountered when N_I is less than 40.

$$N_I = \frac{V^2 \sqrt{C_D}}{C_v g D(s - 1)} \quad \text{(C11.40)}$$

Newitt et al.[35] have developed the following formula based on their experiments:

$$\frac{i - i_w}{i_w C_v} = 66(s - 1)\frac{gD}{V^2} \quad \text{(C11.41)}$$

Babcock[36] has proposed the following formula based on his tests:

$$\frac{i - i_w}{i_w C_v} = 60.6(s - 1)\frac{gD}{V^2} \quad \text{(C11.42)}$$

Friction Loss for Intermediate Regime. Most of the formulas for heterogeneous flow and saltation regime are applicable to uniform particles. For a mixture of two or more size fractions, it is necessary to determine an average particle diameter for use in various formulas. Use of a weighted-average diameter or drag coefficient was illustrated in the previous section.

For slurries containing finely ground particles, a better approach is to divide solid particles into a fraction that is carried in homogeneous flow and a fraction that is carried in heterogeneous or saltation regimes. The friction losses for each fraction are computed separately, using appropriate formulas, and then added to obtain the total slurry friction loss. This approach also allows the use of nonnewtonian flow properties of the slurry in computing homogeneous friction loss. Wasp et al.[37] have successfully used this approach for correlating coal slurry data.

The method of Wasp et al.[38] is an iterative procedure which works as follows:

1. Divide the total size fraction into a heterogeneous part and a homogeneous part, using the following formula:

$$\log \frac{C}{C_A} = -1.8 \left(\frac{w}{\beta \kappa u^*} \right) \tag{C11.43}$$

where C = volume fraction of solids at $0.98D$ from bottom
C_A = volume fraction of solids at pipe axis
$u^* = \sqrt{\tau_w g_c / \rho}$ = friction velocity, ft/s (m/s)
w = settling velocity of particles, ft/s (m/s)
β = ratio of mass-transfer coefficient to momentum-transfer coefficient (about 1)
κ = Von Karman constant = 0.4

It is assumed that for each size fraction, the fraction C/C_A is homogeneously distributed and that the remainder is heterogeneously distributed. At the start of the iteration, the slurry may be assumed to be homogeneous to compute the initial value of the wall shear stress.

2. Compute the friction losses for the homogeneous part, using the rheological properties of the slurry. Compute the friction loss for the heterogeneous part, using Durand's formula [Eq. (C11.38)]. The sum of the two parts gives an initial estimate of the slurry friction losses.

3. Determine the C/C_A values of each size fraction based on the value of friction loss estimated in step 2.

4. Based on these new values of C/C_A, determine the fraction of solids in the homogeneous phase and in the heterogeneous phase.

5. Recompute the friction loss for slurry as in step 2. This provides a new estimate of slurry friction losses. The iteration is continued until the new estimate closely agrees with the previous estimate. The computational procedure is suitable for analysis using a digital computer.

Lazarus[39] has described a method of estimating friction loss as well as determining whether a sliding or stationary bed of solids exists for slurries containing a mixture of coarse and fine particles.

Wilson and coworkers[40] have developed a mechanistic approach for estimating deposition velocity and friction losses for mixtures of fine and coarse slurries. Gillis et al.[41] have described a method of estimating friction losses for such slurries based on Wilson's mechanistic approach.

Friction Loss in Vertical Pipes. In vertical pipe flow, there is an absence of a concentration gradient. The slurry flow may be treated as homogeneous flow. For coarse particles, the friction loss for slurry has been found to be the same as that for water at the same velocity. With fine particles, the viscosity of the slurry should be considered in computing the friction losses. The friction factor for slurry is estimated using the equations presented earlier.

Friction Losses in Inclined Pipes. Worster and Denny[42] have proposed the following equation relating the friction loss for inclined pipes with that for horizontal pipe for heterogeneous slurries:

$$i_\theta = i_w + (i - i_w)\cos\theta \tag{C11.44}$$

where i_θ = friction loss in inclined pipe
θ = angle of inclination of pipe from horizontal

This equation suggests that the friction losses in an inclined pipe are the same with both up- and downflow and that they are smaller than those in a horizontal pipe.

Experimental evidence presented by Kao and Hwang[43] shows that the friction losses in an inclined pipe with upflow first increase and then decrease after the angle reaches a certain magnitude. In the case of downflow, the friction losses are less than those for horizontal pipe flow.

Friction Losses in Pipe Fittings. Turian et al.[44] have shown that the friction loss in fittings can be approximated by using the relations for single-phase newtonian fluid provided that the density of the liquid is set equal to that of the slurry.

DESIGN FEATURES

System Components

Figure C11.14 shows a sketch of a slurry pipeline system. It includes a slurry preparation plant, pipeline, pump stations, and slurry receiving terminal.

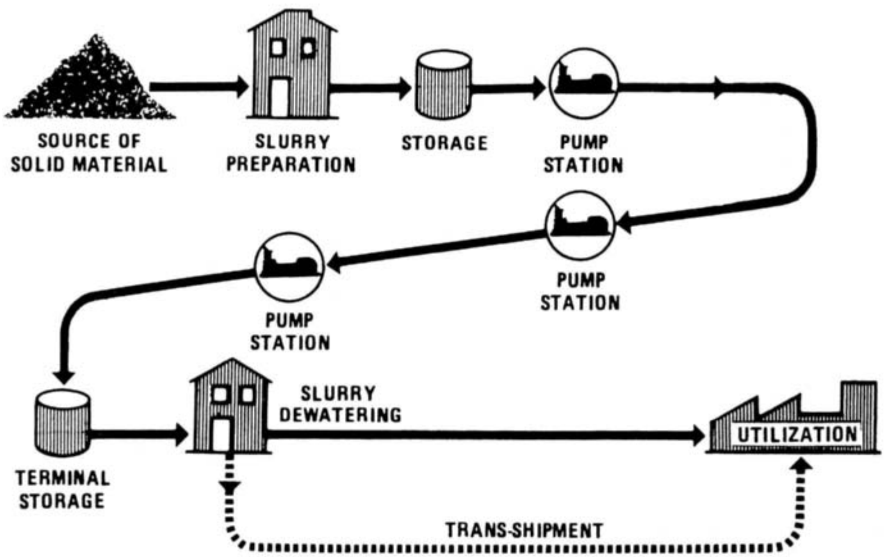

FIGURE C11.14 Sketch of a slurry pipeline system.

