

Construction Financial Management: Solutions

S. L. Tang



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CONSTRUCTION FINANCIAL MANAGEMENT: SOLUTIONS

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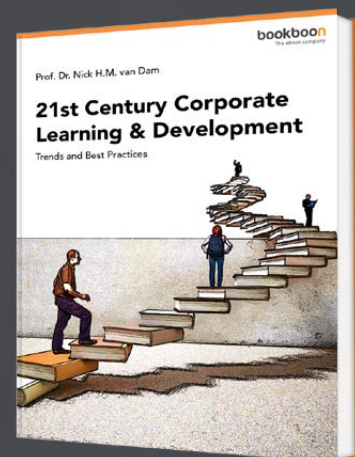
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EXERCISE QUESTIONS FOR CHAPTER 1

Exercise Question 1

Using the company balance sheet shown on Table 2.2 of Chapter 2, calculate for each 2012 and 2011:

- a) the company's equity (or net worth),
- b) working capital, and
- c) current ratio.

Solution:

(a) Company's equity (or net worth)

	2012	2011
Total assets	14,591,105	13,772,652
Total liabilities	9,159,760	8,078,450
Net worth	5,431,345	5,694,202

(b) Working capital

	2012	2011
Current assets	12,697,745	11,685,952
Current liabilities	7,679,247	6,177,005
Working Capital	5,018,498	5,508,947

(c) Current ratio

	2012		2011	
Current assets	12,697,745		11,685,952	
Current liabilities	7,679,247	= 1.65	6,177,005	= 1.89

Exercise Question 2

Based on the project data presented in the table below, calculate for each of the two projects:

- the revenue using the percentage-of-completion method,
- the gross profit to date, using the percentage-of-completion method, and
- the amount of over / under billing for each project.

Project Financial data	Project A	Project B
Contract amount	\$15,000,000	\$15,000,000
Original estimated cost	14,400,000	14,800,000
Amount billed to date	10,700,000	10,700,000
Payments received to date	10,900,000	10,630,000
Cost incurred to date	11,450,000	10,550,000
Forecasted cost to complete	3,000,000	4,100,000
Costs paid to date	9,400,000	9,600,000

Note: some figures are for reference only and are not useful for calculating what are asked for

Solution:

- Revenue using the percentage-of-completion method

		Project A	Project B
% completed =	Cost incurred	11,450,000	10,550,000
	Cost incurred + forecasted cost	14,450,000	14,650,000
		= 79%	= 72%

	Project A	Project B
Revenue = Contract Amount × % completed =	15,000,000 × 79%	15,000,000 × 72%
	= 11,850,000	= 10,800,000

b) Gross Profit using the percentage-of-completion method

	Project A	Project B
Revenue	11,850,000	10,800,000
Cost incurred	11,450,000	10,550,000
Gross Profit	400,000	250,000

c) Under billing

	Project A	Project B
Revenue	11,850,000	10,800,000
Amount billed	10,700,000	10,700,000
Under-billing	1,150,000	100,000

EXERCISE QUESTIONS FOR CHAPTER 2

Exercise Question 1

Base on the Income Statement and the Balance Sheet shown on Tables 2.1 and 2.2 respectively in Chapter 2. Calculate:

- a) the three Profitability Ratios,
- b) the three Liquidity Ratios,
- c) the three Working Capital Ratios,
- d) the two Capital Structure Ratios, and
- e) the seven Activity Ratios.

Solution:

(a) Profitability Ratios

Profitability ratios measure the construction company's ability to earn profit from its operation. The three most commonly used profitability ratios are:

Gross Profit Margin Ratio = Gross profit / Revenue

For 2012, $9,921,256 / 40,875,351 = 24.27\%$

For 2011, $10,319,606 / 34,701,250 = 29.74\%$

(The **goal** for net profit margin ratio is **25% minimum**; if subcontractors (pay-as-paid basis) occupy a significant portion of the cost of revenue, the goal can be reduced to **20% minimum**)

Net Profit Margin Ratio = Net profit before tax / Revenue

For 2012, $1,333,440 / 40,875,351 = 3.26\%$

For 2011, $2,814,730 / 34,701,250 = 8.11\%$

(The **goal** for net profit margin ratio is **5% minimum**)

Return on Equity Ratio = Net profit before tax / Owners' equity

For 2012, $1,333,440 / 5,431,345 = 24.55\%$

For 2011, $2,814,730 / 5,694,202 = 49.43\%$

(The return on equity ratio should be **between 15% and 40%**)

(b) Liquidity Ratios

Liquidity ratios indicate the construction company's ability to pay its obligations as they come due. The three most common liquidity ratios used are shown below.

Current Ratio = Current assets / Current liabilities

For 2012, $12,697,745 / 7,679,247 = 1.65$

For 2011, $11,685,952 / 6,177,005 = 1.89$

(The current ratio should be **higher than 1.3** for a financially healthy construction company)

Acid Test Ratio (or Quick Ratio) = (Cash + Accounts receivables) / Current liabilities

For 2012, $(2,305,078 + 6,124,992) / 7,679,247 = 1.10$

For 2011, $(1,877,676 + 5,837,658) / 6,177,005 = 1.25$

(The acid test ratio or quick ratio should be **higher than 1.1** for a construction company)

Current Assets to Total Assets Ratio = Current assets / Total assets

For 2012, $12,697,745 / 14,591,105 = 87.02\%$

For 2011, $11,685,952 / 13,772,652 = 84.85\%$

(The current assets to total assets ratio should be **between 60% and 80%**)

(c) Working Capital Ratios

These ratios measure how well the construction company is utilizing its working capital. The three most commonly used working capital ratios are shown below.

Working Capital Turnover = Revenue / Working capital

For 2012, $40,875,351 / (12,697,745 - 7,679,247) = 8.14$ times

For 2011, $34,701,250 / (11,685,952 - 6,177,005) = 6.30$ times

(The working capital turnover should be **between 8 and 12 times per year**)

Net Profit to Working Capital Ratio = Net profit before tax / Working capital

For 2012, $1,333,440 / (12,697,745 - 7,679,247) = 26.57\%$

For 2011, $2,814,730 / (11,685,952 - 6,177,005) = 51.09\%$

(The net profit to working capital ratio should be **between 40% and 60%**)

Degree of Fixed Asset Newness = Net depreciable fixed assets / Total depreciable fixed assets

For 2012, 1,893,360 / 3,945,260 = 47.99%

For 2011, 2,086,700 / 3,750,100 = 55.64%

(The degree of fixed asset newness should be **between 40% and 60%**)

(d) Capital Structure Ratios

Capital structure ratios indicate the ability of the construction company to manage liabilities. These ratios also indicate the approach that the company prefers to finance its operation. The two major capital structure ratios are:

Debt to Equity Ratio = Total liabilities / Owners' equity

For 2012, 9,159,760 / 5,431,345 = 1.69

For 2011, 8,078,450 / 5,694,202 = 1.42

(The debt to equity ratio should **be lower than 2.5**)

Leverage = Total assets / Owners' equity

For 2012, 14,591,105 / 5,431,345 = 2.69

For 2011, 13,772,652 / 5,694,202 = 2.42

Or

Leverage = Total assets / Owners' equity
= (Total liabilities + Owners' equity) / Owners' equity
= (Total liabilities / Owners' equity) + 1
= Debt to Equity Ratio + 1

For 2012, 1.69 + 1 = 2.69

For 2011, 1.42 + 1 = 2.42

(The leverage should **be lower than 3.5**. Some construction companies prefer to use leverage of 3.5 or close to it but some conservative ones prefer to use a lower leverage. This relates to, of course, the use of a higher or lower debt to equity ratio by the company.)

(e) Activity Ratios

Activity ratios indicate whether or not the construction company is using its assets effectively, and if yes, how effective they are. There are quite a number of activity ratios, and the seven commonly used ones are shown below.