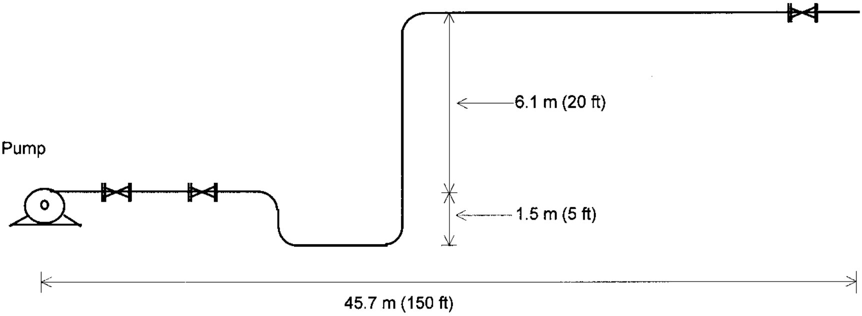

FIGURE C11.15 Sketch of pumping system for Example C11.10.

Slurry handling operations in a preparation plant may involve pipelining of slurry from one processing unit to another. Example C11.11 illustrates the design of such a piping system.

Example C11.11. Determine the pipe size and pumping requirements for the slurry piping system schematically shown in Fig. C11.15. The design basis for the system is as follows:

Maximum flow rate	2000 gpm (455 m^3/h)
Minimum flow rate	1000 gpm (227.5 m^3/h)
Slurry concentration, wt%	50
Specific gravity of solids	3.0
Slurry specific gravity	1.5

Estimated deposition velocity in 12-in (0.3048-m) pipe is 4.0 ft/s (1.22 m/s). The average value of C_D for use in Eq. (C11.37) is 50. Viscosity of water = 1 m·Pa·s (2×10^{-5} lbf·s/ft^2). Use steel pipe with a roughness of 4.57×10^{-5} m (0.00015 ft).

Solution. Assuming that the deposition velocity varies as the square root of the pipe inside diameter [refer to Eq. (C11.30)], estimate the pipe diameter that will give a velocity greater than the deposition velocity at the minimum flow rate of 1000 gpm (227.5 m^3/h).

$$\text{Minimum flow} = 1000 \text{ gpm } (227.5 \text{ m}^3/\text{h})$$
$$= 2.23 \text{ ft}^3/\text{s } (0.063 \text{ m}^3/\text{s})$$

Let D = pipe ID, ft (m)

$$\text{Deposition velocity} = 4D^{1/2} \text{ ft/s}$$

Let the velocity at the minimum flow rate be 1 ft/s (0.3 m/s) above the estimated deposition velocity.

Thus,

$$\frac{2.23}{(\pi/4)D^2} = 4\sqrt{D} + 1$$

$$D = 0.79 \text{ ft } (0.241 \text{ m})$$

Try 10.75-in (273-mm) outside-diameter pipe with 0.25-in (6.4-mm) wall thickness and 0.375-in (9.5-mm) thick rubber lining.

$$\begin{aligned}\text{Pipe ID} &= 10.75 - 2(0.25 + 0.375)\\ &= 9.5 \text{ in}\\ &= 0.79 \text{ ft } (0.241 \text{ m})\end{aligned}$$

The computations of friction loss and pumping head requirement are summarized in Table C11.3.

Pipe Wall Thickness

The pipe wall thickness must be sufficient to withstand the expected maximum pressure in the pipe and expected corrosion-erosion effects on the pipe wall during the intended operating lifetime. Most pipelines are designed to have a service life of at least 10 years. An example based on the ASME B31.11 Code, with enhancements based on typical service conditions, follows:

The pipe wall thickness is computed by using the following equation:

$$t = \frac{pD}{2S} + c \tag{C11.45}$$

where t = pipe wall thickness, in (mm)
p = maximum design pressure in pipe, psi (Pa)
S = maximum allowable design stress, psi (Pa)
c = allowance for corrosion-erosion, inch (mm)

The maximum allowable pipe stress is given as follows, per ASME B31.11:

$$S = 0.8E \times \text{ specified minimum yield strength of pipe}$$

where E = weld joint factor, as summarized in Table C11.4.

Example C11.12. Determine the wall thickness required at point A in the pipeline shown in Fig. C11.16. A 12-in (324-mm) steel pipe is used. The pipe is manufactured in accordance with API 5LX. The minimum yield strength of the pipe is 52,000 psi (358.5 MPa). The design hydraulic gradient is shown in Fig. C11.16. The design head at point A is 800 ft (244 m). The slurry specific gravity is 1.5. The metal loss due to corrosion-erosion is estimated to be 4 mils/yr (0.1 mm/yr), and the design life of the pipeline is 25 yr.

Solution. Maximum design pressure at A

$$\text{Maximum design pressure at } A = 800 \text{ ft slurry}$$
$$= \frac{800 \times 1.5}{2.31 \text{ psi}}$$
$$= 520 \text{ psi } (3589 \text{ kPa})$$

$$\text{Maximum allowable stress} = 0.8 \times \text{minimum yield strength of pipe steel}$$
$$= 0.8 \times 52{,}000 \text{ psi}$$
$$= 4160 \text{ psi } (286.8 \text{ MPa})$$

$$\text{Corrosion allowance } C = 0.004 \times 25 \text{ in}$$
$$= 0.1 \text{ in } (2.5 \text{ mm})$$

$$\text{Wall thickness } t = \frac{pD}{2S} + c$$
$$= \frac{4160(2.75)}{2(4160)} + 0.1$$
$$= 0.18 \text{ in } (4.53 \text{ mm})$$

The next-higher commercially available pipe wall thickness is 0.188 in (4.78 mm).

Corrosion-Erosion Control

In a slurry pipeline, metal loss is expected to be a result of corrosion with possible erosion of the corrosion products taking place simultaneously. Under some conditions, mechanical abrasion will play a part in producing the metal loss.

Erosive wear (abrasive) is governed by the size, shape, and angularity of the solids, slurry concentration, and velocity of flow. In a slurry pipeline, these parameters are interdependent to some extent. For example, use of large solids requires an increase in minimum transportation velocity. It has been found that above some critical velocity, the abrasive wear increases as the cube of slurry velocity. Wear also increases as the size of the solid particles increases. Thus, by reducing the size of the solids, the abrasive wear can be substantially reduced due to the combination of lower required velocity and reduction in wear due to smaller particle size. The effect of slurry concentration on the abrasive wear is more complicated.

From experience, it has been found that the metal loss due to abrasion is insignificant if the velocity of flow is less than about 10 ft/s (3 m/s). For long-distance slurry pipelines, velocities in the range of 4 to 6 ft/s (1.2 to 1.8 m/s) result in an optimum design from the standpoint of economics. Thus, when possible, a particle size should be selected so that the slurry is nearly homogeneously suspended at velocities of 4 to 6 ft/s (1.2 to 1.8 m/s).

Corrosion can be controlled by passivating either the anodic or the cathodic reaction at the pipe wall. Elimination of dissolved oxygen and the adjustment of slurry pH can reduce the corrosion rate substantially.[45] In most long-distance slurry

TABLE C11.3 Summary of Hydraulic Computations

Item	Min. flow Metric	Max. flow Metric	Min. flow U.S. Units	Max. flow U.S. Units	Remarks
1. Flow rate, ft^3/s (m^3/s)	0.095	0.126	3.34	4.45	Given
2. Velocity, ft/s (m/s)	2.1	2.8	6.8	9.1	Flow/area
3. Solids specific gravity	3.00	3.00	3.00	3.00	Given
4. Volume fraction of solids Cv	0.25	0.25	0.25	0.25	Use Eq. (C11.6)
5. $V^2/[2gD(s - 1)CD1/2]$	6.45	11.46	6.45	11.46	
6. $(i - iw)/iw$	1.24	0.52	1.24	0.52	Use Eq. (C11.37)
7. Viscosity of water, $lbf.s/ft^2$ ($Pa \cdot s$)	9.58E-04	9.58E-04	2.00E-05	2.00E-05	Given
8. Density of water, lb/ft^3 (kg/m^3)	1000	1000	62.4	62.4	Given
9. Reynolds no. for water	5.22E + 05	6.95E + 05	5.22E + 05	6.95E + 05	
10. Relative roughness of pipe	0.00019	0.00019	0.00019	0.00019	Roughness/pipe ID
11. Friction factor for water	0.00384	0.00375	0.00384	0.00375	From Fig. C11.6
12. Friction loss for water, ft/ft (m/m)	0.0140	0.0243	0.0140	0.0243	Use Eq. (C11.17)

TABLE C11.3 Summary of Hydraulic Computations *(Continued)*

13. Friction loss for slurry:					
(a) Horizontal pipe: ft(m) water/ft (m)	0.0314	0.0370	0.0313	0.00370	Use Eq. (C11.37)
ft (m) slurry/ft (m)	0.0209	0.0247	0.0209	0.0247	ft slurry = ft water/slurry sp. gr.
(b) Vertical pipe: ft (m) slurry/ft (m)	0.0140	0.0243	0.0140	0.0243	Same as water
14. Length of horizontal pipe, ft (m)	45.7	45.7	150	150	
15. Length of vertical pipe, ft (m)	9.1	9.1	30	30	
16. Friction loss, ft (m) slurry					
(a) Horizontal pipe, ft (m) slurry	0.96	1.13	3.13	3.70	Given
(b) Vertical pipe, ft (m) slurry	0.13	0.22	0.42	0.73	Given
17. Head loss through three valves, ft (m)	0.11	0.20	0.37	0.66	$3 \times V/(2g) \times K$ valve
18. Head loss through four bends, ft (m)	0.40	0.71	1.31	2.33	$4 \times V/(2g) \times K$ bends
19. Total head loss, ft (m) slurry	1.60	2.26	5.23	7.42	Sum of 16 (a), 16 (b), 17, and 18
20. Static head, ft (m) slurry	6.10	6.10	20	20	Elevation (outlet − inlet)
21. Total pumping head ft (m) slurry	7.69	8.36	25.23	27.42	Sum of 19 and 20

TABLE C11.4 Weld Joint Factor

Specification no.	Pipe type*	Weld joint factor: Pipe mfd. before 1959	Weld joint factor: Pipe mfd. after 1959
ASTM A 53	Seamless	1.00	1.00
	Electric resistance-welded	0.85†	1.00
	Furnace lap-welded	0.80	0.80
	Furnace butt-welded	0.60	
ASTM A 106	Seamless	1.00	1.00
ASTM A 134	Electric fusion (arc) welded, single- or double-pass	0.80	0.80
ASTM A 135	Electric resistance-welded	0.85†	1.00
ASTM A 139	Electric fusion-welded, double-submerged arc-welded	0.80	0.80
ASTM A 155	Electric fusion-welded	0.90	1.00
ASTM A 381	Electric fusion-welded, double-submerged arc-welded	—	1.00‡
ASTM A 672	Electric fusion-welded	—	1.00
API 5L	Seamless	1.00	1.00
	Electric resistance-welded	0.85†	1.00
	Electric flash-welded	0.85†	1.00
	Electric induction-welded	—	1.00
	Submerged arc-welded	—	1.00
	Furnace lap-welded	0.80	0.80§
	Furnace butt-welded	0.60	0.60

* Definitions for the various pipe types (weld joints) are given in paragraph 1100.2 of ASME B31.11 Code.