Average Age of Material Inventory = (Material inventory / Materials cost) × 365 days

For 2012, $(942,765 / 20,732,506) \times 365 = 16.60$ days

For 2011, $(761,763 / 15,925,567) \times 365 = 17.46$ days

(The average age of material inventory should be [shorter than 30 days](#))

Average Age of Under Billings = (Under billings / Revenue) × 365 days

For 2012, $(581,221 / 40,875,351) \times 365 = 5.19$ days

For 2011, $(486,472 / 34,701,250) \times 365 = 5.12$ days

(The average age of under billings should be [the shorter the better](#))

Average Age of Accounts Receivable = (Accounts receivable / Revenue) × 365 days

For 2012, $(6,124,992 / 40,875,351) \times 365 = 54.69$ days

For 2011, $(5,837,658 / 34,701,250) \times 365 = 61.40$ days

(The average age of accounts receivable should be [shorter than 45 days](#))

Cash Conversion Period = Average age of material inventory + Average age of under billings
+ Average age of accounts receivable

For 2012, $16.60 + 5.19 + 54.69 = 76.48$ days

For 2011, $17.46 + 5.12 + 61.40 = 83.98$ days

(The cash conversion period should be [shorter than 75 days](#))

Average Age of Accounts Payable = [Accounts payable / (Materials + Subcontracts)] × 365 days

For 2012, $[3,930,309 / (20,732,506 + 6,417,407)] \times 365 = 52.84$ days

For 2011, $[3,481,330 / (15,925,567 + 4,721,312)] \times 365 = 61.54$ days

(The average age of accounts payable should be [shorter than 45 days](#))

Average Age of Over Billings = (Over billings / Revenue) × 365 days

For 2012, $(560,847 / 40,875,351) \times 365 = 5.01$ days

For 2011, $(495,167 / 34,701,250) \times 365 = 5.21$ days

(Usually there is [no guideline](#) on average age of over billings)

Cash Demand Period = Cash conversion period – Average age of accounts payable – Average age of over-billings

For 2012, $76.48 - 52.84 - 5.01 = 18.63$ days

For 2011, $83.98 - 61.54 - 5.21 = 17.23$ days

(The cash demand period should be **shorter than 30 days**)

Exercise Question 2

By referring to the ratios calculated in Exercise Question 1 above, are there any things you would like to add to Section 2.2 of the chapter to remind the new general manager that he has missed but should have considered?

Solution:

This is an open-ended question, and is suitable for group discussion followed by presentation from each group.



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EXERCISE QUESTIONS FOR CHAPTER 3

Exercise Question 1

The pavement of a road requires \$400,000 per year to maintain. The feasibility of a new pavement is being considered for reducing maintenance costs. If the new pavement needs no maintenance in the first three years, then \$200,000 per year for the next seven years, and then \$400,000 per year thereafter, what is the immediate expenditure for the new pavement that is justifiable? (Assume a discount rate of 10% p.a.).

Solution:

The present value of maintaining the new pavement in the first 10 years:

Let PV_3 = the equivalent sum of money at the end of Year 3 for the uniform series of payments of \$200,000 per annum from Yr 4 to Yr 10 (a total of 7 years)

$$\text{Then } PV_3 = 200,000 \times \left[\frac{(1+0.1)^7 - 1}{0.1(1+0.1)^7} \right] = 200,000 \times 4.8684 = 973,680$$

Let PV_0 = Present value of PV_3 = PV of maintaining the new pavement in the first 10 yrs

$$\text{Then } PV_0 = 973,680 \times \left[\frac{1}{(1+0.1)^3} \right] = 973,680 \times 0.7513 = 731,526$$

The present value of maintaining the old pavement for 10 years:

Let this present value be PV_{old}

$$\text{Then } PV_{\text{old}} = 400,000 \times \left[\frac{(1+0.1)^{10} - 1}{0.1(1+0.1)^{10}} \right] = 400,000 \times 6.1446 = 2,457,840$$

The justifiable immediate expenditure $\leq PV_{\text{old}} - PV_0 = 2,457,840 - 731,526 = 1,726,314$

Exercise Question 2

A contractor borrowed \$500,000 from a bank to buy earth-moving equipment with an estimated service life of 10 years. The bank charged the contractor 12% interest p.a. and required him to pay back the loan in 10 years' time.

- Assuming that the contractor paid back the bank in 10 equal instalments (once every year), calculate the amount of each end-of-year payment.
- The contractor at the end of year 4 wished to make an early redemption (i.e. pay all the money that he owed the bank). How much should he pay?
- The bank negotiated with the contractor and reduced the interest rate to 10% p.a. at the beginning of the 5th year in order to attract the contractor to stay borrowing. What would be the contractor's repayment schedule if he chose to pay back the bank in the form of six uniform payments from the end of years 5 to the end of year 10?
- If the bank changed the interest rate back to 12% p.a. at the beginning of the 8th year, what would be the amount of the contractor's last payment (i.e. payment at the end of year 10) if he kept on paying the bank the same instalment as calculated in (c) above at the end of years 8 and 9?

Solution:

- Amount of each end-of-year payment

$$\begin{aligned}
 &= 500,000 \times \left[\frac{0.12(1 + 0.12)^{10}}{(1 + 0.12)^{10} - 1} \right] \\
 &= 500,000 \times 0.1770 \\
 &= \$88,500
 \end{aligned}$$

-

Year	Debt	Yearly payment	Interest	Principal paid	Remaining debt
1	500,000	88,500	$500,000 \times 0.12$ = 60,000	$88,500 - 60,000$ = 28,500	$500,000 - 28,500$ = 471,500
2	471,500	88,500	$471,500 \times 0.12$ = 56,580	$88,500 - 56,580$ = 31,900	$471,500 - 31,900$ = 439,580
3	439,580	88,500	$439,580 \times 0.12$ = 52,750	$88,500 - 52,750$ = 35,750	$439,580 - 35,750$ = 403,830
4	403,830	88,500	$403,830 \times 0.12$ = 48,460	$88,500 - 48,460$ = 40,040	$403,830 - 40,040$ = 363,790

At the end of year 4, the contractor should pay $\$363,790 + \$88,500 = \$452,290$.

- c) If the bank reduces its interest rate to 10% p.a. at the beginning of the 5th year, then the uniform payments from years 5 to 10 (totally 6 years)

$$\begin{aligned}
 &= 363,790 \times \left[\frac{0.10(1+0.10)^6}{(1+0.10)^6 - 1} \right] \\
 &= 363,790 \times 0.2296 \\
 &= \$83,526 \text{ at the end of each year.}
 \end{aligned}$$



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d)

Year	Debt	Yearly payment	Interest	Principal paid	Remaining debt
5	363,790	83,526	$363,790 \times 0.10$ = 36,379	$83,526 - 36,379$ = 47,147	$363,790 - 47,147$ = 316,643
6	316,643	83,526	$316,643 \times 0.10$ = 31,664	$83,526 - 31,664$ = 51,862	$316,643 - 51,862$ = 264,781
7	264,781	83,526	$264,781 \times 0.10$ = 26,478	$83,526 - 26,478$ = 57,048	$264,781 - 57,048$ = 207,733
--- Interest rate changes to 12% p.a. ---					
8	207,733	83,526	$207,733 \times 0.12$ = 24,928	$83,526 - 24,928$ = 58,598	$207,733 - 58,598$ = 149,135
8	149,135	83,526	$149,135 \times 0.12$ = 17,896	$83,526 - 17,896$ = 65,630	$149,135 - 65,630$ = 83,505
10	83,505	83,526	$83,505 \times 0.12$ = 10,021	$83,526 - 10,021$ = 73,505	$83,505 - 73,505$ = 10,000

At the end of year 10, the contractor has to pay $\$10,000 + \$83,526 = \$93,526$

Exercise Question 3

There are two alternatives to construct a storage house. Both serve the purpose of allowing construction materials to be stored in the house. However, due to different construction methods (one is made of wood and the other made of bricks), different life spans and cash flow patterns are associated with each alternative as follows:

	Alternative 1 (wood)	Alternative 2 (bricks)
Life	10 years	15 years
Initial capital cost	\$900,000	\$1,300,000
Operation and maintenance cost	\$80,000 p.a.	\$20,000 p.a.

Assuming the discount rate to be 16% p.a., choose the better alternative by:

- the present value method, and
- the equivalent annual cost method.

(Hints: compare the alternatives based on the same number of years, i.e. 30 years)

Solution:

Let us simplify the writing of $\left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$ to $(uscrf)^{n,i}$ to denote “uniform series capital recovery factor” for a period of n at a discount rate of i , and simplify the writing of $\left[\frac{1}{(1+i)^n} \right]$ and $\left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$ to $(pvf)^{n,i}$ and $(uspvf)^{n,i}$ respectively to denote “present value factor” and “uniform series present value factor” for a period of n at a discount rate of i .

a) present value method (compare 30 yrs)

	Alternative 1	Alternative 2
i) initial capital	\$900,000	\$1,300,000
ii) present value of OMR costs over 30 yrs	$80,000 \times (uspvf)^{n=30,i=16\%}$ $= 80,000 \times 6.1772$ $= 494,176$	$20,000 \times (uspvf)^{n=30,i=16\%}$ $= 20,000 \times 6.1772$ $= 123,544$
iii) present value of 1 st renewal	$900,000 \times (pvf)^{n=10,i=16\%}$ $= 900,000 \times 0.2267$ $= \$204,030$	$1,300,000 \times (pvf)^{n=15,i=16\%}$ $= 1,300,000 \times 0.1079$ $= 140,270$
iv) present value of 2 nd renewal	$900,000 \times (pvf)^{n=20,i=16\%}$ $= 900,000 \times 0.0514$ $= \$46,260$	Nil
Present value of total costs	\$1,644,466	\$1,563,814

Alternative 2 is a better choice.

b) equivalent annual cost method

	Alternative 1	Alternative 2
i) equivalent annual cost of initial cost	$900,000 \times (\text{uscrf})^{n=10, i=16\%}$ $= 900,000 \times 0.2069$ $= \$186,210$	$1,300,000 \times (\text{uscrf})^{n=15, i=16\%}$ $= 1,300,000 \times 0.1794$ $= \$233,220$
ii) annual OMR cost	\$80,000	\$20,000
Total equivalent annual costs	\$266,210	\$253,220

Alternative 2 is a better choice.