† A weld joint factor of 1.0 may be used for electric resistance-welded or electric flash-welded pipe manufactured prior to 1959 where

1. Pipe furnished under this classification has been subjected to supplemental tests and/or heat treatment as agreed to by the purchaser, and such supplemental tests and/or heat treatment demonstrates the strength characteristics of the weld to be equal to the minimum tensile strength specified for the pipe; *or*
2. Pipe has been tested as required for a new pipeline in accordance with paragraph 1137.4.1 of the ASME B31.11 Code.

‡ For classes and grades that have been hydrostatically and nondestructively tested to specification requirements.

§ Manufacture was discontinued and process deleted from API 5L in 1962.

pipelines carrying mineral concentrate, the slurry pH is adjusted to 9.0 or higher, using lime to reduce the corrosion rate.

Slack Flow and Its Control

Slack flow occurs when the available static head between the discharge point of the pipeline and a given point in the pipeline exceeds the friction loss between

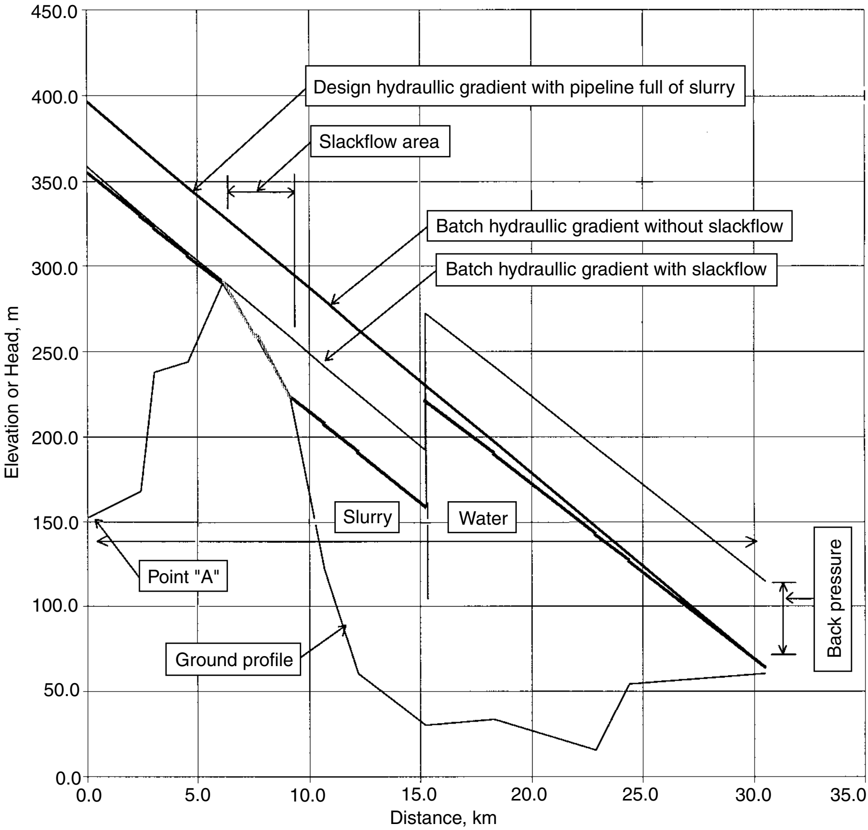

FIGURE C11.16 Pipeline profile and hydraulic gradient showing design gradient, batch gradient, and slack flow area.

those points at a given flow rate. It usually occurs just downstream of peaks where the hydraulic gradient intersects the ground profile. For example, Fig. C11.16 shows slack flow area.

In slack flow areas, the velocity of flow is governed by the pipe slope. The velocity in the slack flow area is higher than that in the fully packed pipe section. The section of pipe which flows full and is under pressure is called *fully packed.* In slack flow sections the pipe flows partly full, and pressure generally drops below atmospheric. Erosion takes place in the slack flow area owing to higher flow velocity as well as possible cavitation at the point where the pipeline changes from slack flow to a packed flow condition.

Because of the higher density of slurry compared to water, slack flow can occur when water is displaced by slurry. For example, Derammelaere and Chapman[46] have described the development of slack flow in the Samarco iron concentrate pipeline during operation of the pipeline in batch mode. This is illustrated in Fig.

C11.16, where the hydraulic gradients for pipe full of slurry as well as for batch operation are shown.

Slack flow can be avoided by using a smaller-diameter pipe section or orifice chokes. The velocity of flow in the smaller-diameter pipe should not exceed about 10 ft/s (3 m/s); otherwise, erosion may occur. Either orifice chokes or a combination of orifice chokes and smaller-diameter pipe may be used to achieve flexibility and economy.

Storage

It is very seldom that the flow of solids in a slurry pipeline matches its rate of production at the mine site or its rate of utilization at the terminal facility. To provide means by which the pipeline can operate efficiently by transporting solids in a nearly continuous fashion, storage facilities are required at both ends of the line. The type and amount of storage are determined by the specific system operating parameters as well as the characteristics of the material being transported.

Tanks

For live slurry storage at the head and tail ends of a long-distance pipeline system, agitated tanks are typically utilized. The number and size to be used are determined by comparing the plant and pipeline availabilities in addition to normal engineering criteria such as allowable soil loadings and an economic analysis of available sizes. If the pipeline is to be operated in batch mode, for the first few years of operation, because of low throughput requirements, the size of the tanks may be determined by the minimum batch length chosen for the system. This in turn is a function of pipeline length.

Agitated storage may also be required at intermediate pump stations. The purpose would be to facilitate pipeline section reconnection during restart operations as well as hold slurry which has been flushed from station piping during pump change-out.

Ponds

Ponds are utilized for the storage of high volumes of solids over intermediate to long periods. Ponds are generally used when the storage requirement exceeds 24 h of pipeline flow. Their prime advantage is that no energy is required to maintain continuous suspension of the solids.

Slurry storage ponds can be classified into two basic types: semiactive and dead storage. Semiactive ponds are equipped for recovery and reslurrying on short notice. Depending on the design, it may be necessary to remove all the stored material before refilling the pond.

The major consideration in pond design is recovery of a uniform solids particle size distribution. Segregation of the solids can occur during the filling or recovery operation. The resulting coarse and fine slurry slugs can be very difficult to handle in the downstream processes. Three major recovery methods have been utilized: mechanical, dredge, and the Marconaflo technique.

Mechanical recovery uses conventional earthmoving equipment to remove the settled solids. Prior to the start of recovery operations, the bulk of the free water must be removed from the pond by evaporation, natural drainage, sand points, or an underdrain system.

Dredge recovery requires the maintenance of a water layer over the settled solids to float the dredge. The recovery dredge may be maintained on site or moved in for the recovery operation, depending on whether semiactive or dead storage is required.

The Marconaflo technique[47] was originally developed to remove settled slurries from the hold of ore ships. A high-pressure water jet on a rotating head undercuts and reslurries the solids, and the material flows into an underdrain system for pumping away.

Slurry Pumps

Centrifugal as well as positive displacement pumps are available for pumping slurry. The maximum pressure capability and maximum particle size limits for different types of pumps are shown in Fig. C11.17.[48]

The positive displacement pumps can be divided into piston, diaphragm, and plunger pumps. Piston pumps can be used for relatively less abrasive materials while the diaphragm and plunger pumps are used for handling abrasive slurries at high pressures. The initial capital costs and maintenance costs of positive displacement pumps are higher than those of centrifugal pumps, but their hydraulic efficiency is 85 percent, compared to about 60 to 70 percent for the centrifugal slurry pumps. The flow rate per pump is limited in the case of positive displacement pumps while

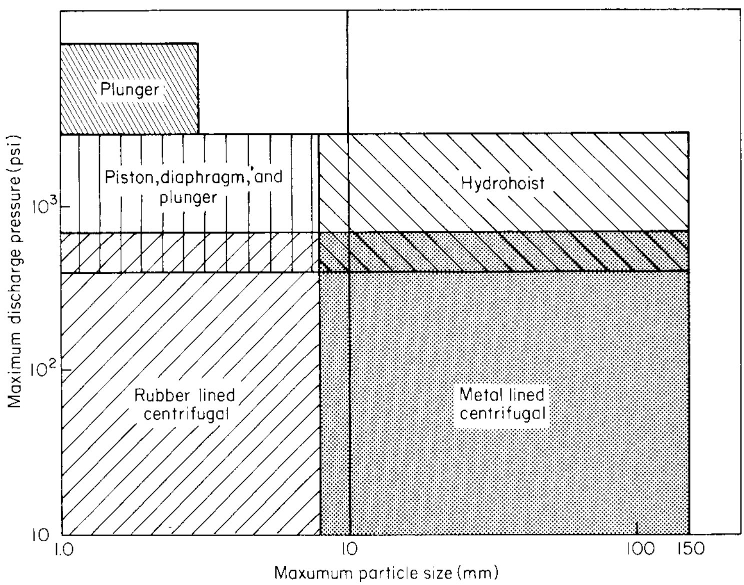

FIGURE C11.17 Maximum particle size and pressure capabilities of different types of pumps.

the head developed per pump is limited to about 30 m in the case of centrifugal slurry pumps.

Centrifugal Slurry Pumps. Centrifugal pumps are extensively used for pumping slurry under relatively low pressure. The main advantages of these pumps are as follows:

1. High flow rates can be achieved with a single unit at a relatively low initial cost. Pumps capable of pumping 25,000 gpm (5680 m^3/h) are available.
2. Very few moving and wearing parts are involved.
3. They are simple to operate and maintain.
4. There is no practical restriction on the maximum size of solids that can be handled.
5. The flow through the pump is pulse-free.
6. They require little space.
7. Valves are not required for the operation of the pump.

Centrifugal pumps have the following shortcomings:

1. The maximum discharge head is limited to less than 130 ft (40 m) for a single-stage pump. With several pumps in series, a maximum pressure of about 750 psi (5.17 MPa) can be achieved.
2. The flow rate of the pump is governed by the system pressure.
3. Attrition of friable solids may occur due to the high velocity of flow through the pump. Attrition can become important if the slurry has to pass through a number of pumps.
4. Seal liquid is required for good packing life. The seal liquid dilutes the slurry. The amount of dilution could become significant when the slurry passes through a number of pumps.
5. Centrifugal slurry pumps are made robust because of the abrasion of parts coming in contact with slurry, resulting in a low pump efficiency.

The centrifugal pump parts exposed to wear from slurry are the casing, impeller, and gland seal. Mechanical shaft seals often are ineffective in slurry installations. A seal that incorporates a liquid flush to keep solids from entering the gland is necessary.

To obtain good service, the casing and impeller should be lined with abrasion-resistant material. Both rubber-lined and ‘‘Ni-hard’’ (metal-lined) units are used extensively. The size of the solids to be pumped determines the type of pump to be selected. Rubber-lined pumps are generally used with particles up to about 0.375 in (9.5 mm), and Ni-hard pumps are used for coarser slurries. However, if material with sharp cutting edges is being pumped, such as crushed glass, Ni-hard pumps can be used even for relatively fine solids. Rubber-lined pump parts usually have longer service life with fine materials. Coal is one material for which Ni-hard line pumps have shown better parts life than rubber-lined units.

To obtain good pump parts life, it is good practice to limit the impeller tip speed to less than 4000 ft/min (1220 m/min). The pump parts life on units running faster than this speed drops in proportion to the square of the impeller tip speed.

Reciprocating Pumps. Reciprocating pumps have several desirable features:

- The flow rate of the pump is independent of system pressure.
- They can meet any reasonable system discharge pressure requirement. Pumps capable of producing 2300 psi (15.8 MPa) pressure have been used in magnetite pipelines. Units capable of discharge pressures greater than 5,000 psi (34.5 MPa) are available.
- The overall efficiency of the pump, including drivetrain, is relatively high—on the order of 85 percent.
- Pipeline flow rate can be determined without the use of a flowmeter.