Exercise Question 4

We have seen from Section 3.5 (or Example 3.6) of Chapter 3 that no matter which method we use to pay back the loan, the present values of the payments are always the same, equal to the principal sum. Prove mathematically to show that such a phenomenon is always true.

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Solution:

Let P = principal borrowed,
 A = principal amortization in year i ,
 $R = 1 + i$ (i = borrowing interest rate, % p.a.), and
 n = number of years.

Then, from Tables 3.1, 3.2 and 3.3 in the book, we can derive that:

$$\text{PV of the first year installment} = \frac{A_1 + PR - P}{R}$$

$$\text{PV of the second year installment} = \frac{A_2 + (P - A_1)R - (P - A_1)}{R^2}$$

$$\text{PV of the third year installment} = \frac{A_3 + (P - A_1 - A_2)R - (P - A_1 - A_2)}{R^3}$$

•
•
•

PV of the n th year installment

$$= \frac{A_n + (P - A_1 - \dots - A_{n-1})R - (P - A_1 - \dots - A_{n-1})}{R^n}$$

Now we use mathematical induction to prove that this mathematical expression is true:

$$\begin{aligned} & \frac{A_1 + PR - P}{R} + \frac{A_2 + (P - A_1)R - (P - A_1)}{R^2} + \frac{A_3 + (P - A_1 - A_2)R - (P - A_1 - A_2)}{R^3} \\ & + \dots + \frac{A_n + (P - A_1 - \dots - A_{n-1})R - (P - A_1 - \dots - A_{n-1})}{R^n} \\ & = \frac{(A_1 + A_2 + \dots + A_n) + PR^n - P}{R^n} \end{aligned}$$

First, we must prove that the above mathematical expression is true when $n = 2$.

When $n = 2$,

$$\begin{aligned} \text{LHS} &= \frac{A_1 + PR - P}{R} + \frac{A_2 + (P - A_1)R - (P - A_1)}{R^2} \\ &= \frac{A_1 R + PR^2 - PR + A_2 + PR - A_1 R - P + A_1}{R^2} \\ &= \frac{(A_1 + A_2) + PR^2 - P}{R^2} = \text{RHS} \end{aligned}$$

So, the mathematical expression is true when $n = 2$.

Next, we must also prove that if the mathematical expression is true when $n = k$, it will also be true when $n = k + 1$.

When $n = k + 1$,

LHS

$$\begin{aligned}
 &= \frac{A_1 + PR - P}{R} + \frac{A_2 + (P - A_1)R - (P - A_1)}{R^2} \\
 &+ \frac{A_3 + (P - A_1 - A_2)R - (P - A_1 - A_2)}{R^3} + \dots \\
 &+ \frac{A_k + (P - A_1 - \dots - A_{k-1})R - (P - A_1 - \dots - A_{k-1})}{R^k} \\
 &+ \frac{A_{k+1} + (P - A_1 - \dots - A_k)R - (P - A_1 - \dots - A_k)}{R^{k+1}} \\
 &= \frac{(A_1 + A_2 + \dots + A_k) + PR^k - P}{R^k} \quad \text{(as it is assumed true for } n = k) \\
 &+ \frac{A_{k+1} + (P - A_1 - \dots - A_k)R - (P - A_1 - \dots - A_k)}{R^{k+1}} \\
 &= \frac{1}{R^{k+1}} (A_1R + A_2R + \dots + A_kR + PR^{k+1} - PR + A_{k+1} + PR - A_1R - A_2R - \dots \\
 &- A_kR - P + A_1 + \dots + A_k) \\
 &= \frac{(A_1 + A_2 + \dots + A_{k+1}) + PR^{k+1} - P}{R^{k+1}} = \text{RHS}
 \end{aligned}$$

So, we have proved that if the case $n = k$ is true, then the case $n = k+1$ shall be true. In addition, it has been proved true when $n = 2$. Therefore, the mathematical expression is true.

Hence, by applying the mathematical expression to the sum of PVs from year 1 to year n , the sum of the present values of all the annual installments

$$\begin{aligned}
 &= \frac{(A_1 + A_2 + \dots + A_n) + PR^n - P}{R^n} \\
 &= \frac{P + PR^n - P}{R^n} \quad \text{(as } P = A_1 + A_2 + \dots + A_n) \\
 &= P
 \end{aligned}$$

The proof is thus completed.

Exercise Question 5

We have seen from Section 3.6 (or Example 3.7) of Chapter 3 that the number is zero at the bottom-right corner of the table in Part a) of the example. Prove mathematically to show that this is always true for all cases.

Solution:

In Example 3.7, the periodical payment A is calculated by (see Eq 3.5 of Chapter 3):

$$A = P_1 \times \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] \quad (1)$$

where P_1 is the principal amount borrowed at the beginning.

This time we do a backward proof. We first assume that the remaining principal at the end of period n is zero. Then, the unpaid principal at the beginning of period n can be calculated as shown below:

$$P_n = \frac{A}{(1+i)} \quad (2)$$



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where P_n is the unpaid principal at the beginning of period n , and A is the final payment at the end of period n , which consists of the interest (iP_n) and the unpaid principal P_n in that period.

Hence, the unpaid principal (P_{n-1}) at the beginning of period $n-1$ can be calculated as follows:

$$P_{n-1} = P_n + (A - iP_n) \quad (3)$$

Note that iP_{n-1} and $(A - iP_{n-1})$ equal the interest payment and the principal amortization respectively paid at the end of period $n-1$. Substituting (2) into (3) and by algebraic transformation, we obtain:

$$P_{n-1} = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} \quad (4)$$

Similarly, the unpaid principal at the beginning of period $n-2$ is:

$$P_{n-2} = P_{n-1} + (A - iP_{n-1}) \quad (5)$$

where iP_{n-2} and $(A - iP_{n-2})$ equal the interest and the principal amortization paid at the end of period $n-2$, respectively. Substituting (4) into (5), we have:

$$P_{n-2} = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} \quad (6)$$

So, we continue to find the earlier unpaid principals, and we can finally obtain the unpaid principal at the beginning of period 1:

$$P_1 = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \dots + \frac{A}{(1+i)^n} \quad (7)$$

From Eq 3.7 of Chapter 4, (7) can also be written as:

$$P_1 = A \times \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \quad (8)$$

We can observe that (8) is identical to (1), and (8) is derived from (1). Since we have derived (8) based on the assumption that the remaining principal at the end of period n is zero, the proof is thus completed.

Note

The above proof also proves that the total amortization (sum of all amortizations) is equal to the principal amount P_1 .

EXERCISE QUESTIONS FOR CHAPTER 4

Exercise Question 1

An appraisal of three alternative investments, A, B and C is being made and the minimum desirable rate of return is 10% p.a. on its invested capital. The details of the investments are shown below.

	Investment A	Investment B	Investment C
Initial cost	\$900,000	\$1,600,000	\$3,100,000
Salvage value	Nil	Nil	\$300,000
Net annual income	\$368,000	\$612,000	\$846,000
Life	4 years	4 years	5 years

- Find the IRR of each investment.
- Find the NPV of each investment. Compare the ranking with (a) above.
- Use the Incremental IRR method to determine which investment is the best to invest.

Solution:

- The NCF, IRR and IRR ranking of each investment:

End of year	Investment A	Investment B	Investment C
0	-900,000	-1,600,000	-3,100,000
1	368,000	612,000	846,000
2	368,000	612,000	846,000
3	368,000	612,000	846,000
4	368,000	612,000	846,000
5			1,146,000
IRR	23.06%	19.48%	13.49%
Ranking	1 st	2 nd	3 rd

b) The NPV and NPV ranking of each investment

	Investment A	Investment B	Investment C
NPV	266,510	339,958	293,282
Ranking	3 rd	1 st	2 nd

c) The Incremental IRR Analysis

First, compare Investment A and Investment B:

End of year	NCF of A	NCF of B	B minus A
0	-900,000	-1,600,000	-700,000
1	368,000	612,000	244,000
2	368,000	612,000	244,000
3	368,000	612,000	244,000
4	368,000	612,000	244,000

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Incremental IRR = 14.76% p.a. > 10% p.a. So, B is better than A.

A is out.