The following disadvantages are associated with this type of unit:

- The maximum flow rate per pump is limited to less than about 3900 gpm. Furthermore, this capacity is only available at relatively low discharge pressures. Therefore, a large number of pumps operating in parallel are needed to handle the high flow rates and working pressures found in large long-distance systems. For example, seven 1,250-hp units are used at each pump station of the Samarco system.
- Initial capital costs and maintenance costs are usually high. Skilled labor is required for operation and maintenance.
- Variable-speed drives are needed to vary flow rates.
- The flow through the pump is pulsating, which requires greater attention to station piping design to avoid vibration and fatigue problems.
- The maximum size of particles that can be pumped is restricted by the check valve seal requirements.

For material with a maximum particle size of less than 0.1 in (2.5 mm) and with discharge pressures up to about 2000 psi (13.8 MPa), either piston or plunger pumps can be used. Slurry with a maximum particle size of 0.1 to 0.25 in (2.5 to 6 mm) may also be handled with these types of pumps if special design pump valves are used.

The decision to use piston or plunger pumps is usually based on the results of a Miller abrasivity test. Material with a Miller number below 30 can be handled using piston pumps, and material with a Miller number above 60 should be pumped with plunger units. Between these values, the type of pump to use is based upon other considerations:

- Piston pumps can be of double-acting design so that about twice the flow rate can be obtained for the same physical pump size.
- Plunger pumps are more adapatable to flushing and lubrication. A flushed stuffing box can prolong parts life. However, a flush fluid free of solids must be provided, and some dilution of the slurry will result.

Piston Pumps. Figure C11.18 shows the fluid end of a conventional piston pump. With such a unit, major wear occurs on the pistons, valves, cylinder liners, piston rods, and packing.

To reduce maintenance, several variations of piston pumps have been developed that limit the number of working parts in contact with the slurry. One such type is the diaphragm pump, the fluid end of which is sketched in Fig. C11.19. By pumping

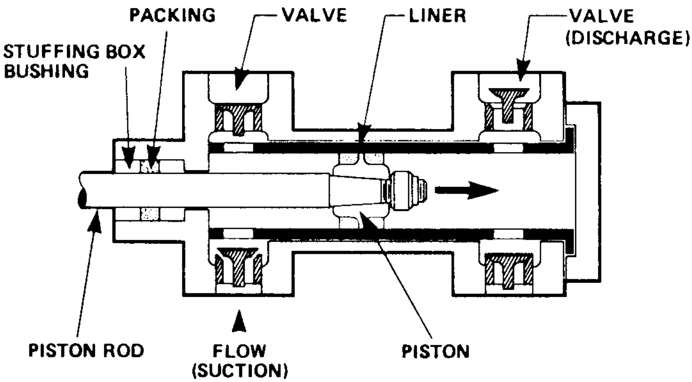

FIGURE C11.18 Plunger pump fluid end.

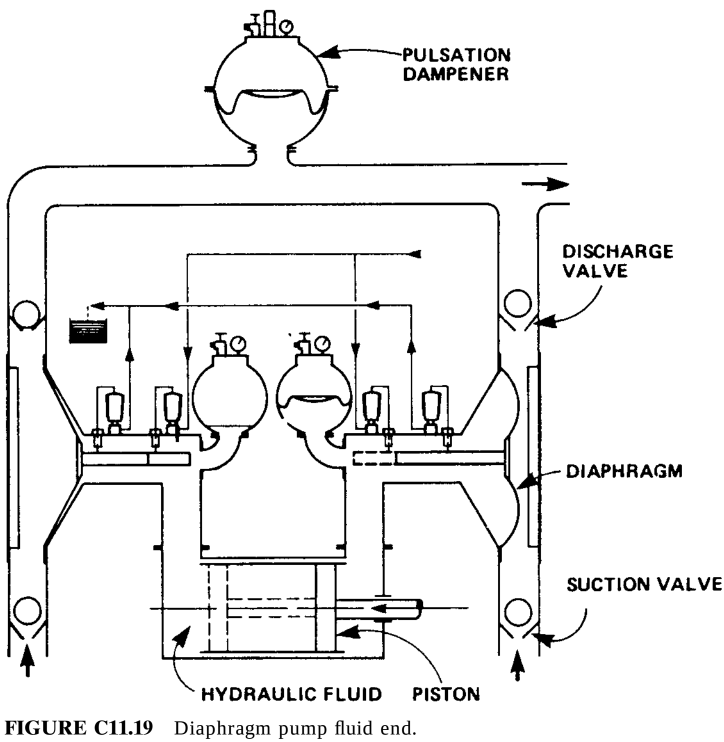

FIGURE C11.19 Diaphragm pump fluid end.

hydraulic fluid with a piston pump, the diaphragm is alternately squeezed and expanded. With this type of pump, the valves and diaphragm are the only parts that experience major wear. One disadvantage is the possibility of diaphragm failure. However, this can be minimized through careful pump design by limiting the amount of diaphragm flexing and by selecting the membrane elastomer carefully. In addition, suppliers offer visual and audible monitoring systems to detect diaphragm failure. Units are available that will limit the amount of slurry entering the propelling fluid chamber.

Plunger Pumps. Because abrasive slurries can greatly reduce the life of the pistons and cylinder liners of conventional piston units, plunger pumps are often used. This type of pump maintains a clear liquid barrier between the plunger and packing by means of a flushing system, as shown in Fig. C11.20. Major wear is limited to the plunger, valves, and packing.