Next, compare Investment B and Investment C:

End of year	NCF of B	NCF of C	C minus B
0	-1,600,000	-3,100,000	-1,500,000
1	612,000	846,000	234,000
2	612,000	846,000	234,000
3	612,000	846,000	234,000
4	612,000	846,000	234,000
5	0	1,146,000	1,146,000

Incremental IRR = 9.04% p.a. < 10% p.a. So, B is better than C.

Therefore, B is the best overall.

Exercise Question 2

Search the following from libraries or the internet:

Tang, S.L. and Tang, H. John (2003). "The variable financial indicator IRR and the constant economic indicator NPV". *Engineering Economist*, Volume 48, Number 1, pages 69–78.

Then read it and see if you have understood more on Section 4.4 of this chapter.

Solution:

The following is a condensed (simplified) version for readers' easy reference.

ECONOMIC INDICATOR NPV AND FINANCIAL INDICATOR IRR

S.L. Tang and H. John Tang

Abstract

The NPV (net present value) and the IRR (internal rate of return) are the two most common and important indicators for capital investments. These two indicators, however, have intrinsic differences between one and the other. It has been proposed since 1991 that **NPV is an economic indicator** and **IRR a financial indicator**. The former (NPV) indicates the economic value of an investment from a society's point of view and the latter (IRR) indicates the financial return of an investment from a private investor's point of view. The value of IRR varies with the change of financial arrangement of an investment. The NPV, however, remains constant no matter how the financial arrangement changes. This is a very fundamental discovery on NPV and IRR, and a very useful knowledge and great contribution to the field of Economics, Finance and Management.

Introduction

The evaluation of net present value (NPV) and internal rate of return (IRR) is well developed and documented in many publications. Some representative ones are Muro (1998 Chapters 4 and 5) and Lang and Merino (1993 Chapters 6 and 7). Although NPV and IRR are both discount cash flow methods, they have intrinsic differences between one and the other. Tang (1991, 2003 Chapter 5) and Robinson & Cook (1996) illustrated that the ranking of investment alternatives is not necessarily the same obtained by the two methods. Differences in rankings between NPV and IRR are further exhibited by Asguith and Bethel (1995), who reported that IRR may be preferred to NPV under certain circumstances. Evans and Forbes (1993) also reckoned that IRR is more cognitively efficient than NPV, because IRR is expressed as a percentage (or a rate of return) while NPV is just a monetary value cognitively inefficient to decision makers. Other researchers, such as Lefley and Morgan (1998), and particularly the academicians (Evans and Forbes, 1993), however, took the view that NPV is more conceptually "correct" despite the fact that IRR is more popular than NPV, and that NPV is more theoretically sound as IRR may be too "capricious" or "fickle" and may not rank some projects in the same order as NPV.

The authors, on the contrary, have the view that all such controversies and arguments about NPV and IRR might not be necessary if their basic functions could be better defined. It has been pointed out by Battaglio *et al.* (1996) that IRR is meant for a consumer's point of view and NPV for a banker's point of view. This is close to the true definition as consumers usually have relatively limited money and banks relatively unlimited money. The authors, however, reckon that an even more fundamental definition can be given. The definition is: NPV is an economic indicator and IRR a financial indicator. In other words, NPV gives the society's point of view and IRR the private investor's point of view. Because IRR functions as a financial indicator, its value varies with the change of financial arrangement (e.g. change of equity-loan ratio) of a capital investment. NPV, however, does not vary when financial arrangement varies, because it functions as an economic indicator. This is analogous to the fact that the Financial-IRR changes but the Economic-IRR remains constant for investing in a capital project whose financial arrangement changes (Tang 2003 Chapter 8).

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An Illustrative Example

If an investor has an all-equity case investment in which an initial capital outlay of \$10,000 leads to a receipt of \$5,000 each year for three years as shown in Table 1, the investor obtains an IRR of 23.4% p.a.

End of Year	Cash-out	Cash-in
0	10,000	
1		5,000
2		5,000
3		5,000

Table 1 Cash Flows of An Investment

If the investor uses \$4,000 as equity and borrows \$6,000 as loan (paying 10% interest per annum), making a total of \$10,000 serving as the initial capital outlay of **the same investment** as above, then the net annual receipts in the next three years, having deducted the three annual principal amortizations and the annual interest payments from the gross annual receipts, are calculated to be \$2,400, \$2,600 and \$2,800 as shown in Table 2.

End of Year	(1) Amount borrowed	(2) Principal amortization	(3) Balance of Principal unpaid	(4) Interest (10%)	(5) Total annual Payment (2) + (4)	(6) Net annual receipt 5,000 - (5)
0	6,000		6,000			
1		2,000	4,000	600	2,600	5,000-2,600=2,400
2		2,000	2,000	400	2,400	5,000-2,400=2,600
3		2,000	0	200	2,200	5,000-2,200=2,800

Table 2 Net Annual Receipts for Uniform Principal Amortization of \$2,000 for Loan \$6,000

In other words, the investor will obtain \$2,400, \$2,600 and \$2,800 in Years 1, 2 and 3 respectively if he initially invests \$4,000 (the equity). The cash flows of the investment become those as shown in Table 3. It should be noted Table 1 and Table 3 represent **the same investment**, although they have different financial arrangements. Any one of the two should have no advantage over the other as far as the **economic value of the investment** is concerned.

End of Year	Cash-out	Cash-in
0	4,000	
1		2,400
2		2,600
3		2,800

Table 3 Cash Flows with \$4,000 as Equity and \$6,000 as Loan

The IRR calculated from the cash flows of Table 3 is 41.2% p.a., which is higher than 23.4% p.a. as calculated from those shown in Table 1. This indicates that the financial arrangement of \$4,000 equity and \$6,000 loan is **financially better** than the all-equity case, although the **economic value of the investment is unchanged** (see below). If the NPV method is used to compare these two sets of cash flows, namely, Tables 1 and 3, or (-10,000, 5,000, 5,000, 5,000) and (-4,000, 2,400, 2,600, 2,800) respectively, using $i = 10\%$ p.a. (the borrowing interest rate), the NPVs will be the same as shown in the following calculations:

NPV of the cash flows of Table 1

$$= -10,000 + \frac{5,000}{(1+i)} + \frac{5,000}{(1+i)^2} + \frac{5,000}{(1+i)^3} = 2,434.26$$

NPV of the cash flows of Table 3

$$= -4,000 + \frac{2,400}{(1+i)} + \frac{2,600}{(1+i)^2} + \frac{2,800}{(1+i)^3} = 2,434.26$$

Now, a further case is to be investigated. If the investor uses only \$1,000 as equity and borrows \$9,000 as loan at 10% p.a. interest rate, the cash flow will become (-1,000, 1,100, 1,400, 1,700), and the IRR calculated will be 113.1% p.a. This is much higher than 23.4% p.a. (calculated from Table 1) and also 41.2% p.a. (calculated from Table 3). The NPV, however, is the same, that is, \$2,434.26.

It can be seen that the IRR becomes higher if a bigger loan is borrowed, although the economic value (indicated by the NPV) of the investment remains the same in all situations. This is because the borrowing interest rate 10% p.a. is smaller than the all-equity case IRR 23.4% p.a. If the borrowing interest rate is not 10% p.a. but higher than the all-equity IRR, then an exactly opposite effect will occur, that is, the higher the loan is, the lower the IRR will be.

The Mathematical Proof

The example illustrated above that NPV is a constant value in all situations can be proved mathematically. Mathematical Induction is used in the proof.

If we assume that:

C	=	all-equity case initial capital outlay,
I_i	=	all-equity case cash-in in year i ,
B	=	loan amount,
A_i	=	principal amortization in year i ,
R	=	$1 + i$ (i = borrowing interest rate, % per period), and
n	=	total number of periods,

then, from Table 2, we can derive that the net annual receipts (cash-in in Table 3) are as follows:

$$\begin{aligned}
 \text{Cash-in in year 1} &= I_1 - (A_1 + BR - B) \\
 \text{Cash-in in year 2} &= I_2 - [A_2 + (B - A_1)R - (B - A_1)] \\
 \text{Cash-in in year 3} &= I_3 - \{A_3 + [B - (A_1 + A_2)]R - [B - (A_1 + A_2)]\} \\
 &\vdots \\
 \text{Cash-in in year } n &= I_n - \{A_n + [B - (A_1 + \dots + A_{n-1})]R - [B - (A_1 + \dots + A_{n-1})]\}
 \end{aligned}$$



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Now, we use mathematical induction to prove that the following mathematical expression is true:

$$\begin{aligned} & \frac{I_1 - (A_1 + BR - B)}{R} + \frac{I_2 - [A_2 + (B - A_1)R - (B - A_1)]}{R^2} + \frac{I_3 - \{A_3 + [B - (A_1 + A_2)]R - [B - (A_1 + A_2)]\}}{R^3} + \dots \\ & \dots + \frac{I_n - \{A_n + [B - (A_1 + A_2 + \dots + A_{n-1})]R - [B - (A_1 + A_2 + \dots + A_{n-1})]\}}{R^n} \\ & = \frac{(I_1 R^{n-1} + I_2 R^{n-2} + \dots + I_n) - [(A_1 + A_2 + \dots + A_n) + BR^n - B]}{R^n} \end{aligned}$$

It can be proved easily that the above mathematical expression is true when $n = 2$.