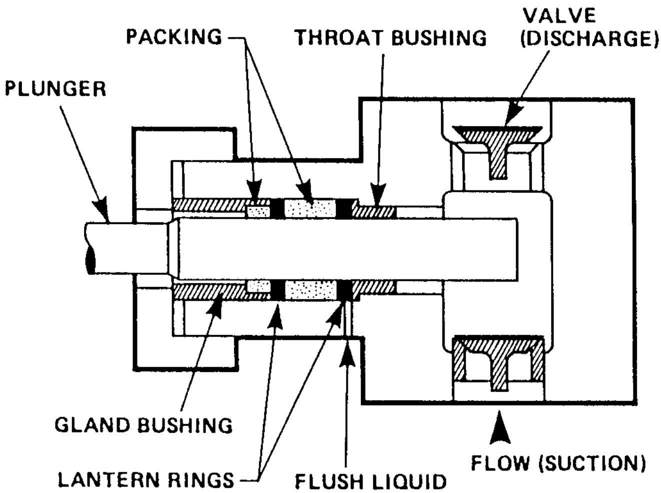

FIGURE C11.20 Plunger pump fluid end.

Hydrohoists. The limitation of maximum particle size for positive displacement pumps and the limitation of pump discharge pressure for centrifugal pumps can be eliminated by using a hydrohoist system. Figure C11.21 shows a sketch of a hydrohoist pumping system. The hydrohoist consists of one or more chambers that can be filled with solids either in dry form (lock-hopper) or as a slurry. Once the chamber is filled with solids, water under high pressure is admitted to the chamber to push the solids into the pipeline. A major application is found in vertical transportation of coarse ore from deep mines. The advantages of these devices are as follows:

- Large solids can be pumped under high pressure without having them pass through a number of centrifugal pumps. The attrition of solids is thereby reduced.
- A high-pressure water pump, having relatively high efficiency compared to a centrifugal slurry pump, is used.

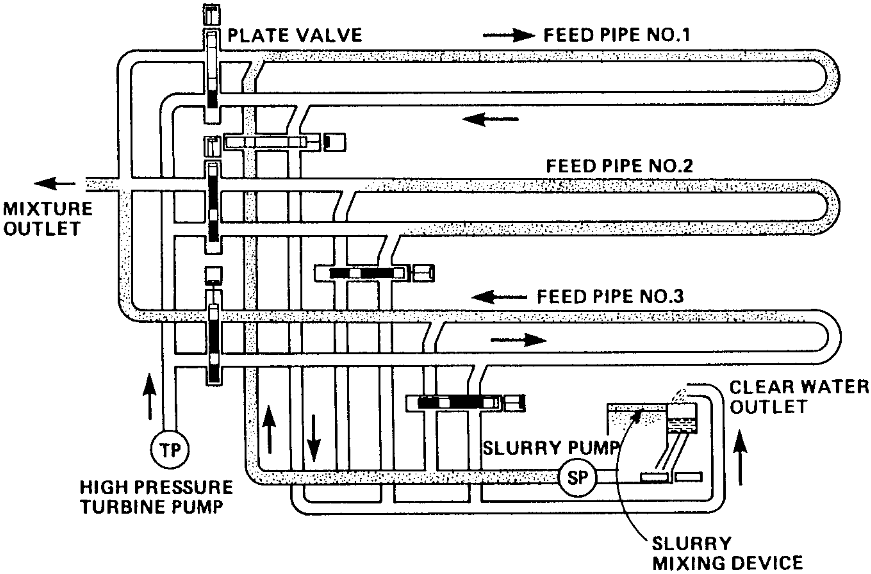

FIGURE C11.21 Sketch of a hydrohoist.

- The number of moving parts exposed to slurry is considerably less than that with reciprocating units.
- The life of slurry check valves is extended because of the reduced frequency of operation compared to reciprocating pumps; normally the valves open and close in the presence of water.
- A larger capacity can be obtained with a hydrohoist than with a reciprocating pump operating at the same discharge pressure.

The disadvantages of the hydrohoist system are as follows:

- A sophisticated control system is needed to open and close the slurry valves.
- The flow rate of this device depends upon the system discharge pressure since a high-pressure centrifugal water pump is used.

Pump Power. The pump power is computed from the following equation:

$$\mathrm{HP} = \frac{QP}{F \times \text{efficiency}} \tag{C11.46}$$

where HP = pumping power, kW (hp)
Q = flow rate, m^3/h (gpm)
P = increase in pressure across the pump, Pa (psi)
F = conversion factor, 1714 hp/(gpm·psi) {3.6×10^6 kW/[m^3/(h·Pa)]}

Example C11.13. Select the type, number, and power of pumps based on the following:

Flow rate	3000 gpm (680 m^3/h)
Rise in head across pump(s)	260 ft (80 m)
Specific gravity of slurry	1.5

Solution. The pressure rise across the pumps is 260 ft × 1.5 × 62.4/144 psi = 169 psi (1.177 MPa). Select centrifugal slurry pumps. Assume three pumps operating in series to get a total head rise of 260 ft (80 m).

Assuming a pump efficiency of 70 percent, the power per pump is obained as follows:

$$\text{HP} = \frac{3000 \times 169/3}{1714 \times 0.7}$$

$$= 141 \qquad \text{Use 150-hp motors.}$$

$$\text{Power} = \frac{680(1.17)/3}{3.6(0.7)}$$

$$= 106\,\text{kW}$$

Life of Pump Parts. Typical parts life for the different types of slurry pumps is given in Table C11.5. The actual life depends upon the pump speed, discharge pressure, and abrasivity of the slurry.

TABLE C11.5 Pump Parts Life

	Part life			
Pump part	Piston	Plunger	Diaphragm	Centrifugal
Piston Rod	3000	n/a	n/a	n/a
Plunger sleeve	n/a	700	n/a	n/a
Piston liner	4000	n/a	n/a	n/a
Packing	6000	400	n/a	1500
Diaphragm	n/a	n/a	8000	n/a
Valve seat	1000	800	5000	n/a
Valve body	1000	800	3000	n/a
Valve insert	1000	500	3000	n/a
Impeller	n/a	n/a	n/a	6000
Casing	n/a	n/a	n/a	6000

Selection of Valves

There are many manufacturers and types of valves offered for low-pressure slurry service. These designs are backed by years of experience in the minerals and dredging industries. Selection of valves for use in positive displacement slurry pump stations introduces some new parameters besides those that must be considered in selecting low-pressure valves. These include vibration of the piping and packing of solids into the void spaces in the valves.

Generally speaking, in high-pressure duty where a full opening is not required, lubricated plug valves have been applied. Depending on the service, the wetted parts of these valves should be hardfaced to minimize abrasive wear during the opening or closing cycle. Gate or ball valves have been used where a full round opening is required. Both types introduce the problem of removing solids from the body cavity after the valve has been operated in slurry. Again, it may be necessary to hardface some wetted parts to extend the valve life.

Slurry valves require regular lubrication and maintenance to remain operable. An important feature in selecting valves is the ease of disassembly for maintenance. Some ball valves, for instance, have to be sent to the factory to renew the seats or seals. Some maintenance problems may simply be a matter of solids finding their way into the valve workings, with the solution simply being disassembly and cleanup of the valve. Some slurries (such as limestone) may have some cementation properties. Therefore, it is good practice to regularly lubricate and exercise valves that are otherwise infrequently used.

Abrasion Control

Pipe abrasion is a major concern in coarse solids pipelines. In temporary systems, the pipeline may be replaced periodically. For example, the phosphate mining industry in Florida replaces temporary steel pipes after about 2 years of operation.[49] Use of special pipes, nonferrous pipes, and steel pipes lined with abrasion-resistant linings can significantly increase the life of pipes carrying coarse solids. Example C11.12 addresses the subject of corrosion-erosion control for long-term service.

Nonferrous Pipes. For low pressure applications, a number of nonferrous materials can be considered for transporting coarse solids.[50]

Polyethylene. Ultrahigh-molecular-weight polyethylene has been extensively used for tailings line construction. For sizes up to NPS 14 (DN 350), it is available with pressure ratings up to 250 psi (1725 kPa). This material is a major improvement over unlined carbon steel or wood, and under good conditions it lasts many times longer than the material it replaces.

Polybutylene. Polybutylene pipe is a flexible thermoplastic pipe that has been developed relatively recently. Depending upon the diameter, it is normally available with pressure ratings up to 200 psi (1380 kPa), although higher-pressure pipes have been made. Polybutylene has two major advantages compared to polyethylene: the tensile strength is higher, and it is less affected by extremes of temperature.

Polyurethane. This type of pipe has been used for some in-plant systems, in particular, coal cleaning plants. The polyurethane is spun-cast into flanged spools up to NPS 12 (DN 300).

Other Nonferrous Materials. Polyvinylchloride (PCV) polypropylene and acrylonitrile-butadiene-styrene (ABS) have been made into pipe but have poor wear resistance compared to polyethylene and polybutylene. They should not be considered for long-life slurry applications.

Fiberglass pipe is available with ceramic chips or tile embedded into the inner surface. This material is quite expensive, and once the chips are broken out, the epoxy matrix that holds the chips has very poor wear resistance and the pipe quickly fails. Also, joint systems designed to take high pressure are prone to failure. Abrasive particles attack the groove left at the joint, and early failure may result.

Internally Lined Steel. Many of the materials discussed in the previous section can also be utilized as a lining within a steel outer shell. By utilizing such a system, the inherent pressure limitations of nonferrous materials can be overcome. In most cases, the use of a composite pipe leads to difficulty in joining the pipe sections. Most systems require the use of steel flanges with the lining material serving as the gasket material.

In Situ Lining of Steel Pipe. Two methods of in situ internal lining of welded steel pipe have been developed. In one method a plastic pipe of slightly smaller diameter than the inside diameter of steel pipe is pulled inside, and the annular space between the two pipes is grouted with cement. In another method a high-density polyethylene pipe of slightly larger outside diameter than the inside diameter of the steel pipe is installed by pulling it after compressing it to reduce its outside diameter. The pipe expands and presses against the steel pipe upon release of pulling forces.

These liners are pulled inside the steel pipe, which is welded prior to pulling the liner. The liner can be pulled to a distance of 1 km depending upon the pipe size and the number of bends in the steel pipe.

In either method, best mechanical design and economic results are obtained when lining is planned for in the beginning. In an operating pipeline these lining methods can be used to extend service life.

Instrumentation

The instrumentation used and the major parameters measured and controlled are as follows:

Pressure. Both the suction and discharge pressures at a pump station are normally measured and controlled. Suction pressure is controlled to prevent cavitation. The discharge pressure is controlled so as not to exceed the maximum allowable pressure for the pump unit or the pipeline system. Bourdon-type pressure gages with an isolation diagram are generally used.

Density. The pumping head requirement varies with the changes in slurry density. If the slurry density is not controlled within the pipeline design limits, the pipeline system can be overpressurized or the flow velocity could drop below the deposition critical velocity.

Slurry density is normally measured by radiation gages. Density meters may be needed at each pump station in a multiple-station system. The density meters are normally installed on the suction side of a pump station.

Density is controlled by adding dilution water to high-density slurry. If the density is too low, the slurry can be returned to the preparation plant for concentrating.

Flow Rate. Because slurries are often abrasive by nature, some in-line flow-measuring devices such as orifice plates and pitot tubes are not applicable. For primary slurry flowmetering, magnetic flowmeters are considered the best choice. Ultrasonic flowmeters sometimes are used in applications including oil slurries where magnetic flowmeters are unsuitable.

Slurry Tank Level Measurement. A number of different systems have been used for recording the level in slurry storage tanks.

Conductance probes for level alarms and a pressure sensor mounted in the wall of the tank are used to obtain a rough indication of level of slurry in the tank. This procedure has proved satisfactory although the accuracy of the level measurement with a pressure sensor is affected by changes in slurry density. A way to compensate for this effect is available but not commonly used. Sonic devices provide direct measurement of tank level.

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