Next, we must also prove that if the mathematical expression is true when $n = k$, then it will also be true when $n = k + 1$. So, when $n = k + 1$,

$$\begin{aligned} \text{LHS} &= \frac{I_1 - (A_1 + BR - B)}{R} + \frac{I_2 - [A_2 + (B - A_1)R - (B - A_1)]}{R^2} + \dots \\ & \dots + \frac{I_k - \{A_k + [B - (A_1 + \dots + A_{k-1})]R - [B - (A_1 + \dots + A_{k-1})]\}}{R^k} \\ & \quad + \frac{I_{k+1} - \{A_{k+1} + [B - (A_1 + \dots + A_k)]R - [B - (A_1 + \dots + A_k)]\}}{R^{k+1}} \\ &= \frac{(I_1 R^{k-1} + I_2 R^{k-2} + \dots + I_k) - (A_1 + A_2 + \dots + A_k) + BR^k - B}{R^k} \\ & \quad + \frac{I_{k+1} - \{A_{k+1} + [B - (A_1 + \dots + A_k)]R - [B - (A_1 + \dots + A_k)]\}}{R^{k+1}} \\ & \hspace{15em} (\text{since it is assumed true for } n = k) \\ &= \frac{(I_1 R^k + I_2 R^{k-1} + \dots + I_{k+1}) - (A_1 + A_2 + \dots + A_{k+1}) + BR^{k+1} - B}{R^{k+1}} = \text{RHS} \end{aligned}$$

So, the case $k+1$ is true if the case k is assumed true. Therefore, the mathematical expression is true.

Now, let us go back to look at the evaluation of NPV of the cash flows shown in Tables 3. The calculation of the NPV can be generalised as follows:

$$\begin{aligned} \text{NPV} &= -(C - B) + \frac{I_1 - (A_1 + BR - B)}{R} + \frac{I_2 - [A_2 + (B - A_1)R - (B - A_1)]}{R^2} + \dots \\ & \dots + \frac{I_n - \{A_n + [B - (A_1 + A_2 + \dots + A_{n-1})]R - [B - (A_1 + A_2 + \dots + A_{n-1})]\}}{R^n} \end{aligned}$$

$$\begin{aligned}
&= -(C - B) + \frac{(I_1 R^{n-1} + I_2 R^{n-2} + \dots + I_n) - [(A_1 + A_2 + \dots + A_n) + BR^n - B]}{R^n} \\
&= -C + \frac{I_1}{(1+i)} + \frac{I_2}{(1+i)^2} + \dots + \frac{I_n}{(1+i)^n} \quad \text{which is the original definition of NPV.}
\end{aligned}$$

It can be seen that NPV is independent of B (i.e., the amount of loan) and is only dependent on the all-equity initial capital outlay C and the all-equity cash-in I_i . Hence, the NPV is a constant value.

Conclusion

It has been shown that the NPV of a capital investment is constant in all situations and is not affected by the variation of financial arrangements. NPV is an **economic indicator** for a capital investment, as the economic value of an investment does not change when its financial arrangement changes. The IRR, however, is suitable for use as a **financial indicator**, since private investors usually like to play around with financial arrangements to optimise their rates of return.

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Further information

The above describes the discovery of a fundamental difference of the functions between NPV and IRR. The first author proposed such a difference the first time in 1991 in his book published by McGraw-Hill (Tang 1991). He and H. John Tang then provided a mathematical proof for it in 2003, which was published by *The Engineering Economist* (Tang and Tang 2003). This discovery is believed to be a great contribution to the field of Economics, Finance & Management.

Acknowledgement

The author thanks his son H. John Tang for helping him solve a fast knot in the mathematical proof. John is a very intelligent young man and the author is proud to be his father.



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EXERCISE QUESTIONS FOR CHAPTER 5

Exercise Question 1

In what cash flow situations should inflation adjustment be made or not be made? Examples should be given in your answer.

Solution:

End of year	NCF	
0	NCF_0	(based on money value of Year 2012)
1	NCF_1	(based on money value of Year 2012)
2	NCF_2	(based on money value of Year 2012)
:	:	:
:	:	:
10	NCF_{10}	(based on money value of Year 2012)

Inflation adjustment is NOT necessary for the above “**constant base year prices**” case.

Apparent (nominal) rate of return is the rate calculated using current year prices, for example:

End of year	NCF	
0	NCF_0	(based on money value of Year 2002)
1	NCF_1	(based on money value of Year 2003)
2	NCF_2	(based on money value of Year 2004)
:	:	:
:	:	:
10	NCF_{10}	(based on money value of Year 2012)

Inflation adjustment is necessary for this second case.

Exercise Question 2

The following cash flows of a project are the *actual transactions* in the respective years as indicated. The average inflation rate for those years was 3% p.a. Calculate the real IRR.

End of year	Cash out	Cash in
0 (2008)	\$102,500	
1 (2009)		\$28,900
2 (2010)		\$31,000
3 (2011)		\$34,600
4 (2012)		\$38,500
5 (2013)		\$43,300

Solution:

Let i = real IRR, and i' = apparent (nominal) IRR

End of year	NCF of B
0 (2008)	-\$102,500
1 (2009)	\$28,900
2 (2010)	\$31,000
3 (2011)	\$34,600
4 (2012)	\$38,500
5 (2013)	\$43,300

i' is found to be 19.63% p.a.

$$i' = (1 + i)(1 + f) - 1$$

$$0.1963 = (1 + i)(1 + 0.03) - 1$$

Solving, $i = 16.15\%$ p.a.

Exercise Question 3

There is a proposed project of constructing a self-financed highway, so the toll paid by the vehicles should be sufficient to recover the initial capital cost and all other costs that will be incurred over the life of the project. Establish a suitable charge on each vehicle passing the highway with the use of the following numerical values which are estimated based on 2014 prices.



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1	Capital cost (construction cost and professional fees)	\$500,000,000
2	Annual operation and maintenance cost	\$15,000,000
3	Annual administration cost	\$20,000,000
4	Replacement cost of autopay ticket machine	\$4,000,000 every 5 years
5	Estimated annual traffic flow	10,000 vehicles per day in the first 5 years; the growth rate of traffic flow is 20% every 5 years
6	Life of project	25 years
7	Desirable rate of return (real)	8% p.a.
8	Estimated average inflation rate in the next 25 years	3% p.a. (averaged)

Solution:

Let us simplify the writing of $\left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$ to $(uspvf)^{n,i}$ to denote “uniform series present value factor” for a period of n and at a discount rate of i , and simplify the writing of $\left[\frac{1}{(1+i)^n} \right]$ to $(pvf)^{n,i}$ to denote “present value factor” for a period of n and at a discount rate of i .

1.	Initial costs	\$500,000,000
2.	PV of annual OMR costs $= 15,000,000 \times (uspvf)^{n=25,i=8\%} =$	160,125,000
3.	PV of annual administration costs $= 20,000,000 \times (uspvf)^{n=25,i=8\%} =$	213,500,000
4.	PV of replacement costs for ticket machine $= 4,000,000 \times (pvf)^{n=5,i=8\%} + 4,000,000 \times (pvf)^{n=10,i=8\%}$ $+ 4,000,000 \times (pvf)^{n=15,i=8\%} + 4,000,000 \times (pvf)^{n=20,i=8\%} =$	6,694,000
	Present value of total costs =	\$880,319,000

Number of vehicles passing the tunnel each year in the 1 st five years (i.e. year 1 to year 5)	$= 10,000 \times 365$ $= 3,650,000$
Number of vehicles passing the tunnel each year in the 2 nd five years (i.e. year 6 to year 10)	$= (10,000 \times 1.2) \times 365$ $= 4,380,000$
Number of vehicles passing the tunnel each year in the 3 rd five years (i.e. year 11 to year 15)	$= (12,000 \times 1.2) \times 365$ $= 5,256,000$
Number of vehicles passing the tunnel each year in the 4 th five years (i.e. year 16 to year 20)	$= (14,400 \times 1.2) \times 365$ $= 6,307,200$
Number of vehicles passing the tunnel each year in the 5 th five years (i.e. year 21 to year 25)	$= (17,280 \times 1.2) \times 365$ $= 7,568,640$

If y = toll paid by each vehicle, then,

PV of income in the 1 st five years $= 3,650,000y \times (uspvf)^{n=5, i=8\%} =$	\$14,573.355y
PV of income in the 2 nd five years $= 4,380,000y \times (uspvf)^{n=5, i=8\%} \times (pvf)^{n=5, i=8\%} =$	\$11,902,350y
PV of income in the 3 rd five years $= 5,256,000y \times (uspvf)^{n=5, i=8\%} \times (pvf)^{n=10, i=8\%} =$	\$9,720,544y
PV of income in the 4 th five years $= 6,307,200y \times (uspvf)^{n=5, i=8\%} \times (pvf)^{n=15, i=8\%} =$	\$7,937,605y
PV of income in the 5 th five years $= 7,568,640y \times (uspvf)^{n=5, i=8\%} \times (pvf)^{n=20, i=8\%} =$	\$6,482,042y
Total =	\$50,615,897y

$$\text{Set } 880,319,000 = 50,615,897y$$

$$\text{then, } y = \frac{880,319,000}{50,615,897} = \$17.39$$

The toll paid by each vehicle initially is \$17.39 in order to have a rate of return of 8% p.a. A toll of, say, \$18.00 is reasonable if one wishes to make a slightly higher rate of return than 8% p.a.

Note: The effect of inflation (3% p.a. on average) has not been taken account of in the above calculation. It is in fact not necessary to do so because constant base year prices are used (based on money value of year 2014). It is easy to see that they are constant base year prices. If we look at an example, say, the annual operation and maintenance cost, it is \$15,000,000 every year. How can it be \$15,000,000 without change for 25 years? So it must be a constant base year price – based on the money value of year 2014 alone. In practice, the cost of \$15,000,000 will increase at a rate of 3% p.a. (on compound basis) in the next 25 years. Inflation indeed may influence both revenues and costs to differing degrees, but such impacts are only of short duration. Usually, its effect on both revenues and costs will cancel out each other in the long run to regain a balance.

Therefore, if $f = 3\%$ p.a., simply also increase the toll 3% p.a. on average in principle (on compound basis, with skillful price-increase technique of course) to adjust the effect of inflation.

Mathematically:

$$\begin{aligned}\text{Toll after } n \text{ years} &= \$17.39 \times (1 + 3\%)^n \\ \text{or} &= \$18.00 \times (1 + 3\%)^n \text{ if a higher return than } 8\% \text{ p.a. is desired} \\ &\text{where } n = 1, 2, \dots 25.\end{aligned}$$

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EXERCISE QUESTIONS FOR CHAPTER 6

Exercise Question 1

A contractor is considering whether he could drop an item and replace it with another from his precast concrete yard. These are his current data:

Precast concrete	p	v	% of total sales
Roof panel	\$600	\$400	30%
Window panel	\$1,000	\$600	20%
Seawall panel	\$2,000	\$1,200	50%

Total fixed cost per year is \$7,500,000 and total sales this year is \$25,000,000.

The change being considered is to drop the window panel production and replace it with wall panel production with following percentages of total sales:

Precast concrete	p	v	% of total sales
Roof panel	\$600	\$400	50%
Wall panel	\$1,600	\$700	10%
Seawall panel	\$2,000	\$1,200	40%

Total fixed cost per year is \$7,500,000 and the estimated total sales next year is \$26,000,000. Is this change viable?

Solution:

Profit on the present product line is calculated as follows:

$$\begin{array}{rcl}
 \frac{\$600 - \$400}{\$600} \times 30\% & = & 0.10 \\
 \frac{1000 - 600}{1000} \times 20\% & = & 0.08 \\
 \frac{2000 - 1200}{2000} \times 50\% & = & 0.20 \\
 \hline
 \Sigma = 0.38 & & \text{(contribution = 38\%)}
 \end{array}$$

$$\text{Contribution in \$} = \$25,000,000 \times 0.38 = \$9,500,000$$

$$\text{Profit} = \$9,500,000 - \$7,500,000 = \$2,000,000$$

Profit on the proposed product line:

$$\begin{array}{rcl}
 \frac{\$600 - \$400}{\$600} \times 50\% & = & 0.17 \\
 \frac{1600 - 700}{1600} \times 10\% & = & 0.06 \\
 \frac{2000 - 1200}{2000} \times 40\% & = & 0.16 \\
 \hline
 \Sigma = 0.39 & & \text{(contribution = 39\%)}
 \end{array}$$

$$\text{Contribution in \$} = \$26,000,000 \times 0.39 = \$10,140,000$$

$$\text{Profit} = \$10,140,000 - \$7,500,000 = \$2,640,000 > \$2,000,000$$

The proposed change appears to be attractive.

Exercise Question 2

A flood control pumping station is being designed. Three schemes are proposed and the itemized costs of each scheme are shown below:

Scheme number		Scheme A	Scheme B	Scheme C
Pump	Cost of pumps (\$)	120,000	180,000	280,000
	Life (years)	15	15	20
	Maintenance (\$/year)	10,000	15,000	15,000
Pipe	Cost of pipes (\$)	220,000	180,000	120,000
	Life (years)	30	30	30
	Cost of pumping (\$/hour)	12.00	9.00	8.00

What is the most economical range of pumping time in hours/year for each scheme?
(Take $i = 18\%$ p.a. and maximum pumping hours in a year = $24 \times 365 = 8,760$ hours)

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Solution:

Scheme A

Equivalent annual cost of installation and maintenance

$$= \$120,000 \times \left[\frac{0.018(1+0.18)^{15}}{(1+0.18)^{15}-1} \right] + \$220,000 \times \left[\frac{0.18(1+0.18)^{30}}{(1+0.18)^{30}-1} \right] + \$10,000$$

$$= \$120,000 \times 0.1964 + \$220,000 \times 0.1813 + \$10,000 = \$73,454$$

Scheme B

Equivalent annual cost of installation and maintenance

$$= \$180,000 \times \left[\frac{0.018(1+0.18)^{15}}{(1+0.18)^{15}-1} \right] + \$180,000 \times \left[\frac{0.18(1+0.18)^{30}}{(1+0.18)^{30}-1} \right] + \$15,000$$

$$= \$180,000 \times 0.1964 + \$180,000 \times 0.1813 + \$15,000 = \$82,986$$

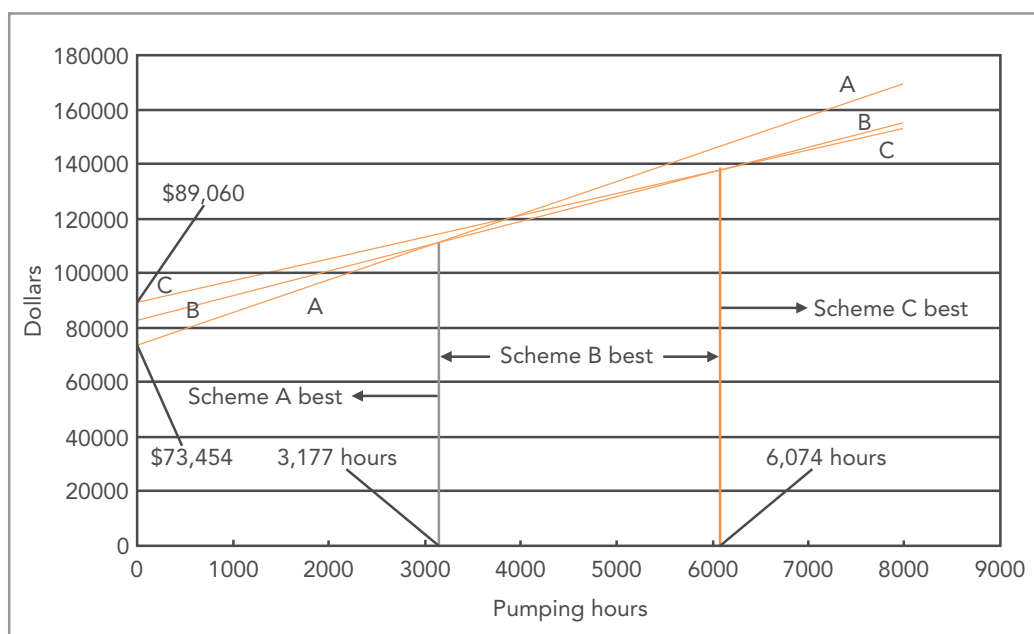
Scheme C:

Equivalent annual cost of installation and maintenance

$$= \$280,000 \times \left[\frac{0.18(1+0.18)^{20}}{(1+0.18)^{20}-1} \right] + \$120,000 \times \left[\frac{0.18(1+0.18)^{30}}{(1+0.18)^{30}-1} \right] + \$15,000$$

$$= \$280,000 \times 0.1868 + \$120,000 \times 0.1813 + \$15,000 = \$89,060$$

The break-even chart is:



EXERCISE QUESTIONS FOR CHAPTER 7

Exercise Question 1

A construction company is considering setting up a new precast concrete yard. To do so, a new equipment, which has a life of four years and costs \$1,000,000 has to be purchased. The revenue generated is estimated to be \$500,000 per year. The total expenses associated with this precast concrete yard are estimated to be \$120,000 per year. The profit tax rate is 25%.

- a) Assuming straight line depreciation and ignoring salvage values, carry out a financial analysis for the company. (You are expected to create an income statement and derive a cash flow table for IRR calculation, and then to compute the IRR).
- b) Carry out a new financial analysis and find the new IRR if the total capital of \$1,000,000 is made up of \$250,000 equity and \$750,000 loan. The loan has to be paid back in three years (i.e. principal amortization of \$250,000 per year) at an interest rate of 8% p.a.
- c) Compare the internal rates of return in (a) and (b) above and give comments on them. If the NPV method at a discount rate of 8% p.a. (the borrowing interest rate) is used in (a) and (b), what results will be expected? And why?

Solution:

a) Income Statement of "All-equity case":

	Year 1	Year 2	Year 3	Year 4
Revenue	500,000	500,000	500,000	500,000
Cost				
Depreciation	250,000	250,000	250,000	250,000
Other expenses	120,000	120,000	120,000	120,000
Total	370,000	370,000	370,000	370,000
Profit before tax	130,000	130,000	130,000	130,000
Tax (25%)	32,500	32,500	32,500	32,500
Profit after tax	97,500	97,500	97,500	97,500
Add depreciation	250,000	250,000	250,000	250,000
Cash flow	347,500	347,500	347,500	347,500

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The cash flow table for calculating IRR is:

End of year	Cash out	Cash in
0	1,000,000	
1		347,500
2		347,500
3		347,500
4		347,500

IRR = 14.61% p.a.

b) “Loan + Equity” case:

	Year 1	Year 2	Year 3	Year 4
Revenue	500,000	500,000	500,000	500,000
Cost				
Depreciation	250,000	250,000	250,000	250,000
Other expenses	120,000	120,000	120,000	120,000
Total	370,000	370,000	370,000	370,000
Gross profit	130,000	130,000	130,000	130,000
Interest payment	60,000	40,000	20,000	0
Profit before tax	70,000	90,000	110,000	130,000
Tax (25%)	17,500	22,500	27,500	32,500
Profit after tax	52,500	67,500	82,500	97,500
Add depreciation	250,000	250,000	250,000	250,000
Cash flow before amortization	302,500	317,500	332,500	347,500
Amortization	250,000	250,000	250,000	0
Cash flow after amortization	52,500	67,500	82,500	347,500

The following table shows the calculation of “Interest payment” row for the above table:

End of Year	Principal borrowed	Principal amortization	Balance unamortized	Interest (8%)
0	750,000		750,000	
1		250,000	500,000	60,000
2		250,000	250,000	40,000
3		250,000	0	20,000

The cash flow table for calculating IRR:

End of year	Cash out	Cash in
0	250,000	
1		52,500
2		67,500
3		82,500
4		347,500

IRR = 28.17% p,a,

$$\begin{aligned} \text{NPV at 8\%} &= -1,000,000 + \frac{347,500}{1.08} + \frac{347,500}{1.08^2} + \frac{347,500}{1.08^3} + \frac{347,500}{1.08^4} \\ &= 150,964 \end{aligned}$$

$$\begin{aligned} \text{NPV at 8\%} &= -250,000 + \frac{52,500}{1.08} + \frac{67,500}{1.08^2} + \frac{82,500}{1.08^3} + \frac{347,500}{1.08^4} \\ &= 177,396 \end{aligned}$$

Apparently, The two NPVs differ to a certain extent. It is true that the two NPV values appear not to be close enough to each other, but in theory they should have values very similar to each other (see Exercise Question 2 of Chapter 4). However, the tax deduction has contributed to the disturbance of mathematics and has resulted in a difference between the two NPVs. The bigger the loan is, the bigger the interest payments are, and that will result in a bigger disturbance of the mathematics and hence less stable NPVs. Readers may verify it by using other loan values such as \$450,000 and \$600,000.

EXERCISE QUESTIONS FOR CHAPTER 8

Exercise Question 1

The following table lists the cumulative monthly costs incurred by a contractor and the corresponding monthly payments which are received from the owner of a project.



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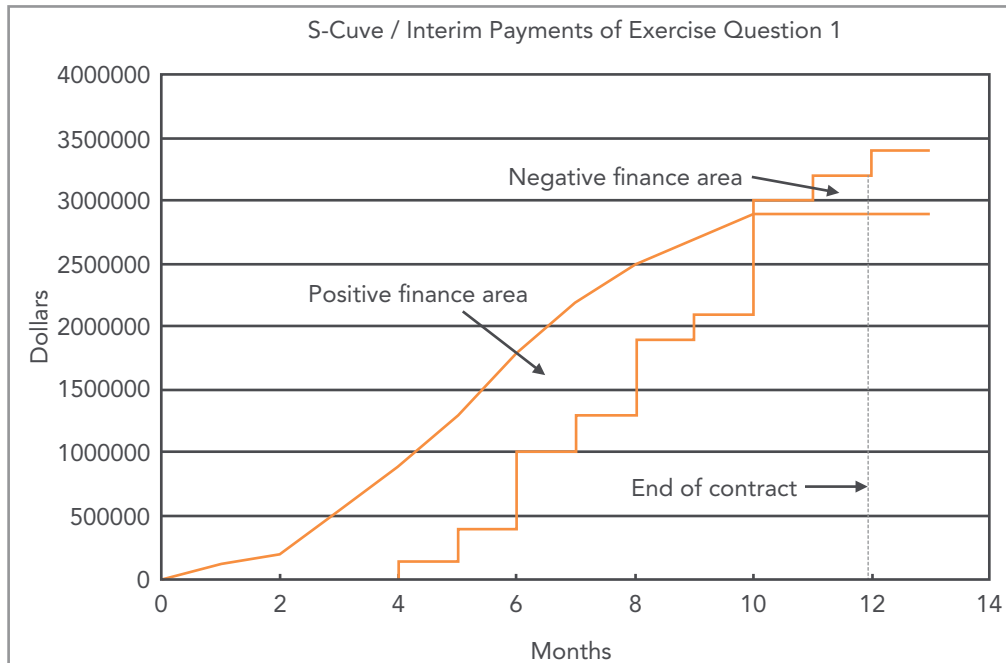
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End of month	Cumulative expenditure	Cumulative revenue
0	0	0
1	120,000	0
2	200,000	0
3	540,000	0
4	900,000	140,000
5	1,300,000	400,000
6	1,800,000	1,000,000
7	2,200,000	1,300,000
8	2,500,000	1,900,000
9	2,700,000	2,100,000
10	2,900,000	3,000,000
11	2,900,000	3,200,000
12	2,900,000	3,400,000

Calculate the cost to the contractor of borrowing the working capital necessary to finance the project if the annual rate of interest he has to pay is 12%.

Solution:

From the data given, the S-Curve and the cumulative interim payment graph can be drawn as follows:



The positive finance area, from Month 0 to Month 10, is equal to 6,537,000 \$-month. The negative finance area, from Month 10 to Month 12, is equal to 400,000 \$-month. The total (or net) finance area is therefore 6,137,000 \$-month.

Since the interest rate is 12% per annum, or 1% per month, the interest charge therefore is \$61,370. This amount is the cost to the contractor of borrowing the working capital necessary to finance the project. By the way, we can see from the graph that the maximum working capital required by the project occurs at the end of Month 6 and the amount is \$1,400,000 (i.e. \$1,800,000 – \$400,000).

Exercise Question 2

The expenditure per month for a small-medium scale building project scheduled for 5 months are given below. The mark-up is 5%. The retention money of the billed cost is 10% if the cumulative payment is less than half of the total value of work (contract value) and then 0% thereafter. The retention money will be released at the end of month 6, one month after the completion of the contract. Payment is billed at the end of each month and received one month later by the contractor. The finance charge is 1% per month.

End of month	1	2	3	4	5
Expenditure (\$)	1,000,000	2,100,000	2,500,000	2,500,000	2,500,000

By plotting the S-Curve and the cumulative interim payment graph, find the peak working capital requirement and the total financial charge on the working capital if it is totally borrowed.



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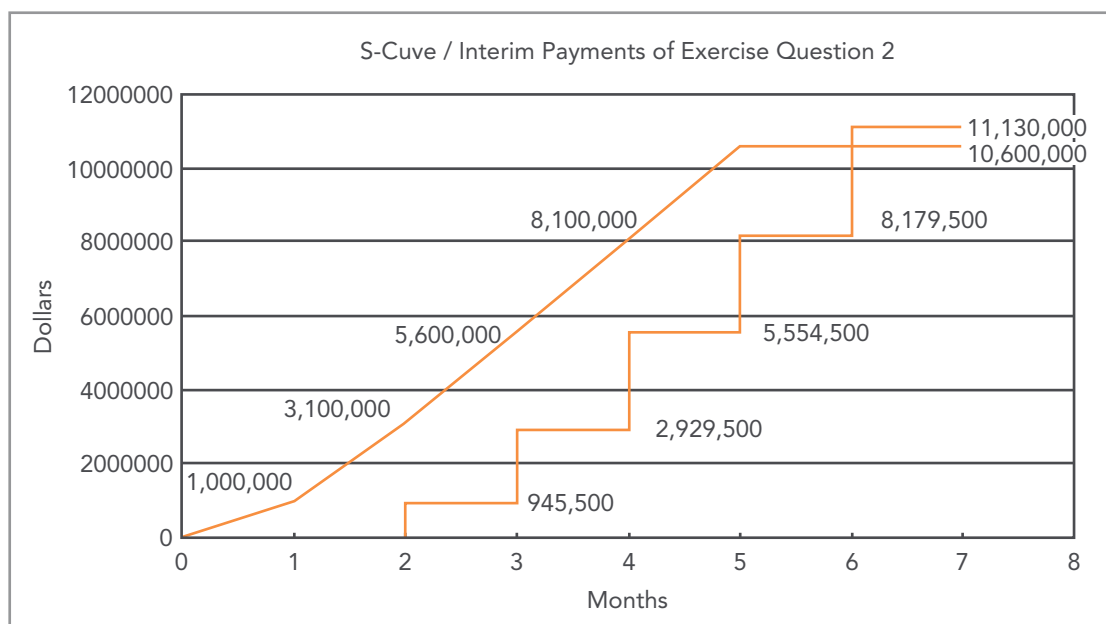
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Solution:

End of month	1	2	3	4	5	6
Expenditure (\$)	1,000,000	2,100,000	2,500,000	2,500,000	2,500,000	
Cumulative expenditure (\$) (i.e. S-Curve)	1,000,000	3,100,000	5,600,000	8,100,000	10,600,000	10,600,000
Mark-up (5%)	50,000	105,000	125,000	125,000	125,000	
Value of work billed (\$)	1,050,000	2,205,000	2,625,000	2,625,000	2,625,000	
Retention money (10%)	105,000	220,500	0	0	0	
Value of work less retention (\$)	945,000	1,984,500	2,625,000	2,625,000	2,625,000	
Interim payment received (\$)	0	945,000	1,984,500	2,625,000	2,625,000	2,950,500 (i.e. 2,625,000 + retentions)
Cumulative interim payment (\$)	0	945,000	2,929,500	5,554,500	8,179,500	11,130,000

From the above table, the S-Curve and the cumulative interim payment graph can be drawn as follows:



The peak capital working capital requirement is \$5,170,500 at the end of Month 4. This figure is obtained by subtracting \$2,929,500 from \$8,100,000.

The contract period is 6 months, from commencement up to the release of retention money. There is only positive finance area in that period, and it is found to be approximately 16.0 million\$-month. Since the finance charge is 1% per month, the total financial charge on the working capital is therefore approximately \$0.16 million (i.e. $16.0 \times 1\%$), or \$160,000.

Exercise Question 3

The following table shows the value of work a contractor completes and the profit distribution for a 10-month construction contract.

End of month	1	2	3	4	5	6	7	8	9	10
Value of work (million \$)	2	3	4	7	7	7	6	5	4	1
Profit (% of value of work)	15	15	10	10	10	10	10	10	5	5

Interim measurements are made monthly and payments of the amount certified less 10% retention are paid the contractor one month later. 50% of the retention money is released one month after substantial completion (i.e. end of month 11, since completion is at end of month 10), and the other 50% retention is released 6 months later (the defect liability period is 6 months).

However, the client desires to decrease his administration duties and proposes to the contractor that interim payments be made bi-monthly. The contractor wishes to know the financial implications of this proposal. In other words, he wants to know how much more he has to pay for the cost of the working capital.

Assuming the contractor can delay paying his expenditures by an average of one month due to the availability of trade credits, calculate the extra financial charge for borrowing the working capital if interest rate is at 9% p.a. and interim payments are made bi-monthly.

Solution:

The calculation of the cumulative expenditure (S-Curve) and the cumulative interim payment graph are shown in the table below. (All in million dollars)

End of month	1	2	3	4	5	6	7	8	9	10	11	17
Value of work	2	3	4	7	7	7	6	5	4	1		
Cumulative value	2	5	9	16	23	30	36	41	45	46		
Cumulative value – retention money	1.8	4.5	8.1	14.4	20.7	27.0	32.4	36.9	40.5	41.4		
Cumulative interim payment (monthly basis)	0	1.8	4.5	8.1	14.4	20.7	27.0	32.4	36.9	40.5	$41.4 + 2.3^* = 43.7$	$43.7 + 2.3^* = 46.0$
Cumulative interim payment (bi-monthly basis)			4.5		14.4		27.0		36.9		43.7	46.0
% of profit	15%	15%	10%	10%	10%	10%	10%	10%	5%	5%		
Expenditure (monthly basis)	1.7	2.55	3.6	6.3	6.3	6.3	5.4	4.5	3.8	0.95		
Cumulative expenditure	1.7	4.25	7.85	14.15	20.45	26.75	32.15	36.65	40.45	41.4		
S-Curve		1.7	4.25	7.85	14.15	20.45	26.75	32.15	36.65	40.45	41.4	

* 2.3 is half of the total retention money, equal to 5% of cumulative value of work ($46 \times 5\% = 2.3$)



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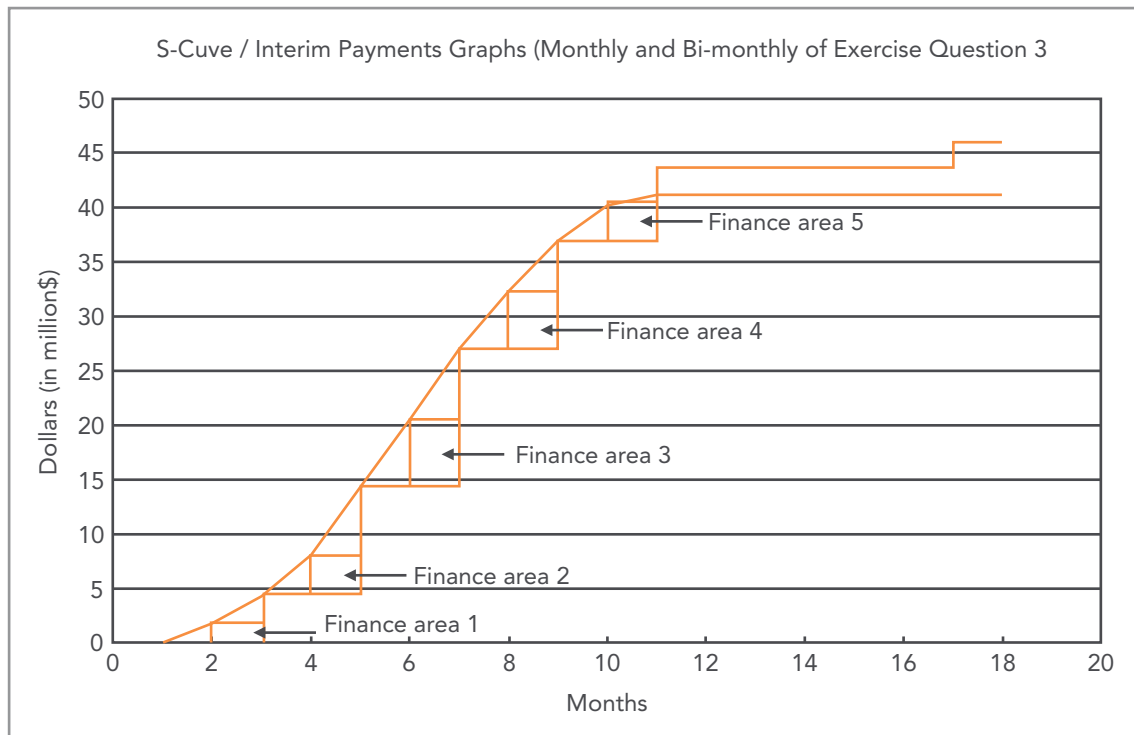
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The S-Curve and the two cumulative interim payment graphs (monthly and bi-monthly) can be drawn as below:



Finance areas 1, 2, 3, 4 and 5 represent extra financing required if interim payments are paid to the contractor bi-monthly by the client.

Total area of finance areas 1, 2, 3, 4 and 5

$$= 1.7 \times 1 + (8.1 - 4.5) \times 1 + (20.7 - 14.4) \times 1 + (32.4 - 27.0) \times 1 + (40.5 - 36.9) \times 1$$

$$= 20.6 \text{ million\$-month}$$

The interest charge on the extra finance needed to fund bi-monthly interim payments

$$= 20.6 \times 0.09/12$$

$$= 0.1545 \text{ million dollars}$$

$$= \$154,500$$

